Lab 6

Problem 1

Propose an optimal importance sampling estimator within the class of N(0, σ^2) importance density for estimating the exception of standard normal random distribution. Using this importance estimator, estimate the expectation based on 10^4 samples.

```r
set.seed(123)

n <- 10^4  # Number of samples

# Optimal sigma for importance sampling
sigma_opt <- sqrt(2)

# Generate uniform random variables
U1 <- runif(n/2)
U2 <- runif(n/2)

# Apply Box-Muller Transform
Z1 <- sqrt(-2 * log(U1)) * cos(2 * pi * U2)
Z2 <- sqrt(-2 * log(U1)) * sin(2 * pi * U2)

#  N(0,1)
Z <- c(Z1, Z2)

# Scale to get N(0, sigma^2)
X <- sigma_opt * Z

# Standard normal PDF
f_x <- function(x) dnorm(x, mean=0, sd=1)

# Importance density PDF
g_x <- function(x) dnorm(x, mean=0, sd=sigma_opt)


h_x <- f_x(X) / g_x(X)

# Importance sampling estimator for E[X]
IS_estimate <- mean(X * h_x)
cat("Importance Sampling Estimate for E[X]:", IS_estimate, "\n")
```

```
Importance Sampling Estimate for E[X]: -0.008226473
```

Problem 2

Estimate the variance of Gamma(2,4) using importance sampling estimator using optimal importance density (10^4 samples). Estimate the variance of the importance sampling estimator also.

```
shape <- 2
rate <- 4

# Gamma PDF
f_gamma <- function(x) dgamma(x, shape=shape, rate=rate)

# Optimal importance density: Gamma(shape+2, rate)
shape_opt <- shape + 2
rate_opt <- rate

# Generate Gamma(shape_opt, rate_opt) using sum of exponentials
U_gamma <- matrix(runif(n * shape_opt), nrow=n, ncol=shape_opt)  # Generate k U(0,1)
X_gamma <- -rowSums(log(U_gamma)) / rate_opt  # Sum of exponentials

# Importance density PDF
g_gamma <- function(x) dgamma(x, shape=shape_opt, rate=rate_opt)


h <- f_gamma(X_gamma) / g_gamma(X_gamma)

# Estimate E[X^2]
E_X2_IS <- mean((X_gamma^2) * h)
E_X_IS <- mean(X_gamma * h)  # Estimate E[X]

# Variance estimate
Var_X_IS <- E_X2_IS - E_X_IS^2
cat("Importance Sampling Estimate for Variance of Gamma(2,4):", Var_X_IS, "\n")
```

```
Importance Sampling Estimate for Variance of Gamma(2,4): 0.1271765
```

```
# Variance of the importance sampling estimator
Var_IS_estimator <- var((X_gamma^2) * h) / n
cat("Variance of the Importance Sampling Estimator:", Var_IS_estimator, "\n")
```

```
Variance of the Importance Sampling Estimator: 6.264367e-37
```