

$\boxed{N=16}$ — sample size

$x_0 x_1 \dots x_{15} \rightarrow f_0 f_1 \dots f_{15}$ ~ what we want
sample spectrum

$$\begin{aligned} \text{DFT: } f_k &= \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} nk} = \sum_{m=0}^{\frac{N}{2}-1} x_{2m} e^{-\frac{2\pi i}{N} 2mk} + \sum_{m=0}^{\frac{N}{2}-1} x_{2m+1} e^{-\frac{2\pi i}{N} (2m+1)k} \\ &= \underbrace{\sum_{m=0}^{\frac{N}{2}-1} x_{2m} e^{-\frac{2\pi i}{N/2} mk}}_{E_k} + e^{-\frac{2\pi i}{N} k} \underbrace{\sum_{m=0}^{\frac{N}{2}-1} x_{2m+1} e^{-\frac{2\pi i}{N/2} mk}}_{O_k} \end{aligned}$$

$$f_k = E_k + e^{-\frac{2\pi i}{N} k} O_k, \quad k = 0, \frac{N}{2}-1$$

$$f_{k+\frac{N}{2}} = E_k - e^{-\frac{2\pi i}{N} k} O_k, \text{ apparently}$$

Here E_k and O_k are DFTs of even and odd indexed sample values respectively — they're completely legit and independent DFTs that can be calculated in the same manner:

$$E_k = E_{E_k} + O_{E_k} \cdot e^{-\dots}$$

$$O_k = E_{O_k} + O_{O_k} \cdot e^{-\dots}$$

$$, \quad k = 0, \frac{N}{4}-1$$

note that range for k is decremented on each iteration

Given $N=16$, which sample values contribute to calculation of different even & odd DFTs?

$E: 0, 2, 4, 6, 8, 10, 12, 14$

$O: 1, 3, 5, 7, 9, 11, 13, 15$

$E_E: 0, 4, 8, 12$

$O_E: 2, 6, 10, 14$

$E_O: 1, 5, 9, 13$

$O_O: 3, 7, 11, 15$

$E_{EE}: 0, 8$

$O_{EE}: 4, 12$

$E_{OE}: 2, 10$

$O_{OE}: 6, 14$

$E_{EO}: 1, 9$

$O_{EO}: 5, 13$

$E_{OO}: 3, 11$

$O_{OO}: 7, 15$

O_{EO} , for example, should be read as

"DFT of odd indices of even indices of odd indices"

$x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10} x_{11} x_{12} x_{13} x_{14} x_{15}$

$$O_{EE_0} = x_4 + e^{\frac{2\pi i}{2} \cdot 0} x_{12}$$

$$O_{EE_1} = x_4 - e^{\frac{2\pi i}{2} \cdot 0} x_{12}$$

depth=1

$E_{EE_0} E_{EE_1} E_{EO_0} E_{EO_1} E_{OE_0} E_{OE_1} E_{OO_0} E_{OO_1} O_{EE_0} O_{EE_1} O_{EO_0} O_{EO_1} O_{OE_0} O_{OE_1} O_{OO_0} O_{OO_1}$

depth=2

$E_{E_0} E_{E_1} E_{E_2} E_{E_3} E_{O_0} E_{O_1} E_{O_2} E_{O_3} O_{E_0} O_{E_1} O_{E_2} O_{E_3} O_{O_0} O_{O_1} O_{O_2} O_{O_3}$

offset=2 offset=2

shift=4

depth=3

$E_0 E_1 E_2 E_3 E_4 E_5 E_6 E_7 O_0 O_1 O_2 O_3 O_4 O_5 O_6 O_7$

offset=4

shift=8

depth=4

$f_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} f_{13} f_{14} f_{15}$

offset=8