Cosmic Birefringence and Hemispherical Asymmetry from Axial Torsion on K_4

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Abstract

We derive parameter-free predictions for cosmic birefringence and CMB hemispherical asymmetry from the spectral action on K_4 developed in the companion paper [1]. The internal spectral triple on K_4 fixes the Chern–Simons coupling of an axial torsion pseudoscalar to gravity, giving a cosmic birefringence angle

$$\alpha_0 = \frac{(2-\sqrt{3})^2}{10}\, F(m_A) \leq 0.41^\circ,$$

where $F(m_A)$ is the cosmological evolution factor and $(2-\sqrt{3})^2 = 0.0718$ is the inverse eigenvalue ratio of K_4. This is consistent with the measured $0.30^\circ \pm 0.11^\circ$ (Planck/ACT, 3σ) and will be tested at $> 8\sigma$ by LiteBIRD.

The contorsion coupling to photon geodesics vanishes identically for axial torsion ($K^\mu{}_{\nu\rho}\, p^\nu p^\rho = 0$), eliminating the direct temperature modulation. A separate-universe Boltzmann computation gives the shape function $r_{TT}(\ell) \approx 2.2\,\ell^{-1.3}$ for $\ell \lesssim 15$, crossing zero near $\ell \sim 20$, with the correct amplitude ($A \sim 0.08$) at low ℓ but tension at $\ell_{\max} = 64$. The EB shape is exactly $r_{EB} = 1$.

We state five parameter-free, falsifiable predictions for LiteBIRD and CMB-S4: (1) birefringence dipole aligned with TT asymmetry, (2) dipolar amplitude $\delta\alpha/\alpha_0 = (2-\sqrt{3})^2$, (3) EB BiPoSH parity alternation $(-1)^{\ell+1}$, (4) TT/EB shape split with sign change at $\ell \sim 20$, and (5) TB/EB ratio $= C_\ell^{TE}/C_\ell^{EE}$.

Contents

1 Introduction

The isotropic cosmic birefringence angle has been measured at $\alpha_0 = 0.30° \pm 0.11°$ by Minami and Komatsu [4] using Planck HFI polarization data, with corroboration from independent analyses [5]. The significance stands at approximately 3σ. Meanwhile, the CMB hemispherical power asymmetry—a dipolar modulation of the temperature power spectrum—has been detected at 3.3σ by Planck [7, 8] with amplitude $A = 0.066 \pm 0.021$ at $\ell_{\max} = 64$.

We show that both signals can originate from a single mechanism: a cosmological axial torsion pseudoscalar whose coupling constants are fixed by the spectral action on K_4. The mathematical framework is developed in the companion paper [1], which proves that Lorentzian signature $(1, 3)$ emerges generically from the Pfaffian structure of antisymmetric matrices on four vertices, with the key structural numbers—the eigenvalue ratio $R = 7 + 4\sqrt{3}$ and its inverse $A_0^* = (2 - \sqrt{3})^2$—determined by the K_4 combinatorics.

Within the Einstein–Cartan extension of general relativity, the axial part of the torsion tensor behaves as a pseudoscalar field $\mathcal{A}$ with a gravitational Chern–Simons coupling. The K_4 spectral action fixes the coupling constants that are normally free parameters:

(i) The kinetic coefficient $\alpha_T = 1/8$ (from the K_4 heat kernel).

(ii) The parity ratio $b_0/a_0 = -1/5$ (from the chirality-weighted trace).

(iii) The amplitude scale $A_0^* = (2 - \sqrt{3})^2$ (from the D^2 eigenvalue ratio).

Together, these yield the birefringence prediction $\alpha_0 \le 0.41°$ with no adjustable parameters beyond the torsion mass m_A, which is a hierarchy problem shared with all ultralight scalar models.

The paper is organized as follows. Section 2 summarizes the relevant results from Paper I. Section 3 derives the birefringence angle. Section 4 develops the shape functions and states five falsifiable predictions. Section 5 confronts the predictions with Planck data. Section 6 assesses the quadrupole–octupole alignment. Section 7 discusses the product geometry and grading violation. Section 8 summarizes the status and outlook.

2 Summary of the K_4 Spectral Action

We collect the results from Paper I [1] that are needed for the cosmological applications.

The Dirac operator. Each four-edge subgraph σ of the complete graph K_4 carries a real skew-symmetric matrix $D_\sigma \in \mathfrak{so}(4)$, with entries $D_{ij} = w_e$ for edge $e = (i, j)$ and $D_{ji} = -w_e$.

Lorentzian signature. The spectral action $I[w] = \operatorname{Tr}\exp(D^2/2)$ has Hessian with signature $(1, 3)$ for 13 of 15 subgraphs, with failures at $\operatorname{Pf}(D) = 0$. The critical scale is $s_{\text{crit}} \approx 1.233$.

Key structural numbers. For hub-spoke subgraphs (12 of the 13 Lorentzian subgraphs), the D^2 eigenvalues split into pairs with ratio

$$R = \frac{|\mu_1|}{|\mu_2|} = \frac{2 + \sqrt{3}}{2 - \sqrt{3}} = 7 + 4\sqrt{3} \approx 13.93. \tag{1}$$

The inverse ratio

$$A_0^* = 1/R = (2-\sqrt{3})^2 \approx 0.0718 \tag{2}$$

sets the scale of the birefringence amplitude and hemispherical asymmetry.

Heat kernel coefficients. The internal spectral triple on K_4 has eigenvalues $\lambda_n = n$ with degeneracy $D(n) = 2n$ for $n = 1, 2, 3, 4$. The chirality γ_5 assigns grading $(-1)^{n+1}$. The traces are

$$a_0 = \mathrm{Tr}(\mathbf{1}) = \sum_{n=1}^{4} 2n = 20, \tag{3}$$

$$b_0 = \mathrm{Tr}(\gamma_5) = 2 - 4 + 6 - 8 = -4. \tag{4}$$

The parity ratio $b_0/a_0 = -1/5$ is a purely combinatorial invariant of K_4.

3 Cosmic Birefringence from Axial Torsion

3.1 Spectral Action Couplings

The spectral action on $M^4 \times K_4$ yields the effective gravitational Lagrangian with torsion:

$$\mathcal{L} = \frac{1}{16\pi G} R + \frac{\alpha_T}{2} (\partial_\mu \mathcal{A})^2 - \frac{m_A^2}{2} \mathcal{A}^2 + \frac{\beta}{4} \frac{\mathcal{A}}{f_A} \tilde{R}R + \cdots \tag{5}$$

where $\tilde{R}R = \varepsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\mu\nu} R_{\gamma\delta}{}^{\mu\nu}$ is the Pontryagin density. The coupling constants are fixed by the K_4 traces:

$$\alpha_T = \frac{1}{8}, \qquad \frac{\beta}{\alpha_T} = \frac{b_0}{a_0} = -\frac{1}{5}, \qquad f_A = \frac{M_P}{\sqrt{\alpha_T}} = 2\sqrt{2}\, M_P. \tag{6}$$

These three numbers are fixed by the K_4 eigenvalue structure and Newton's constant.

3.2 The Birefringence Angle

The Chern–Simons term rotates the CMB polarisation plane by $\alpha_0 = \Delta\mathcal{A}/f_A$. Setting the initial displacement $\mathcal{A}_i = A_0^* f_A/2$ with $A_0^* = (2-\sqrt{3})^2$:

$$\boxed{\alpha_0 = \frac{(2-\sqrt{3})^2}{10} F(m_A) = 0.411^\circ \times F(m_A),} \tag{7}$$

where $F(m_A)$ is the cosmological evolution factor measuring the fractional field change between recombination and today.

The coefficient combines two structural numbers: the eigenvalue ratio $(2-\sqrt{3})^2 = 0.0718$ and the parity coefficient $|b_0/a_0| = 1/5$.

Matching the central observed value $\alpha_0 = 0.30^\circ$ requires $F = 0.729$, achieved at $m_A = 2.7\, H_0 \approx 3.9 \times 10^{-33}$ eV. The firm upper bound is $\alpha_0^{\max} = 0.411^\circ$ (for $F \to 1$), which LiteBIRD will test at $> 8\sigma$ significance with its projected $\sigma(\alpha_0) \sim 0.05^\circ$.

3.3 The Mass Is Not Predicted

The torsion mass $m_A \sim H_0$ must arise from a non-perturbative mechanism analogous to the shift symmetry protecting the QCD axion mass. The spectral action at face value gives $m_A \sim M_P$, far too heavy. The ultralight mass $m_A \sim 10^{-33}$ eV is the same hierarchy problem faced by all ultralight scalar models. The framework's prediction is the *coefficient*, not the mass: $\alpha_0 < 0.41°$ for any $m_A \gtrsim H_0$. Any future measurement of $\alpha_0 > 0.5°$ would exclude the framework.

4 Falsifiable Predictions

The axial torsion field couples to the CMB through two physically distinct operators: the contorsion correction to photon geodesics (modifying temperature) and the Chern–Simons gravitational term (rotating polarisation). Because these operators have different spin and derivative structures, they produce BiPoSH coefficients with different multipole dependence—a mechanism split that cannot be mimicked by a single scalar modulation field.

4.1 The m-Sum Cancellation Theorem

Proposition 4.1 (m-sum cancellation)**.** *The ratio of tensor ($s = 2$) to scalar ($s = 0$) BiPoSH coefficients is m-independent:*

$$\frac{A^{(2)}_{10}(\ell, \ell{+}1)}{A^{(0)}_{10}(\ell, \ell{+}1)} = \frac{\begin{pmatrix} \ell & 1 & \ell{+}1 \\ 2 & 0 & -2 \end{pmatrix}}{\begin{pmatrix} \ell & 1 & \ell{+}1 \\ 0 & 0 & 0 \end{pmatrix}}. \tag{8}$$

This is verified to machine precision ($< 10^{-14}$) for $\ell = 2$–60. The ratio approaches unity for large ℓ: it is 0.745 at $\ell = 2$, 0.983 at $\ell = 10$, and 0.999 at $\ell = 50$.

The exact Gaunt formula for the m-summed squared coupling integral is

$$G(\ell) = \sum_m \left| \int Y^*_{\ell m} Y_{10} Y_{\ell+1,m} \, d\Omega \right|^2 = \frac{\ell + 1}{4\pi}. \tag{9}$$

4.2 Contorsion Decoupling from Null Geodesics

Proposition 4.2 (Contorsion–photon decoupling)**.** *For purely axial torsion $T^\mu{}_{\nu\rho} = \frac{1}{3}\varepsilon^\mu{}_{\nu\rho\sigma}\mathcal{A}^\sigma$, the contorsion coupling to null geodesics vanishes identically:*

$$K^\mu{}_{\nu\rho}\, p^\nu p^\rho = \tfrac{2}{3}\, \varepsilon^\mu{}_{\nu\rho\sigma}\, \mathcal{A}^\sigma\, p^\nu p^\rho = 0. \tag{10}$$

Proof. The Levi-Civita symbol $\varepsilon^\mu{}_{\nu\rho\sigma}$ is totally antisymmetric in (ν, ρ, σ), hence antisymmetric in (ν, ρ). The photon momentum product $p^\nu p^\rho$ is symmetric in (ν, ρ). Contraction of an antisymmetric tensor with a symmetric one vanishes identically. □

This result eliminates the direct contorsion coupling to CMB temperature anisotropies. The leading temperature modulation from a dipolar torsion gradient comes instead from the integrated Sachs–Wolfe (ISW) effect: the torsion stress-energy modifies the gravitational potential Φ, and the time-varying $\dot{\Phi}$ generates a dipolar ISW signal.

4.3 Shape Functions

The physical shape function $r(\ell) \equiv \partial \ln C_\ell / \partial \ln \mathcal{A}_0$ is computed by the *separate-universe* method: a superhorizon torsion gradient $\delta\mathcal{A}/\mathcal{A} = w_0 \cos\theta$ causes each direction $\hat{n}$ to see a slightly different cosmology, so we evaluate $C_\ell(\mathcal{A}_0 \pm \delta\mathcal{A})$ using a mini-Boltzmann solver (Sachs–Wolfe + Doppler + ISW line-of-sight integration with Eisenstein–Hu transfer function) and finite-difference. The contorsion vanishing theorem (Proposition 4.2) eliminates any direct geodesic coupling; the entire TT response comes through three physical channels operating simultaneously:

(i) *ISW channel* (dominates at $\ell \lesssim 10$): increasing Ω_A deepens the late-time potential decay, boosting C_ℓ at low ℓ.

(ii) *Distance channel*: increasing Ω_A decreases the comoving distance D_*, compressing angular scales. For $m_A/H_0 = 2.7$: $\partial \ln D_*/\partial \ln \mathcal{A}_0 = -0.27$.

(iii) *Growth channel*: changing Ω_A modifies the growth function $D_+(a)$, affecting the potential at recombination and peak heights.

ℓ	$r_{TT}(\ell)$	$r_{EB}(\ell)$	r_{EB}/r_{TT}	Dominant mechanism
2	+2.19	1	0.46	ISW + distance
5	+1.41	1	0.71	ISW
10	+0.66	1	1.52	ISW (declining)
15	+0.24	1	4.3	ISW $\approx$ distance
20	−0.03	1	—	zero crossing
30	−0.34	1	—	distance (negative)

The TT shape function is positive and steep at low ℓ (power-law $r_{TT} \propto \ell^{-1.3}$ for $\ell \lesssim 15$), crosses zero near $\ell \sim 20$, and turns negative at higher ℓ where the distance compression dominates. All values are for $m_A/H_0 = 2.7$ (best-fit torsion mass); other masses give qualitatively similar shapes with the zero crossing shifting to higher ℓ for larger m_A. The EB shape function is exactly $r_{EB} = 1$ for all ℓ, since the Chern–Simons rotation angle is independent of angular scale.

4.4 Five Pre-Registered Predictions

Prediction 1: Birefringence dipole direction. The TT asymmetry direction is measured at $(l, b) \approx (225°, -27°)$. The dipolar component of cosmic birefringence, when measured, must point in the same direction.

Prediction 2: Dipolar birefringence amplitude.

$$\frac{\delta\alpha}{\alpha_0} = A_0^* = (2 - \sqrt{3})^2 \approx 0.072. \tag{11}$$

If $\alpha_0 \approx 0.3°$, then $\delta\alpha \approx 0.02°$. This is not a free parameter.

Prediction 3: EB BiPoSH parity alternation.

$$A_{10}^{EB}(\ell, \ell+1) \propto (-1)^{\ell+1}. \tag{12}$$

A scalar modulation field produces no such alternation.

Prediction 4: TT/EB shape split. The ratio r_{EB}/r_{TT} grows from ~ 0.5 at $\ell = 2$ through unity at $\ell \approx 10$ and diverges near the TT zero crossing at $\ell \sim 20$:

$$r_{EB}(\ell)/r_{TT}(\ell) \approx 0.46,\ 0.71,\ 1.5,\ 4.3 \quad \text{at } \ell = 2,\ 5,\ 10,\ 15. \tag{13}$$

For $\ell > 20$, r_{TT} changes sign while $r_{EB} = 1$ remains positive, so the EB and TT modulations are *anti-correlated* at high ℓ—a distinctive signature of the two-channel mechanism that no single-field scalar modulation can produce.

Prediction 5: TB/EB ratio.

$$\frac{A_{10}^{TB}(\ell, \ell+1)}{A_{10}^{EB}(\ell, \ell+1)} = \frac{C_\ell^{TE}}{C_\ell^{EE}}. \tag{14}$$

This is independently measurable, providing a zero-parameter consistency check.

Predictions 1 and 2 are testable with LiteBIRD (launch 2032, data ∼2035). Prediction 3 requires EB BiPoSH detection in individual ℓ-bins (CMB-S4 combined with LiteBIRD). Prediction 4 requires both TT and EB shape functions. Prediction 5 functions as an internal consistency check.

5 Shape Function Constraints and Tension with Planck Data

5.1 The Observed Signal

The Planck analysis fits a dipolar modulation $\Delta T(\hat{n})/T = A_0\, \hat{d} \cdot \hat{n}$ assuming ℓ-independent amplitude. The fitted amplitude decreases with $\ell_{\max}$:

$\ell_{\max}$	$A(\ell_{\max})$	$A/A(64)$
64	0.066	1.000
128	0.054	0.818
256	0.040	0.606
512	0.024	0.364
1024	0.012	0.182

5.2 Comparison with the Boltzmann Shape Function

The separate-universe Boltzmann computation (Section 4.3) gives a shape function $r_{TT}(\ell)$ that starts positive ($r_{TT}(2) \approx 2.2$), decays as $\sim\ell^{-1.3}$, crosses zero near $\ell \sim 20$, and turns negative at higher ℓ. The $\ell_{\max}$-averaged prediction is:

$\ell_{\max}$	$\hat{A}_{\text{pred}}$	A_{obs}	pred/obs
5	0.12	—	—
10	0.08	—	—
20	0.03	—	—
64	−0.03	0.066	−0.5

At low ℓ ($\ell_{\max} \lesssim 10$), the torsion framework predicts the *correct order of magnitude* ($A \sim 0.07$–0.12) and the observed *declining trend* of $A(\ell_{\max})$ is qualitatively reproduced by the zero crossing. However, the prediction turns negative at $\ell_{\max} \gtrsim 30$, while the data remain positive through $\ell_{\max} = 1024$. This discrepancy could arise from limitations of the simplified Boltzmann code (Eisenstein–Hu transfer function, no Silk damping evolution, no polarization feedback) or from genuine tension with the data.

5.3 Interpretation

Three interpretations remain possible:

(i) *Partial match.* The torsion framework successfully predicts $A(\ell_{\max} \lesssim 10) \sim 0.08$ and the declining $A(\ell_{\max})$ trend. The zero-crossing prediction at $\ell \sim 20$ is sensitive to the transfer function approximation; a full CLASS/CAMB computation with torsion perturbations would determine whether it persists, shifts to higher ℓ, or softens.

(ii) *Statistical contribution.* The 3σ detection at $\ell_{\max} = 64$ includes a look-elsewhere effect. The torsion contributes $A \sim 0.08$ at $\ell \lesssim 10$, and the observed persistence to $\ell = 64$ may reflect a statistical fluctuation on top of the torsion signal.

(iii) *Different origin at high ℓ.* The low-ℓ signal ($\ell \lesssim 15$) is from torsion; the high-ℓ continuation has a separate origin. The birefringence predictions remain the primary observational test.

We regard the match at low ℓ as encouraging but the high-ℓ discrepancy as an open problem requiring full Boltzmann code refinement.

5.4 The Shape Function as a Constraint

The separate-universe computation demonstrates that a superhorizon dark-energy gradient affects C_ℓ through *three* distinct channels (ISW, distance, and growth), not just the ISW effect assumed in earlier analyses. The resulting $r_{TT}(\ell)$ with its zero crossing is qualitatively different from a pure power-law shape and provides a template for future CMB-S4 BiPoSH analyses. Combined with the exact EB prediction $r_{EB} = 1$ (Chern–Simons), the framework offers a two-channel signature that is distinctive of the axial-torsion mechanism.

6 Quadrupole–Octupole Alignment

The preferred axes of the CMB quadrupole ($\ell = 2$) and octupole ($\ell = 3$) are aligned to within $\sim 2^\circ$ in the Planck data. Because the dipolar torsion modulation couples ℓ to $\ell + 1$, it is natural to ask whether the $\ell = 2 \leftrightarrow \ell = 3$ coupling can produce the observed alignment. We compute this exactly and find that it cannot.

The predicted cross-correlation coefficient is $\rho_{23} = 0.021$, shifting the alignment statistic by $\Delta S = 2.9 \times 10^{-4}$ (0.002σ of the null distribution). Reproducing the observed alignment would require $\rho_{23} \gtrsim 0.5$, corresponding to order-unity modulation of the CMB.

A further obstacle is directional: the hemispherical asymmetry dipole points toward $(l, b) \approx (225^\circ, -27^\circ)$, while the quadrupole–octupole axis points toward $(l, b) \approx$

$(240°, 63°)$—a separation of 89°. The two anomalies are associated with different directions on the sky.

The near-m-independence of the Gaunt coupling ($G_{\pm 2}/G_0 = 0.745$) means that the dipolar modulation does not preferentially enhance the zonal ($m = 0$) component that would create axial alignment. The torsion framework does not explain the quadrupole–octupole alignment: the effect is too weak by a factor of ~ 24, in the wrong direction by 89°, and structurally incapable of the required mode coupling.

7 Product Geometry and Grading Violation

The full Dirac operator on the product geometry $K_4 \times F$ is

$$D_{\text{full}}(t) = D_{\text{space}}(t) \otimes \mathbf{1}_{N_F} + \gamma \otimes D_F, \tag{15}$$

where $D_{\text{space}}(t) = (1 - t)D_{\text{seq}} + t\, D_{\text{hub}}$ interpolates from the Hamiltonian cycle ($t = 0$, where the grading γ exists) to the hub-spoke ($t = 1$), and D_F is the finite Dirac operator encoding Yukawa couplings.

The grading $\gamma = \text{diag}(+1, -1, +1, -1)$ anticommutes with D_{seq} (bipartite) but not with D_{hub} (non-bipartite). The anticommutator $C_{\text{grad}} = \{D_{\text{hub}}, \gamma\}$ is a rank-2 Hermitian matrix with eigenvalues $\{-2, 0, 0, +2\}$, coupling only the same-chirality vertices.

Squaring:

$$D_{\text{full}}(t)^2 = D_{\text{space}}(t)^2 \otimes \mathbf{1} + \mathbf{1} \otimes D_F^2 + t \cdot C_{\text{grad}} \otimes D_F. \tag{16}$$

The cross-term $t \cdot C_{\text{grad}} \otimes D_F$ is the unique coupling between spacetime geometry and internal particle physics, vanishing exactly at the bipartite point $t = 0$.

The Seeley–DeWitt coefficients $a_k(t) = \text{Tr}(D_{\text{full}}(t)^k)$ are all maximized at $t = 0$ and minimized at intermediate t: the cross-term drives the geometry toward lower total curvature. Whether the symmetric vacuum ($t = 0$) is destabilized depends on the ratio f_2/f_0 of spectral action moments and the Yukawa coupling strength, with larger Yukawas favoring destabilization.

8 Discussion

8.1 What Is Established

The cosmic birefringence prediction $\alpha_0 \leq 0.41°$ combines two structural numbers from the K_4 spectral action—the eigenvalue ratio $(2-\sqrt{3})^2$ and the parity coefficient $|b_0/a_0| = 1/5$—with no free parameters beyond the torsion mass. The prediction is consistent with the observed $0.30° \pm 0.11°$ and provides a firm upper bound that LiteBIRD will test decisively.

The contorsion vanishing theorem eliminates a physically important channel: axial torsion does not directly deflect photons. The temperature modulation comes entirely through the ISW effect, distance compression, and growth modification—a three-channel structure that produces the distinctive zero-crossing shape function.

The five predictions are parameter-free and falsifiable. The EB parity alternation (Prediction 3) is the most distinctive: no simple scalar modulation can produce $(-1)^{\ell+1}$ alternation in the EB BiPoSH coefficients. If confirmed by CMB-S4, this would be strong evidence for a pseudoscalar origin of the birefringence.

8.2 What Remains Open

The shape function tension at $\ell_{\text{max}} = 64$ is a genuine concern. The zero crossing at $\ell \sim 20$ is computed with a simplified Boltzmann solver (Eisenstein–Hu transfer function); a full CLASS or CAMB computation with torsion perturbations is needed to determine whether the crossing persists, shifts, or softens. This is the most important technical refinement.

The torsion mass $m_A \sim H_0$ is a hierarchy problem. The spectral action gives $m_A \sim M_P$; the ultralight value requires a protecting symmetry analogous to the QCD axion's shift symmetry. This is not specific to our framework—it is shared by all ultralight dark energy models.

The quadrupole–octupole alignment is not explained. We have been explicit about this failure, which is too weak by a factor of ~ 24 and in the wrong direction.

8.3 Summary of Killed Claims

(i) *Hemispherical asymmetry as direct contorsion effect*: killed by the contorsion vanishing theorem. The effect comes through ISW/distance/growth instead.

(ii) *Quadrupole–octupole alignment from torsion*: too weak by a factor of 24, wrong direction by 89°.

(iii) *Torsion mass prediction*: not predicted; hierarchy problem acknowledged.

8.4 Outlook

The primary test is LiteBIRD (launch 2032). With projected sensitivity $\sigma(\alpha_0) \sim 0.05°$, it will measure α_0 at 6σ if the central value holds, and will test the upper bound 0.41° at $> 8\sigma$. The dipolar birefringence measurement (Prediction 1) requires full-sky polarization data at the same sensitivity level.

CMB-S4 combined with LiteBIRD will enable EB BiPoSH analysis in individual ℓ-bins, testing the parity alternation (Prediction 3) and shape split (Prediction 4). These are the smoking-gun signatures of the axial-torsion mechanism.

Full Boltzmann integration (CLASS/CAMB with torsion perturbations) is needed to resolve the shape function tension and determine whether the zero crossing at $\ell \sim 20$ is physical or an artifact of the simplified transfer function.

Acknowledgments

[To be added.]

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