Lab 4

P 1. Use the Acceptance–Rejection method to generate samples from Gamma(4, 5) using an appropriate proposal distribution.

1. Calculate the sample mean and variance and compare it with population mean and variance.
2. Estimate the value of c and compare it with theoretical value.

Theory:

To minimize the value of c (i.e.computations), we choose c such that $c = max(\frac{\beta^{\alpha}x^{\alpha-1}e^{-x(\beta-\lambda)}}{\lambda\Gamma(\alpha)})$. On solving it we get that c_{max} occurs at $x = \frac{\alpha-1}{\beta-\lambda}$.

On solving it we get $\lambda = \frac{\beta}{\alpha}$ & $c = \frac{\alpha^{\alpha}e^{1-\alpha}}{\Gamma(\alpha)}$.

```
# Set parameters for the Gamma distribution
shape <- 4 ##alpha
rate <- 5  ##beta

# Define the target Gamma PDF
gamma_pdf <- function(x) {
  if (x >= 0) {
    return(dgamma(x, shape = shape, rate = rate))
  } else {
    return(0)
  }
}

# Define the proposal Exponential PDF
lambda <- rate/shape
exp_pdf <- function(x) {
  if (x >= 0) {
    return(dexp(x, rate = lambda))
  } else {
    return(0)
  }
}

# Calculate the constant c
alpha <- shape
beta <- rate
c <- (alpha^alpha)*(exp(1-alpha))/gamma(alpha)

# Acceptance-Rejection Method
set.seed(123)
n_samples <- 1000
```

```r
samples <- numeric(0)
count <- 0
counter <- numeric(0)
while (length(samples) < n_samples) {
  count <- count + 1
  v <- runif(1)
  y <- -log(v)/lambda  #Generate from the proposal distribution
  u <- runif(1)  #Generate from Uniform(0,1).
  if (u <= gamma_pdf(y) / (c * exp_pdf(y))) {
    samples <- c(samples, y)
    counter <- c(counter,count)
    count <- 0
  }
}

# Sample mean and variance
sample_mean <- mean(samples)
sample_variance <- var(samples)

# Compare with theoretical mean and variance
theoretical_mean <- shape / rate
theoretical_variance <- shape / rate^2


cat("Sample Mean:", sample_mean, "\n")
```

```
Sample Mean: 0.7923851
```

```r
cat("Sample Variance:", sample_variance, "\n")
```

```
Sample Variance: 0.1619118
```

```r
cat("Theoretical Mean:", theoretical_mean, "\n")
```

```
Theoretical Mean: 0.8
```

```r
cat("Theoretical Variance:", theoretical_variance, "\n")
```

```
Theoretical Variance: 0.16
```

```r
cat("Estimated c:", mean(counter), "\n")
```

```
Estimated c: 2.105
```

```r
cat("Theoretical c:", c, "\n")
```

```
Theoretical c: 2.124248
```

P 2. Generate 10^4 uniform samples from a unit circle. Observe the scatter plot of the samples.

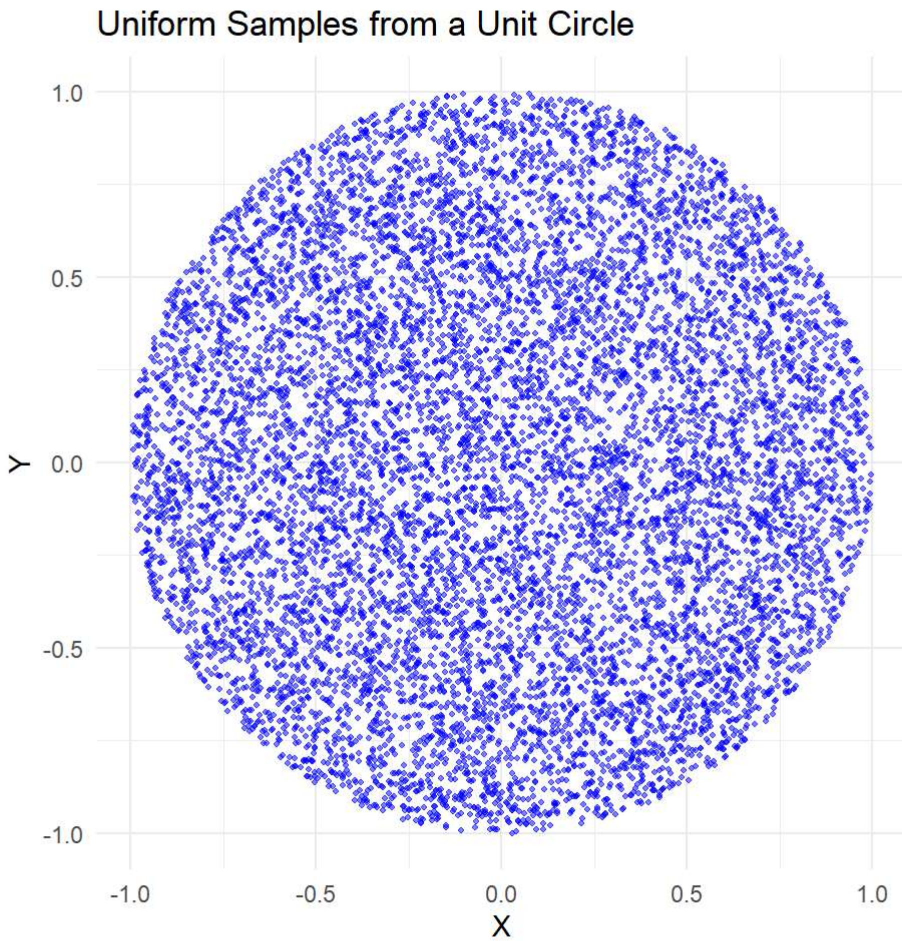
```r
# Set parameters
n_points <- 10000

# Generate uniform samples in a unit circle
set.seed(123)

x<- numeric(0)
y <- numeric(0)

for(i in 1:n_points){
  while (TRUE) {
    a<- runif(1,-1,1)
    b<- runif(1,-1,1)
    if(a^2 + b^2 <=1){
      x <- c(x,a)
      y <- c(y,b)
      break
    }
  }
}


library(ggplot2)
circle_data <- data.frame(x = x, y = y)
ggplot(circle_data, aes(x = x, y = y)) +
  geom_point(color = "blue", alpha = 0.5, size = 0.7) +
  coord_fixed() +
  labs(title = "Uniform Samples from a Unit Circle",
       x = "X",
       y = "Y") +
  theme_minimal()
```


Uniform Samples from a Unit Circle
1.0
0.5
0.0
-0.5
-1.0
Y
-1.0
-0.5
0.0
0.5
1.0
X