

MAT627 Chapter 2.2 Programming Assignment

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1 Finite-Difference Approximation Results

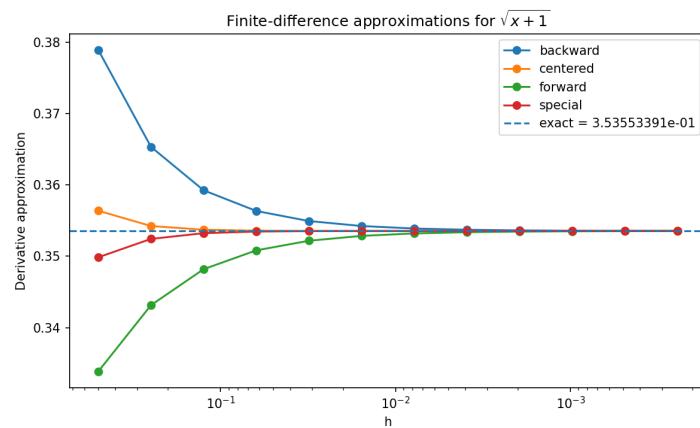


Figure 1: Approximation overlay for $\sqrt{x + 1}$.

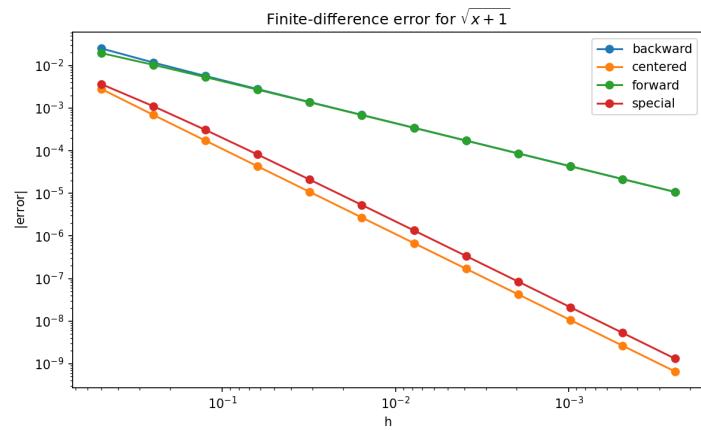


Figure 2: Error overlay for $\sqrt{x+1}$.

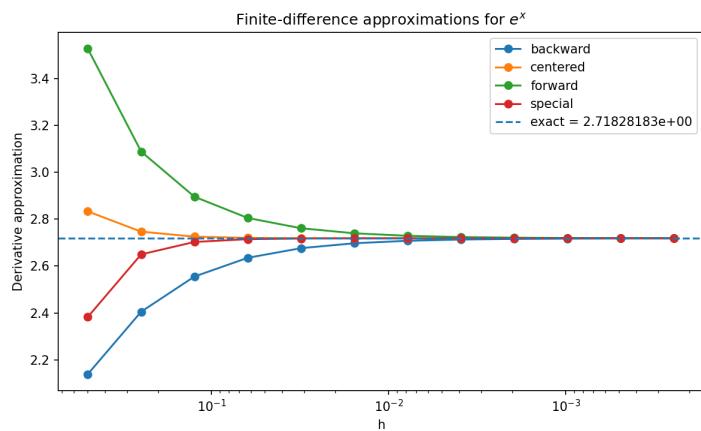


Figure 3: Approximation overlay for e^x .

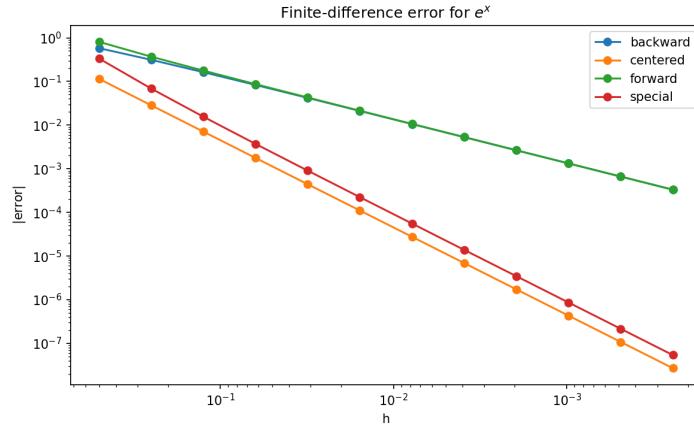


Figure 4: Error overlay for e^x .

Table 1: Backward difference for e^x .

| h^{-1} | $D_h^- f(1)$ | $E_h = f'(1) - D_h^- f(1)$ | $\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$ |
|----------|------------------------|----------------------------|---|
| 2 | 2.139121×10^0 | 5.791607×10^{-1} | — |
| 4 | 2.405127×10^0 | 3.131546×10^{-1} | 8.870888×10^{-1} |
| 8 | 2.555252×10^0 | 1.630296×10^{-1} | 9.417415×10^{-1} |
| 16 | 2.635078×10^0 | 8.320390×10^{-2} | 9.704104×10^{-1} |
| 32 | 2.676248×10^0 | 4.203416×10^{-2} | 9.850890×10^{-1} |
| 64 | 2.697155×10^0 | 2.112640×10^{-2} | 9.925153×10^{-1} |
| 128 | 2.707691×10^0 | 1.059069×10^{-2} | 9.962503×10^{-1} |
| 256 | 2.712980×10^0 | 5.302238×10^{-3} | 9.981233×10^{-1} |
| 512 | 2.715629×10^0 | 2.652845×10^{-3} | 9.990612×10^{-1} |
| 1024 | 2.716955×10^0 | 1.326854×10^{-3} | 9.995305×10^{-1} |
| 2048 | 2.717618×10^0 | 6.635350×10^{-4} | 9.997652×10^{-1} |
| 4096 | 2.717950×10^0 | 3.317945×10^{-4} | 9.998826×10^{-1} |

Table 2: Centered difference for e^x .

| h^{-1} | $D_h f(1)$ | $E_h = f'(1) - D_h f(1)$ | $\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$ |
|----------|------------------------|----------------------------|---|
| 2 | 2.832968×10^0 | -1.146860×10^{-1} | — |
| 4 | 2.746686×10^0 | -2.840405×10^{-2} | 2.013520×10^0 |
| 8 | 2.725366×10^0 | -7.084391×10^{-3} | 2.003381×10^0 |
| 16 | 2.720052×10^0 | -1.770060×10^{-3} | 2.000845×10^0 |
| 32 | 2.718724×10^0 | -4.424503×10^{-4} | 2.000211×10^0 |
| 64 | 2.718392×10^0 | -1.106085×10^{-4} | 2.000053×10^0 |
| 128 | 2.718309×10^0 | -2.765188×10^{-5} | 2.000013×10^0 |
| 256 | 2.718289×10^0 | -6.912953×10^{-6} | 2.000003×10^0 |
| 512 | 2.718284×10^0 | -1.728237×10^{-6} | 2.000001×10^0 |
| 1024 | 2.718282×10^0 | -4.320592×10^{-7} | 2.000001×10^0 |
| 2048 | 2.718282×10^0 | -1.080144×10^{-7} | 2.000005×10^0 |
| 4096 | 2.718282×10^0 | -2.700387×10^{-8} | 1.999985×10^0 |

Table 3: Forward difference for e^x .

| h^{-1} | $D_h^+ f(1)$ | $E_h = f'(1) - D_h^+ f(1)$ | $\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$ |
|----------|------------------------|----------------------------|---|
| 2 | 3.526814×10^0 | -8.085327×10^{-1} | — |
| 4 | 3.088245×10^0 | -3.699627×10^{-1} | 1.127926×10^0 |
| 8 | 2.895480×10^0 | -1.771983×10^{-1} | 1.062015×10^0 |
| 16 | 2.805026×10^0 | -8.674402×10^{-2} | 1.030529×10^0 |
| 32 | 2.761201×10^0 | -4.291906×10^{-2} | 1.015146×10^0 |
| 64 | 2.739629×10^0 | -2.134762×10^{-2} | 1.007543×10^0 |
| 128 | 2.728928×10^0 | -1.064599×10^{-2} | 1.003764×10^0 |
| 256 | 2.723598×10^0 | -5.316064×10^{-3} | 1.001880×10^0 |
| 512 | 2.720938×10^0 | -2.656301×10^{-3} | 1.000940×10^0 |
| 1024 | 2.719610×10^0 | -1.327718×10^{-3} | 1.000470×10^0 |
| 2048 | 2.718946×10^0 | -6.637511×10^{-4} | 1.000235×10^0 |
| 4096 | 2.718614×10^0 | -3.318485×10^{-4} | 1.000117×10^0 |

Table 4: Special forward difference for e^x .

| h^{-1} | $\tilde{D}_h^+ f(1)$ | $E_h = f'(1) - \tilde{D}_h^+ f(1)$ | $\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$ |
|----------|------------------------|------------------------------------|---|
| 2 | 2.382855×10^0 | 3.354271×10^{-1} | — |
| 4 | 2.649675×10^0 | 6.860728×10^{-2} | 2.289566×10^0 |
| 8 | 2.702716×10^0 | 1.556602×10^{-2} | 2.139962×10^0 |
| 16 | 2.714572×10^0 | 3.710289×10^{-3} | 2.068796×10^0 |
| 32 | 2.717376×10^0 | 9.059021×10^{-4} | 2.034105×10^0 |
| 64 | 2.718058×10^0 | 2.238257×10^{-4} | 2.016979×10^0 |
| 128 | 2.718226×10^0 | 5.562881×10^{-5} | 2.008471×10^0 |
| 256 | 2.718268×10^0 | 1.386648×10^{-5} | 2.004231×10^0 |
| 512 | 2.718278×10^0 | 3.461542×10^{-6} | 2.002114×10^0 |
| 1024 | 2.718281×10^0 | 8.647516×10^{-7} | 2.001057×10^0 |
| 2048 | 2.718282×10^0 | 2.161091×10^{-7} | 2.000526×10^0 |
| 4096 | 2.718282×10^0 | 5.401482×10^{-8} | 2.000332×10^0 |

Table 5: Backward difference for $\sqrt{x+1}$.

| h^{-1} | $D_h^- f(1)$ | $E_h = f'(1) - D_h^- f(1)$ | $\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$ |
|----------|---------------------------|----------------------------|---|
| 2 | 3.789374×10^{-1} | -2.538399×10^{-2} | — |
| 4 | 3.653516×10^{-1} | -1.179824×10^{-2} | 1.105348×10^0 |
| 8 | 3.592573×10^{-1} | -5.703958×10^{-3} | 1.048536×10^0 |
| 16 | 3.563595×10^{-1} | -2.806156×10^{-3} | 1.023368×10^0 |
| 32 | 3.549454×10^{-1} | -1.391964×10^{-3} | 1.011473×10^0 |
| 64 | 3.542466×10^{-1} | -6.932446×10^{-4} | 1.005686×10^0 |
| 128 | 3.538993×10^{-1} | -3.459430×10^{-4} | 1.002830×10^0 |
| 256 | 3.537262×10^{-1} | -1.728023×10^{-4} | 1.001412×10^0 |
| 512 | 3.536397×10^{-1} | -8.635892×10^{-5} | 1.000705×10^0 |
| 1024 | 3.535966×10^{-1} | -4.316891×10^{-5} | 1.000352×10^0 |
| 2048 | 3.535750×10^{-1} | -2.158182×10^{-5} | 1.000176×10^0 |
| 4096 | 3.535642×10^{-1} | -1.079025×10^{-5} | 1.000088×10^0 |

 Table 6: Centered difference for $\sqrt{x+1}$.

| h^{-1} | $D_h f(1)$ | $E_h = f'(1) - D_h f(1)$ | $\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$ |
|----------|---------------------------|-----------------------------|---|
| 2 | 3.563940×10^{-1} | -2.840568×10^{-3} | — |
| 4 | 3.542487×10^{-1} | -6.952983×10^{-4} | 2.030475×10^0 |
| 8 | 3.537263×10^{-1} | -1.729292×10^{-4} | 2.007451×10^0 |
| 16 | 3.535966×10^{-1} | -4.317682×10^{-5} | 2.001853×10^0 |
| 32 | 3.535642×10^{-1} | -1.079075×10^{-5} | 2.000462×10^0 |
| 64 | 3.535561×10^{-1} | -2.697470×10^{-6} | 2.000116×10^0 |
| 128 | 3.535541×10^{-1} | -6.743541×10^{-7} | 2.000029×10^0 |
| 256 | 3.535536×10^{-1} | -1.685877×10^{-7} | 2.000007×10^0 |
| 512 | 3.535534×10^{-1} | -4.214690×10^{-8} | 2.000001×10^0 |
| 1024 | 3.535534×10^{-1} | -1.053678×10^{-8} | 1.999992×10^0 |
| 2048 | 3.535534×10^{-1} | -2.634184×10^{-9} | 2.000007×10^0 |
| 4096 | 3.535534×10^{-1} | $-6.585339 \times 10^{-10}$ | 2.000026×10^0 |

Table 7: Forward difference for $\sqrt{x+1}$.

| h^{-1} | $D_h^+ f(1)$ | $E_h = f'(1) - D_h^+ f(1)$ | $\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$ |
|----------|---------------------------|----------------------------|---|
| 2 | 3.338505×10^{-1} | 1.970286×10^{-2} | — |
| 4 | 3.431458×10^{-1} | 1.040764×10^{-2} | 9.207617×10^{-1} |
| 8 | 3.481953×10^{-1} | 5.358100×10^{-3} | 9.578496×10^{-1} |
| 16 | 3.508336×10^{-1} | 2.719802×10^{-3} | 9.782196×10^{-1} |
| 32 | 3.521830×10^{-1} | 1.370383×10^{-3} | 9.889231×10^{-1} |
| 64 | 3.528655×10^{-1} | 6.878497×10^{-4} | 9.944135×10^{-1} |
| 128 | 3.532088×10^{-1} | 3.445943×10^{-4} | 9.971946×10^{-1} |
| 256 | 3.533809×10^{-1} | 1.724651×10^{-4} | 9.985942×10^{-1} |
| 512 | 3.534671×10^{-1} | 8.627462×10^{-5} | 9.992963×10^{-1} |
| 1024 | 3.535102×10^{-1} | 4.314784×10^{-5} | 9.996480×10^{-1} |
| 2048 | 3.535318×10^{-1} | 2.157655×10^{-5} | 9.998239×10^{-1} |
| 4096 | 3.535426×10^{-1} | 1.078894×10^{-5} | 9.999119×10^{-1} |

Table 8: Special forward difference for $\sqrt{x+1}$.

| h^{-1} | $\tilde{D}_h^+ f(1)$ | $E_h = f'(1) - \tilde{D}_h^+ f(1)$ | $\frac{\ln \left \frac{E_{2h}}{E_h} \right }{\ln 2}$ |
|----------|---------------------------|------------------------------------|---|
| 2 | 3.498638×10^{-1} | 3.689565×10^{-3} | — |
| 4 | 3.524410×10^{-1} | 1.112425×10^{-3} | 1.729743×10^0 |
| 8 | 3.532448×10^{-1} | 3.085597×10^{-4} | 1.850087×10^0 |
| 16 | 3.534719×10^{-1} | 8.150493×10^{-5} | 1.920590×10^0 |
| 32 | 3.535324×10^{-1} | 2.096272×10^{-5} | 1.959061×10^0 |
| 64 | 3.535481×10^{-1} | 5.316767×10^{-6} | 1.979205×10^0 |
| 128 | 3.535521×10^{-1} | 1.338884×10^{-6} | 1.989519×10^0 |
| 256 | 3.535531×10^{-1} | 3.359440×10^{-7} | 1.994738×10^0 |
| 512 | 3.535533×10^{-1} | 8.413957×10^{-8} | 1.997364×10^0 |
| 1024 | 3.535534×10^{-1} | 2.105429×10^{-8} | 1.998670×10^0 |
| 2048 | 3.535534×10^{-1} | 5.266597×10^{-9} | 1.999171×10^0 |
| 4096 | 3.535534×10^{-1} | 1.317798×10^{-9} | 1.998742×10^0 |