

MAT627 Chapter 2.2 Programming Assignment

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1 Finite-Difference Approximation Results

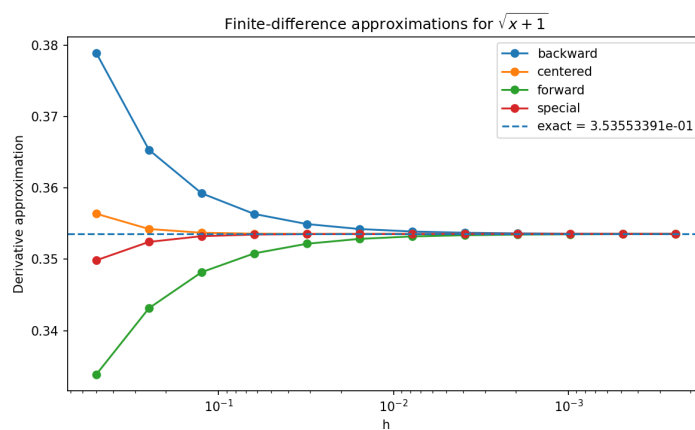


Figure 1: Approximation overlay for $\sqrt{x+1}$.

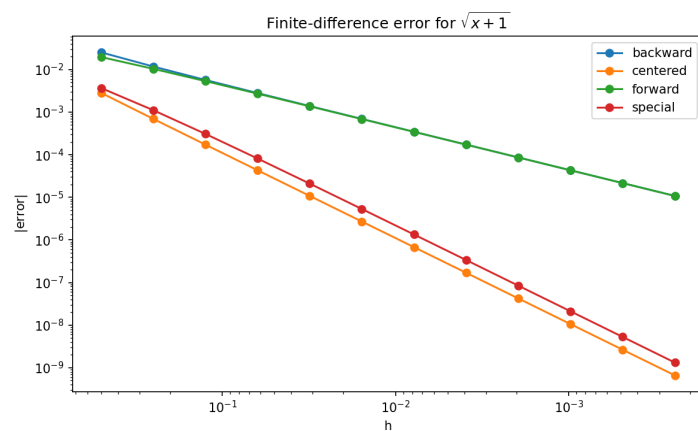


Figure 2: Error overlay for $\sqrt{x+1}$.

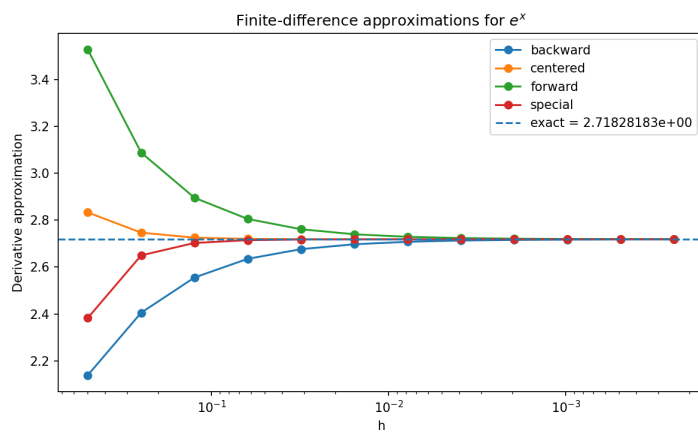


Figure 3: Approximation overlay for e^x .

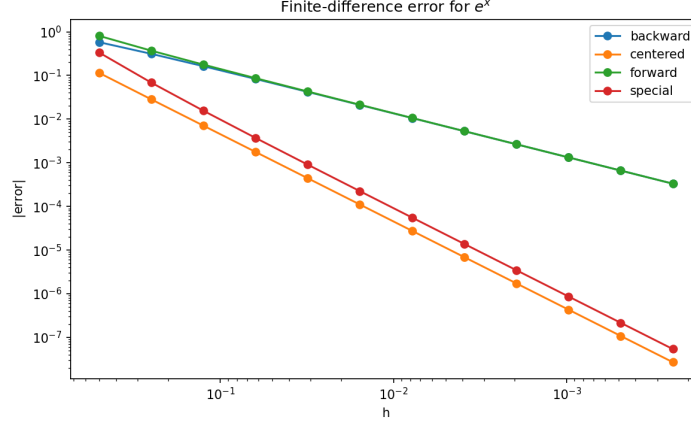


Figure 4: Error overlay for e^x .

Table 1: Backward difference for e^x .

h^{-1}	$D_h^- f(1)$	$E_h = f'(1) - D_h^- f(1)$	$\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$
2	2.139121×10^0	5.791607×10^{-1}	—
4	2.405127×10^0	3.131546×10^{-1}	8.870888×10^{-1}
8	2.555252×10^0	1.630296×10^{-1}	9.417415×10^{-1}
16	2.635078×10^0	8.320390×10^{-2}	9.704104×10^{-1}
32	2.676248×10^0	4.203416×10^{-2}	9.850890×10^{-1}
64	2.697155×10^0	2.112640×10^{-2}	9.925153×10^{-1}
128	2.707691×10^0	1.059069×10^{-2}	9.962503×10^{-1}
256	2.712980×10^0	5.302238×10^{-3}	9.981233×10^{-1}
512	2.715629×10^0	2.652845×10^{-3}	9.990612×10^{-1}
1024	2.716955×10^0	1.326854×10^{-3}	9.995305×10^{-1}
2048	2.717618×10^0	6.635350×10^{-4}	9.997652×10^{-1}
4096	2.717950×10^0	3.317945×10^{-4}	9.998826×10^{-1}

Table 2: Centered difference for e^x .

h^{-1}	$D_h f(1)$	$E_h = f'(1) - D_h f(1)$	$\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$
2	2.832968×10^0	-1.146860×10^{-1}	—
4	2.746686×10^0	-2.840405×10^{-2}	2.013520×10^0
8	2.725366×10^0	-7.084391×10^{-3}	2.003381×10^0
16	2.720052×10^0	-1.770060×10^{-3}	2.000845×10^0
32	2.718724×10^0	-4.424503×10^{-4}	2.000211×10^0
64	2.718392×10^0	-1.106085×10^{-4}	2.000053×10^0
128	2.718309×10^0	-2.765188×10^{-5}	2.000013×10^0
256	2.718289×10^0	-6.912953×10^{-6}	2.000003×10^0
512	2.718284×10^0	-1.728237×10^{-6}	2.000001×10^0
1024	2.718282×10^0	-4.320592×10^{-7}	2.000001×10^0
2048	2.718282×10^0	-1.080144×10^{-7}	2.000005×10^0
4096	2.718282×10^0	-2.700387×10^{-8}	1.999985×10^0

Table 3: Forward difference for e^x .

h^{-1}	$D_h^+ f(1)$	$E_h = f'(1) - D_h^+ f(1)$	$\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$
2	3.526814×10^0	-8.085327×10^{-1}	—
4	3.088245×10^0	-3.699627×10^{-1}	1.127926×10^0
8	2.895480×10^0	-1.771983×10^{-1}	1.062015×10^0
16	2.805026×10^0	-8.674402×10^{-2}	1.030529×10^0
32	2.761201×10^0	-4.291906×10^{-2}	1.015146×10^0
64	2.739629×10^0	-2.134762×10^{-2}	1.007543×10^0
128	2.728928×10^0	-1.064599×10^{-2}	1.003764×10^0
256	2.723598×10^0	-5.316064×10^{-3}	1.001880×10^0
512	2.720938×10^0	-2.656301×10^{-3}	1.000940×10^0
1024	2.719610×10^0	-1.327718×10^{-3}	1.000470×10^0
2048	2.718946×10^0	-6.637511×10^{-4}	1.000235×10^0
4096	2.718614×10^0	-3.318485×10^{-4}	1.000117×10^0

Table 4: Special forward difference for e^x .

h^{-1}	$\tilde{D}_h^+ f(1)$	$E_h = f'(1) - \tilde{D}_h^+ f(1)$	$\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$
2	2.382855×10^0	3.354271×10^{-1}	—
4	2.649675×10^0	6.860728×10^{-2}	2.289566×10^0
8	2.702716×10^0	1.556602×10^{-2}	2.139962×10^0
16	2.714572×10^0	3.710289×10^{-3}	2.068796×10^0
32	2.717376×10^0	9.059021×10^{-4}	2.034105×10^0
64	2.718058×10^0	2.238257×10^{-4}	2.016979×10^0
128	2.718226×10^0	5.562881×10^{-5}	2.008471×10^0
256	2.718268×10^0	1.386648×10^{-5}	2.004231×10^0
512	2.718278×10^0	3.461542×10^{-6}	2.002114×10^0
1024	2.718281×10^0	8.647516×10^{-7}	2.001057×10^0
2048	2.718282×10^0	2.161091×10^{-7}	2.000526×10^0
4096	2.718282×10^0	5.401482×10^{-8}	2.000332×10^0

Table 5: Backward difference for $\sqrt{x+1}$.

h^{-1}	$D_h^- f(1)$	$E_h = f'(1) - D_h^- f(1)$	$\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$
2	3.789374×10^{-1}	-2.538399×10^{-2}	—
4	3.653516×10^{-1}	-1.179824×10^{-2}	1.105348×10^0
8	3.592573×10^{-1}	-5.703958×10^{-3}	1.048536×10^0
16	3.563595×10^{-1}	-2.806156×10^{-3}	1.023368×10^0
32	3.549454×10^{-1}	-1.391964×10^{-3}	1.011473×10^0
64	3.542466×10^{-1}	-6.932446×10^{-4}	1.005686×10^0
128	3.538993×10^{-1}	-3.459430×10^{-4}	1.002830×10^0
256	3.537262×10^{-1}	-1.728023×10^{-4}	1.001412×10^0
512	3.536397×10^{-1}	-8.635892×10^{-5}	1.000705×10^0
1024	3.535966×10^{-1}	-4.316891×10^{-5}	1.000352×10^0
2048	3.535750×10^{-1}	-2.158182×10^{-5}	1.000176×10^0
4096	3.535642×10^{-1}	-1.079025×10^{-5}	1.000088×10^0

Table 6: Centered difference for $\sqrt{x+1}$.

h^{-1}	$D_h f(1)$	$E_h = f'(1) - D_h f(1)$	$\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$
2	3.563940×10^{-1}	-2.840568×10^{-3}	—
4	3.542487×10^{-1}	-6.952983×10^{-4}	2.030475×10^0
8	3.537263×10^{-1}	-1.729292×10^{-4}	2.007451×10^0
16	3.535966×10^{-1}	-4.317682×10^{-5}	2.001853×10^0
32	3.535642×10^{-1}	-1.079075×10^{-5}	2.000462×10^0
64	3.535561×10^{-1}	-2.697470×10^{-6}	2.000116×10^0
128	3.535541×10^{-1}	-6.743541×10^{-7}	2.000029×10^0
256	3.535536×10^{-1}	-1.685877×10^{-7}	2.000007×10^0
512	3.535534×10^{-1}	-4.214690×10^{-8}	2.000001×10^0
1024	3.535534×10^{-1}	-1.053678×10^{-8}	1.999992×10^0
2048	3.535534×10^{-1}	-2.634184×10^{-9}	2.000007×10^0
4096	3.535534×10^{-1}	$-6.585339 \times 10^{-10}$	2.000026×10^0

Table 7: Forward difference for $\sqrt{x+1}$.

h^{-1}	$D_h^+ f(1)$	$E_h = f'(1) - D_h^+ f(1)$	$\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$
2	3.338505×10^{-1}	1.970286×10^{-2}	—
4	3.431458×10^{-1}	1.040764×10^{-2}	9.207617×10^{-1}
8	3.481953×10^{-1}	5.358100×10^{-3}	9.578496×10^{-1}
16	3.508336×10^{-1}	2.719802×10^{-3}	9.782196×10^{-1}
32	3.521830×10^{-1}	1.370383×10^{-3}	9.889231×10^{-1}
64	3.528655×10^{-1}	6.878497×10^{-4}	9.944135×10^{-1}
128	3.532088×10^{-1}	3.445943×10^{-4}	9.971946×10^{-1}
256	3.533809×10^{-1}	1.724651×10^{-4}	9.985942×10^{-1}
512	3.534671×10^{-1}	8.627462×10^{-5}	9.992963×10^{-1}
1024	3.535102×10^{-1}	4.314784×10^{-5}	9.996480×10^{-1}
2048	3.535318×10^{-1}	2.157655×10^{-5}	9.998239×10^{-1}
4096	3.535426×10^{-1}	1.078894×10^{-5}	9.999119×10^{-1}

Table 8: Special forward difference for $\sqrt{x+1}$.

h^{-1}	$\tilde{D}_h^+ f(1)$	$E_h = f'(1) - \tilde{D}_h^+ f(1)$	$\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$
2	3.498638×10^{-1}	3.689565×10^{-3}	—
4	3.524410×10^{-1}	1.112425×10^{-3}	1.729743×10^0
8	3.532448×10^{-1}	3.085597×10^{-4}	1.850087×10^0
16	3.534719×10^{-1}	8.150493×10^{-5}	1.920590×10^0
32	3.535324×10^{-1}	2.096272×10^{-5}	1.959061×10^0
64	3.535481×10^{-1}	5.316767×10^{-6}	1.979205×10^0
128	3.535521×10^{-1}	1.338884×10^{-6}	1.989519×10^0
256	3.535531×10^{-1}	3.359440×10^{-7}	1.994738×10^0
512	3.535533×10^{-1}	8.413957×10^{-8}	1.997364×10^0
1024	3.535534×10^{-1}	2.105429×10^{-8}	1.998670×10^0
2048	3.535534×10^{-1}	5.266597×10^{-9}	1.999171×10^0
4096	3.535534×10^{-1}	1.317798×10^{-9}	1.998742×10^0