MAT627 Chapter 2.1 Programming Assignment

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Standard vs. Horner Evaluation

Task: evalute the polynomial

$$p(x) = (x-2)^9$$

over the domain $D := \{a + kh\}_{k=0}^{N}$ with $[a, b] = [1.92, 2.08]$ and $h = \frac{b-a}{N}$.

Treat the factored form $(x - 2)^9$ as the "exact" reference and compare against the standard expanded-form evaluation and Horner's method.

Observations Let $\tilde{p}_{\text{std}}$ be the standard evaluation and $\tilde{p}_{\text{H}}$ be Horner evaluation. The computed maximum absolute errors were (using double precision):

N	$\max_{x_k \in D} \lvert p(x_k) - \tilde{p}_{\text{std}}(x_k)\rvert$	$\max_{x_k \in D} \lvert p(x_k) - \tilde{p}_{\text{H}}(x_k)\rvert$
10^3	2.4897×10^{-11}	8.7083×10^{-12}
10^5	3.1605×10^{-11}	1.0442×10^{-11}

In both tests, Horner's method reduces the max error by roughly a factor of 3 compared to the standard expanded evaluation.

Analysis On $[1.92, 2.08]$, the true magnitude is extremely small:

$$\max_{x \in [a,b]} |p(x)| = (0.08)^9 \approx 1.34 \times 10^{-10}.$$

However when expanding the polynomial, we see large coefficents like -4032 and 5376. When calculating the polynomial in expanded form, these large intermediate terms must nearly cancel out to achieve a number $\sim 10^{-10}$. Cancelling near-equal floating point numbers discards significant digits, and what remains is noisy, even with double precision. In the plot, this appears as a band around $p(x)$.

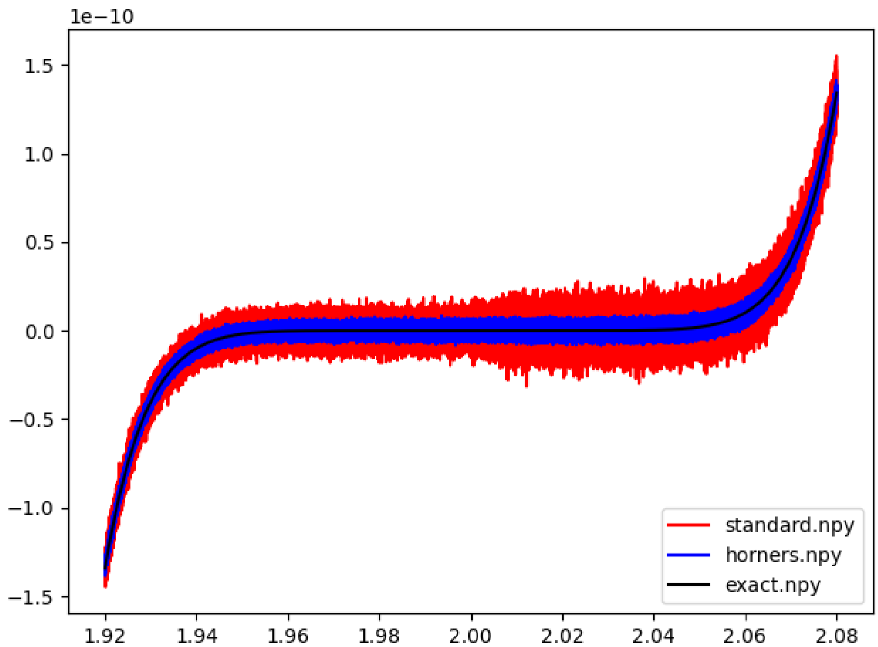

Figure 1:

Horner's method helps reduce error because it actually uses fewer floating-point operations, and it avoids explicitly producing high powers of x^k which typically reduces rounding error accumulation. It isn't perfect though, Horner's method cannot eliminate the underlying cancellation that comes from expanding $p(x)$. This analysis is reflected in the smaller, but still present, band around $p(x)$.