

LAB 5

Problem 1

draw samples from $N(0, 1)$ using Ratio of Uniforms method. Derive the appropriate set D . Using this set D generate samples from $N(0, 1)$.

```
Normal_generator <- function(a,b,c,n){
  nums <- numeric(n)
  for(i in 1:n){
    while(TRUE){
      u1 <- runif(1)
      u2 <- runif(1)

      u<-a*u1
      v<- b + (c-b)*u2
      x <- 1/(2*pi)^(1/4) * exp(-(v/u)^2)/4)
      if(u <= x){
        nums[i]<- v/u
        break
      }
    }
  }
  return (nums)
}
```

```
#For  $N(0,1)$ 

a <- 1/(2*pi)^(1/4)

b <- -(2/(pi * exp(1)^2))^(1/4)

c <- (2/(pi * exp(1)^2))^(1/4)

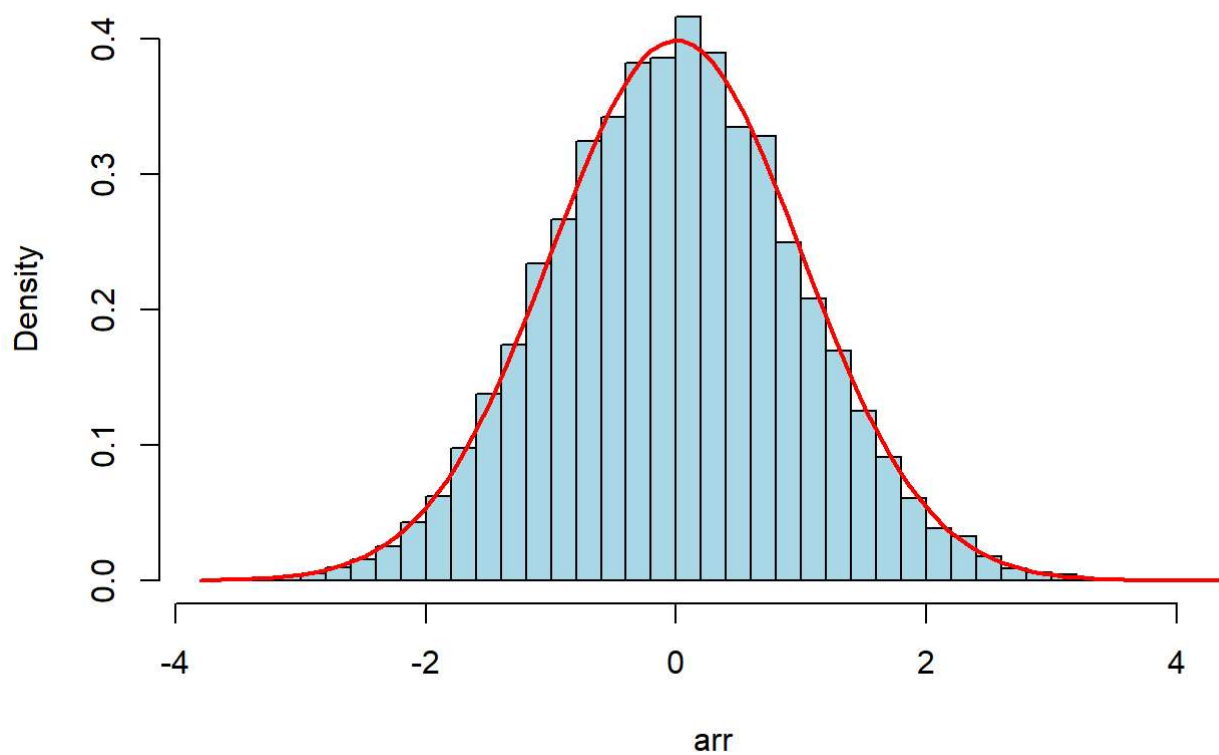
arr <- Normal_generator(a,b,c,10000)

mean(arr)
```

```
[1] -0.01233158
```

```
hist(arr, breaks = 50, probability = TRUE, main = "Histogram of  $N(0,1)$  Samples", col = "lightblue",
curve(dnorm(x), add = TRUE, col = "red", lwd = 2)
```

Histogram of $N(0,1)$ Samples



Problem 2

Using simple Monte Carlo sampling,

A.)

```
n <- 1e5  
  
x <- runif(1e5,min=0.0,max=1.0)  
  
h <- exp(x)  
  
practical_exp <- mean(h)  
  
theoretical_exp <- exp(1) - 1  
cat("Monte Carl estimate:",practical_exp,"\n")
```

Monte Carl estimate: 1.719211

```
cat("Theoretical expectation:",theoretical_exp,"\n")
```

Theoretical expectation: 1.718282

B.)

```
mc_integral2 <- function(n) {  
  x <- runif(n, 0, pi)  
  fx <- sqrt(x^3 + sqrt(x)) - x^2 * sin(4*x)  
  mean(fx) * pi # Multiply by interval length ( $\pi$ )  
}  
ans <- mc_integral2(10000)  
cat("Monte Carlo Integral Result:", ans, "\n")
```

Monte Carlo Integral Result: 10.45133

```
f <- function(x) sqrt(x^3 + sqrt(x)) - x^2 * sin(4*x)  
num_result <- integrate(f, lower = 0, upper = pi)  
cat("In-built Integral function Result:", "\n")
```

In-built Integral function Result:

```
num_result
```

10.51721 with absolute error < 4.9e-11

```
analytical_integral <- function() {  
  term1 <- (pi^2) / 4  
  term2 <- (2/5) * ((pi^(5/4)) * sqrt(1 + pi^(5/2)) + asinh(pi^(5/4)))  
  return(term1 + term2)  
}  
  
I_analytical <- analytical_integral()  
cat("Analytical Integral Result:", I_analytical, "\n")
```

Analytical Integral Result: 10.51721