$$
\begin{aligned}
&\mathcal{L}_S(\mathbf{d}_w, \mathbf{C}^T\mathbf{w}) \\
&= -\log \; \frac{e^{\mathbf{d}_w^T \mathbf{C}^T \mathbf{w}}}{\sum_{\mathbf{d}'_w \in \mathcal{S}_w} e^{{\mathbf{d}'_w}^T \mathbf{C}^T \mathbf{w}}} \\
&\triangleq -\log \; P(\mathbf{d}_w | \mathbf{C}^T \mathbf{w}) \\
&= -\log \frac{e^{\sum_{c \in V_C} d_{w,c} \mathbf{C}_c^T \mathbf{w}}}{\sum_{\mathbf{d}'_w \in \mathcal{S}_w} e^{\sum_{c \in V_C} d'_{w,c} \mathbf{C}_c^T \mathbf{w}}} \\
&= -\log \frac{e^{\sum_{c \in V_C} d_{w,c} \mathbf{C}_c^T \mathbf{w}}}{\sum_{d'_{w,1} \in \mathcal{S}_{w,1}}, \ldots, \sum_{d'_{w,|V_C|} \in \mathcal{S}_{w,|V_C|}} e^{\sum_{c \in V_C} d'_{w,c} \mathbf{C}_c^T \mathbf{w}}} \\
&= -\log \frac{\prod_{c \in V_C} e^{d_{w,c} \mathbf{C}_c^T \mathbf{w}}}{\sum_{d'_{w,1} \in \mathcal{S}_{w,1}}, \ldots, \sum_{d'_{w,|V_C|} \in \mathcal{S}_{w,|V_C|}} \prod_{c \in V_C} e^{d'_{w,c} \mathbf{C}_c^T \mathbf{w}}} \\
&= -\log \frac{\prod_{c \in V_C} e^{d_{w,c} \mathbf{C}_c^T \mathbf{w}}}{\sum_{d'_{w,1} \in \mathcal{S}_{w,1}}, \ldots, \sum_{d'_{w,|V_C|-1} \in \mathcal{S}_{w,|V_C|-1}}, \sum_{d'_{w,|V_C|} \in \mathcal{S}_{w,|V_C|}} \prod_{c \in V_C} e^{d'_{w,c} \mathbf{C}_c^T \mathbf{w}}} \\
&= -\log \frac{\prod_{c \in V_C} e^{d_{w,c} \mathbf{C}_c^T \mathbf{w}}}{\sum_{d'_{w,1} \in \mathcal{S}_{w,1}}, \ldots, \sum_{d'_{w,|V_C|-1} \in \mathcal{S}_{w,|V_C|-1}}, \sum_{d'_{w,|V_C|} \in \mathcal{S}_{w,|V_C|}} e^{d'_{w,|V_C|} \mathbf{C}_{|V_C|}^T \mathbf{w}} \prod_{c \in V_C / \{|V_C|\}} e^{d'_{w,c} \mathbf{C}_c^T \mathbf{w}}} \quad , \\
&= -\log \frac{\prod_{c \in V_C} e^{d_{w,c} \mathbf{C}_c^T \mathbf{w}}}{\sum_{d'_{w,1} \in \mathcal{S}_{w,1}}, \ldots, \sum_{d'_{w,|V_C|-1} \in \mathcal{S}_{w,|V_C|-1}} \prod_{c \in V_C / \{|V_C|\}} e^{d'_{w,c} \mathbf{C}_c^T \mathbf{w}} \left[\sum_{d'_{w,|V_C|} \in \mathcal{S}_{w,|V_C|}} e^{d'_{w,|V_C|} \mathbf{C}_{|V_C|}^T \mathbf{w}} \right]} \\
&= -\log \frac{\prod_{c \in V_C} e^{d_{w,c} \mathbf{C}_c^T \mathbf{w}}}{\sum_{d'_{w,1} \in \mathcal{S}_{w,1}}, \ldots, \prod_{c \in V_C / \{|V_C|, |V_C|-1\}} e^{d'_{w,c} \mathbf{C}_c^T \mathbf{w}} \left[\sum_{d'_{w,|V_C|-1} \in \mathcal{S}_{w,|V_C|-1}} e^{d'_{w,|V_C|-1} \mathbf{C}_{|V_C|-1}^T \mathbf{w}} \right] \left[\sum_{d'_{w,|V_C|} \in \mathcal{S}_{w,|V_C|}} e^{d'_{w,|V_C|} \mathbf{C}_{|V_C|}^T \mathbf{w}} \right]}
\end{aligned}
$$

Through repeating the procedure as line 8 to 10, we can finally get the following

$$
\begin{aligned}
&= -\log \frac{\prod_{c \in V_C} e^{d_{w,c} \mathbf{C}_c^T \mathbf{w}}}{\prod_{c \in V_C} \sum_{d'_{w,c} \in \mathcal{S}_{w,c}} e^{d'_{w,c} \mathbf{C}_c^T \mathbf{w}}} \\
&= -\log \prod_{c \in V_C} \frac{e^{d_{w,c} \mathbf{C}_c^T \mathbf{w}}}{\sum_{d'_{w,c} \in \mathcal{S}_{w,c}} e^{d'_{w,c} \mathbf{C}_c^T \mathbf{w}}} \\
&= -\sum_{c \in V_C} \log P(d_{w,c} | \mathbf{C}_c^T \mathbf{w})
\end{aligned}
$$