LAB 11

Data

```
set.seed(123)
n<- 100
mu_real <- 10
sigma_real <- 2
x<-rnorm(n,mu_real,sigma_real)
```

Closed-form

```
## Closed-form

mu_cl <- mean(x)
sigma_cl <- sqrt(mean((x-mu_cl)^2))
mu_cl
```

```
[1] 10.18081
```

```
sigma_cl
```

```
[1] 1.816481
```

Newton_Raphson Method

```
## Newton_Raphson Method
temp <- 1e3
mu_nr <- 0
sigma_nr<- 1
tol<-1e-8
for(i in 1:temp){
  mu<-mu_nr
  sigma <- sigma_nr
  mu_nr <- mu_nr + mean(x-mu_nr)

  A<- sum((x-mu)^2)
  sigma_nr <- sigma_nr -(-n/(2*sigma_nr) + A/(2*(sigma_nr^2)))/(n/(2*(sigma_nr^2)) - A/(sigm
  if (abs(mu_nr - mu) < tol && abs(sigma_nr - sigma) < tol) break
}
mu_nr
```

```
[1] 10.18081
```

```
sqrt(sigma_nr)
```

```
[1] 1.816481
```

Alternatively,using hessian matrix

```
theta <- c(mu = 0, sigma2 = 1)
tol <- 1e-8
max_iter <- 100

for (i in 1:max_iter) {
  mu <- theta[1]
  sigma2 <- theta[2]

  # Gradient
  grad_mu <- sum(x - mu) / sigma2
  grad_sigma2 <- -n / (2 * sigma2) + sum((x - mu)^2) / (2 * sigma2^2)
  grad <- c(grad_mu, grad_sigma2)

  # Hessian
  H11 <- -n / sigma2
  H12 <- -sum(x - mu) / sigma2^2
  H21 <- H12
  H22 <- n / (2 * sigma2^2) - sum((x - mu)^2) * 2 / (sigma2^3)
  H <- matrix(c(H11, H12, H21, H22), nrow = 2, byrow = TRUE)

  theta_new <- theta - solve(H) %*% grad


  if (max(abs(theta_new - theta)) < tol) break
  theta <- theta_new
}

theta
```

```
          [,1]
[1,] 10.180812
[2,]  3.299602
```

```
sqrt(theta[2])  # to get sigma
```

```
[1] 1.816481
```