Example 7.4 Maximum likelihood estimation - Stochastic volatility models

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```
rm(list=ls(all=TRUE))
library(Quandl)
```

```
## Warning: package 'Quandl' was built under R version 3.2.2

## Loading required package: xts

## Warning: package 'xts' was built under R version 3.2.2

## Loading required package: zoo

## Warning: package 'zoo' was built under R version 3.2.2

##
## Attaching package: 'zoo'
##
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
library(DiffusionRgqd)
# Source data for the S&P500 index (SPX).
quandldata1 <- Quandl("YAHOO/INDEX_GSPC", collapse="weekly",
start_date="1990-01-01",end_date="2015-01-01", type="raw")
St <- rev(quandldata1[,names(quandldata1)=='Close'])
time1 <-rev(quandldata1[,names(quandldata1)=='Date'])
# Source data for the volatility index (VIX).
quandldata2 <- Quandl("YAHOO/INDEX_VIX", collapse="weekly",
start_date="1990-01-01",end_date="2015-01-01", type="raw")
Vt <- rev(quandldata2[,names(quandldata2)=='Close'])
time2 <- rev(quandldata2[,names(quandldata2)=='Date'])

GQD.remove() # Remove the previous model coefficients
```

```
## [1] "Removed :  NA "
```

```
# R_t coefficients:
a00 <- function(t){theta[1]}
a01 <- function(t){-0.5*theta[2]*theta[2]}
c01 <- function(t){theta[2]*theta[2]}
d01 <- function(t){theta[2]*theta[5]*theta[6]}
# V_t coefficients:
```

```r
b00 <- function(t){theta[3]}
b01 <- function(t){-theta[4]}
e01 <- function(t){theta[2]*theta[5]*theta[6]}
f01 <- function(t){theta[5]*theta[5]}

# Create data matrix and numerical time vector :
X <- cbind(log(St),(Vt/100)^2)
time <- cumsum(c(0,diff(as.Date(time1))*(1/365)))

# Some starting parameters for the optimization routine:
theta.start <- c(0,1,1,0.5,1,0)
# Calculate MLEs of the parameter vector:
model_1 <- BiGQD.mle(X,time,mesh=10,theta=theta.start)
```

```
## Compiling C++ code. Please wait.
```

```
## ================================================================
##                    GENERALIZED QUADRATIC DIFFUSON
## ================================================================
## _____________________ Drift Coefficients ______________________
## a00 : theta[1]
## a01 : -0.5*theta[2]*theta[2]
## ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
## b00 : theta[3]
## b01 : -theta[4]
## ___________________ Diffusion Coefficients ____________________
## c01 : theta[2]*theta[2]
## ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
## d01 : theta[2]*theta[5]*theta[6]
## ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
## e01 : theta[2]*theta[5]*theta[6]
## ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
## f01 : theta[5]*theta[5]
##
## __________________________ Model Info __________________________
## Time Homogeneous     : Yes
## Data Resolution      : Homogeneous: dt=0.0192
## # Removed Transits.  : None
## Density approx.      : 4th Ord. Truncation, Bivariate-Saddlepoint
## Elapsed time         : 00:00:13
## ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
## dim(theta)           : 6
## ----------------------------------------------------------------
```

```r
# Retreve parameter estimates and appr. 95% CIs:
GQD.estimates(model_1)
```

```
##          Estimate Lower_95 Upper_95
## theta[1]    0.083    0.031    0.135
## theta[2]    0.770    0.740    0.799
## theta[3]    0.168    0.128    0.207
## theta[4]    3.822    2.816    4.827
```

```
## theta[5]    0.431    0.414    0.447
## theta[6]   -0.671   -0.700   -0.641
```

```r
GQD.remove() # Remove the previous model coefficients
```

```
## [1] "Removed :  a00 a01 b00 b01 c01 d01 e01 f01"
```

```r
# R_t coefficients:
a00 <- function(t){theta[1]}
a02 <- function(t){-0.5*theta[2]*theta[2]}
c02 <- function(t){theta[2]*theta[2]}
d02 <- function(t){theta[2]*theta[5]*theta[6]}
# V_t coefficients:
b00 <- function(t){theta[3]}
b01 <- function(t){-theta[4]}
e02 <- function(t){theta[2]*theta[5]*theta[6]}
f02 <- function(t){theta[5]*theta[5]}

theta.start <- c(0,1,1,1,1,0)
model_2 <- BiGQD.mle(X,time,mesh=10,theta=theta.start)
```

```
## Compiling C++ code. Please wait.
```

```
##  ================================================================
##                    GENERALIZED QUADRATIC DIFFUSON
##  ================================================================
##  _____________________ Drift Coefficients _______________________
##  a00 : theta[1]
##  a02 : -0.5*theta[2]*theta[2]
##  ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##  b00 : theta[3]
##  b01 : -theta[4]
##  ___________________ Diffusion Coefficients _____________________
##  c02 : theta[2]*theta[2]
##  ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##  d02 : theta[2]*theta[5]*theta[6]
##  ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##  e02 : theta[2]*theta[5]*theta[6]
##  ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##  f02 : theta[5]*theta[5]
##
##  __________________________ Model Info __________________________
##  Time Homogeneous    : Yes
##  Data Resolution     : Homogeneous: dt=0.0192
##  # Removed Transits. : None
##  Density approx.     : 4th Ord. Truncation, Bivariate-Saddlepoint
##  Elapsed time        : 00:00:40
##  ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##  dim(theta)          : 6
##  ----------------------------------------------------------------
```

```r
# Compare AIC and BIC vlaues for models 1 and 2:
GQD.aic(list(model_1,model_2))
```

```
##          Convergence p min.likelihood              AIC              BIC    N
## Model 1            0 6      -7852.212       -15692.424       -15661.381 1305
## Model 2            0 6      -7965.957   [=] -15919.914   [=] -15888.871 1305
```

```r
GQD.remove()
```

```
## [1] "Removed :  a00 a02 b00 b01 c02 d02 e02 f02"
```

```r
# R_t coefficients:
a02 <- function(t){-0.5*theta[1]*theta[1]}
c02 <- function(t){theta[1]*theta[1]}
d02 <- function(t){theta[1]*theta[4]*theta[5]}
# V_t coefficients:
b00 <- function(t){theta[2]}
b01 <- function(t){-theta[3]}
e02 <- function(t){theta[1]*theta[4]*theta[5]}
f02 <- function(t){theta[4]*theta[4]}

theta.start <- c(1,1,1,1,0)
model_3 <- BiGQD.mle(X,time,mesh=10,theta=theta.start)
```

```
## Compiling C++ code. Please wait.
```

```
##   ================================================================
##                     GENERALIZED QUADRATIC DIFFUSON
##   ================================================================
##   _____________________ Drift Coefficients _______________________
##   a02 : -0.5*theta[1]*theta[1]
##   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##   b00 : theta[2]
##   b01 : -theta[3]
##   ___________________ Diffusion Coefficients _____________________
##   c02 : theta[1]*theta[1]
##   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##   d02 : theta[1]*theta[4]*theta[5]
##   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##   e02 : theta[1]*theta[4]*theta[5]
##   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##   f02 : theta[4]*theta[4]
##
##   __________________________ Model Info __________________________
##   Time Homogeneous    : Yes
##   Data Resolution     : Homogeneous: dt=0.0192
##   # Removed Transits. : None
##   Density approx.     : 4th Ord. Truncation, Bivariate-Saddlepoint
##   Elapsed time        : 00:00:26
##   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##   dim(theta)          : 5
##   ----------------------------------------------------------------
```

```r
GQD.remove()
```

```
## [1] "Removed :  a02 b00 b01 c02 d02 e02 f02"
```

```r
# R_t coefficients:
a00 <- function(t){theta[1]}
a10 <- function(t){theta[7]}
a02 <- function(t){-0.5*theta[2]*theta[2]}
c02 <- function(t){theta[2]*theta[2]}
d02 <- function(t){theta[2]*theta[5]*theta[6]}
# V_t coefficients:
b00 <- function(t){theta[3]}
b01 <- function(t){-theta[4]}
e02 <- function(t){theta[2]*theta[5]*theta[6]}
f02 <- function(t){theta[5]*theta[5]}

theta.start <- c(0,1,1,1,1,0,0)
model_4 <- BiGQD.mle(X,time,mesh=10,theta=theta.start)
```

```
## Compiling C++ code. Please wait.
```

```
## ================================================================
##                    GENERALIZED QUADRATIC DIFFUSON
## ================================================================
## _____________________ Drift Coefficients _______________________
## a00 : theta[1]
## a10 : theta[7]
## a02 : -0.5*theta[2]*theta[2]
## ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
## b00 : theta[3]
## b01 : -theta[4]
## ___________________ Diffusion Coefficients _____________________
## c02 : theta[2]*theta[2]
## ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
## d02 : theta[2]*theta[5]*theta[6]
## ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
## e02 : theta[2]*theta[5]*theta[6]
## ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
## f02 : theta[5]*theta[5]
##
## __________________________ Model Info __________________________
## Time Homogeneous    : Yes
## Data Resolution     : Homogeneous: dt=0.0192
## # Removed Transits. : None
## Density approx.     : 4th Ord. Truncation, Bivariate-Saddlepoint
## Elapsed time        : 00:00:21
## ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
## dim(theta)          : 7
## ----------------------------------------------------------------
```

```r
GQD.remove()
```

```
## [1] "Removed :  a00 a10 a02 b00 b01 c02 d02 e02 f02"
```

```r
# R_t coefficients:
a00 <- function(t){theta[1]}
a02 <- function(t){-0.5*theta[2]*theta[2]}
c02 <- function(t){theta[2]*theta[2]}
d02 <- function(t){theta[2]*theta[5]*theta[6]}
# V_t coefficients:
b00 <- function(t){theta[3]}
b10 <- function(t){theta[7]}
b01 <- function(t){-theta[4]}
e02 <- function(t){theta[2]*theta[5]*theta[6]}
f02 <- function(t){theta[5]*theta[5]}
theta.start <- c(0,1,1,1,1,0,0)
model_5 <- BiGQD.mle(X,time,mesh=10,theta=theta.start)
```

```
## Compiling C++ code. Please wait.
```

```
## ================================================================
##                   GENERALIZED QUADRATIC DIFFUSON
## ================================================================
##  _____________________ Drift Coefficients _______________________
##  a00 : theta[1]
##  a02 : -0.5*theta[2]*theta[2]
##  ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##  b00 : theta[3]
##  b10 : theta[7]
##  b01 : -theta[4]
##  ___________________ Diffusion Coefficients _____________________
##  c02 : theta[2]*theta[2]
##  ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##  d02 : theta[2]*theta[5]*theta[6]
##  ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##  e02 : theta[2]*theta[5]*theta[6]
##  ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##  f02 : theta[5]*theta[5]
##
##  __________________________ Model Info __________________________
##  Time Homogeneous    : Yes
##  Data Resolution     : Homogeneous: dt=0.0192
##  # Removed Transits. : None
##  Density approx.     : 4th Ord. Truncation, Bivariate-Saddlepoint
##  Elapsed time        : 00:00:44
##  ...   ...   ...   ...   ...   ...   ...   ...   ...   ...   ...
##  dim(theta)          : 7
##  ----------------------------------------------------------------
```

```r
GQD.aic(list(model_1,model_2,model_3,model_4,model_5))
```

```
##         Convergence p min.likelihood           AIC           BIC    N
## Model 1           0 6      -7852.212     -15692.424     -15661.381 1305
## Model 2           0 6      -7965.957 [=] -15919.914 [=] -15888.871 1305
## Model 3           0 5      -7948.554     -15887.109     -15861.239 1305
## Model 4           0 7      -7966.154     -15918.308     -15882.090 1305
## Model 5           0 7      -7936.321     -15858.642     -15822.424 1305
```

```r
GQD.estimates(model_4)
```

```
##          Estimate Lower_95 Upper_95
## theta[1]    0.014   -0.394    0.422
## theta[2]    4.254    4.094    4.414
## theta[3]    0.094    0.066    0.122
## theta[4]    3.042    1.950    4.133
## theta[5]    1.746    1.683    1.809
## theta[6]   -0.647   -0.677   -0.616
## theta[7]    0.017   -0.043    0.077
```