

Module 11 Graphs and Traversals Application Programming Assignment

Problem: You need to design an efficient algorithm which will run in $O(n + m)$ running time that computes for each station, the set of stations it can reach using with maximum 4 links.

Solution: The following algorithm will compute, for any station, the set of stations it can reach using a maximum of 4 links.

Algorithm findStations(G, v):

Input: G , the graph being explored, and v , the starting vertex.

Output: *depthList*, a list of the reachable stations from v using a maximum of 4 links

adjacent \leftarrow empty stack

adjacent.push(v)

depth $\leftarrow 0$

depthList \leftarrow empty list

temp \leftarrow empty stack

vertices $\leftarrow 1$

while *vertices* < the number of vertices in G AND *depth* < 5 **do**

while *adjacent.is*Empty() = false **do**

depthList.add(*adjacent.peak*())

for each edge, e , that is incident to *adjacent.peak*() in G **do**

If e unexplored **then**

 Mark e explored

 Let w be the vertex on the other end of e

temp.push(w)

vertices \leftarrow *vertices* + 1

adjacent.pop()

adjacent \leftarrow *temp*

temp \leftarrow empty stack

depth \leftarrow *depth* + 1

if *depth* < 5 **then**

while *adjacent.is*Empty() = false **do**

depthList.add(*adjacent.pop*())

Output *depthList*

Variables: findStations accepts two parameters: G , the map that is being explored by findStations, and v , the vertex whose stations are being found. findStations also establishes five additional variables: *adjacent*, a stack containing v , which will be used to track the number of adjacent vertices, *depth*, the depth of the current vertices being explored, *depthList*, a list of vertices reachable by v using no more than 4 edges, and *temp*, an empty stack used to store the elements to be placed in *adjacent* for the next iteration.

Explanation: findStations begins by conducting a depth-first search to find the depths of all of the vertices in G , starting with vertex v . Then, as long as the depth was less than 5 and there were more vertices to be found, the depth-first search would keep iterating. The depth-first search would then insert each element found into *depthList*, which would contain a list of all the vertices reachable from v , using no more than 4 edges. findStations finishes by outputting *depthList*.

Runtime: findStations has a runtime of $O(n + m)$. This is because it will iterate over each vertex and edge only once, giving it a runtime of $O(n + m)$, where n is the number of vertices and m is the number of edges.