eBook Tools

Applet

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13.22 Binomial setting? In each situation below, is it reasonable to use a **binomial distribution** for the **random variable** *X*? Give reasons for your answer in each case.

(a) An auto manufacturer chooses one car from each hour's production for a detailed quality inspection. One **variable** recorded is the count *X* of finish defects (dimples, ripples, etc.) in the car's paint.

(b) The pool of potential jurors for a murder case contains 100 persons chosen at random from the adult residents of a large city. Each person in the pool is asked whether he or she opposes the death penalty; *X* is the number who say "Yes."

(c) Joe buys a ticket in his state's Pick 3 lottery game every week; *X* is the number of times in a year that he wins a prize.

Binomial setting? A **binomial distribution** will be approximately correct as a model for one of these two sports settings and not for the other. Explain why by briefly discussing both settings.

(a) A National Football League kicker has made 80% of his field goal attempts in the past. This season he attempts 20 field goals. The attempts differ widely in distance, angle, wind, and so on.

(b) A National Basketball Association player has made 80% of his free-throw attempts in the past. This season he takes 150 free throws. Basketball free throws are always attempted from 15 feet away with no interference from other players.

Show Answer

13.24 Testing ESP. In a test for ESP (extrasensory perception), a **subject** is told that cards the experimenter can see but he cannot contain either a star, a circle, a wave, or a square. As the experimenter looks at each of 20 cards in turn, the **subject** names the shape on the card. A **subject** who is just guessing has **probability** 0.25 of guessing correctly on each card.

(a) The count of correct guesses in 20 cards has a **binomial distribution**. What are *n* and *p*?

(b) What is the **mean** number of correct guesses in many repetitions of the experiment?

(c) What is the **probability** of exactly 5 correct guesses?

13.25 Random stock prices. A believer in the random walk theory of stock markets thinks that an index of stock prices has **probability** 0.65 of increasing in any year. Moreover, the change in the index in any given year is not influenced by whether it rose or fell in earlier years. Let *X* be the number of years among the next 5 years in which the index rises.

(a) *X* has a **binomial distribution**. What are *n* and *p*?

(b) What are the possible values that *X* can take?

(c) Find the **probability** of each value of *X*. Draw a **probability** histogram for the **distribution** of *X*. (See **Figure 13.2** for an example of a **probability** histogram.)

(d) What are the **mean** and **standard deviation** of this distribution? Mark the location of the **mean** on your histogram.

Show Answer

13.26 On the Web. What kinds of Web sites do males aged 18 to 34 visit? About 50% of male Internet users in this age group visit an auction site such as eBay at least once a month.[4] Interview a random **sample** of 12 male Internet users aged 18 to 34.

(a) What is the **distribution** of the number who have visited an online auction site in the past month?

(b) What is the **probability** that exactly 8 of the 12 have visited an auction site in the past month? If you have software, also find the **probability** that at least 8 of the 12 have visited an auction site in the past month.

13.27 The pill. Many women take oral contraceptives to prevent pregnancy. Under ideal conditions, 1% of women taking the pill become pregnant within one year. In typical use, however, 5% become pregnant.[5] Choose at random 20 women taking the pill. How many become pregnant in the next year?

(a) Explain why this is a binomial setting.

(b) What is the **probability** that at least one of the women becomes pregnant under ideal conditions? What is the **probability** in typical use?

Show Answer

13.28 On the Web, continued. A study of Internet usage interviews a random **sample** of 500 men aged 18 to 34. Based on the information in Exercise 13.26, what is the **probability** that at least 235 of the men in the **sample** visit an online auction site at least once a month? (Check that the **Normal approximation** is permissible and use it to find this **probability**. If your software allows, find the exact binomial **probability** and compare the two results.)

13.29 The pill, continued. A study of the effectiveness of oral contraceptives interviews a random **sample** of 500 women who are taking the pill.

(a) Based on the information about typical use in Exercise 13.27, what is the **probability** that at least 25 of these women become pregnant in the next year? (Check that the **Normal approximation** is permissible and use it to find this **probability**. If your software allows, find the exact binomial **probability** and compare the two results.)

(b) We can't use the **Normal approximation** to the **binomial distributions** to find this **probability** under ideal conditions as described in Exercise 13.27. Why not?

Show Answer

13.30 Hitting the fairway. One **statistic** used to assess professional golfers is driving accuracy, the percent of drives that land in the fairway. Driving accuracy for PGA Tour professionals ranges from about 40% to about 75%. Tiger Woods hits the fairway about 60% of the time.[6]

(a) Tiger hits 14 drives in a round. What assumptions must you make in order to use a **binomial distribution** for the count X of fairways he hits? Which of these assumptions is least realistic?

(b) Assuming that a **binomial distribution** can be used, what is the most likely number of fairways that Tiger hits in a round in which he hits 14 drives?

13.31 Genetics. According to genetic theory, the blossom color in the second generation of a certain cross of sweet peas should be red or white in a 3:1 ratio. That is, each plant has **probability** 3/4 of having red blossoms, and the blossom colors of separate plants are independent.

(a) What is the **probability** that exactly 6 out of 8 of these plants have red blossoms?

(b) What is the **mean** number of red-blossomed plants when 80 plants of this type are grown from seeds?

(c) What is the **probability** of obtaining at least 60 red-blossomed plants when 80 plants are grown from seeds? Use the **Normal approximation**. If your software allows, find the exact binomial **probability** and compare the two results.

Show Answer

13.32 False positives in testing for HIV. A rapid test for the presence in the blood of antibodies to HIV, the virus that causes AIDS, gives a positive result with **probability** about 0.004 when a person who is free of HIV antibodies is tested. A clinic tests 1000 people who are all free of HIV antibodies.

(a) What is the **distribution** of the number of positive tests?

(b) What is the **mean** number of positive tests?

(c) You cannot safely use the **Normal approximation** for this **distribution**. Explain why.

13.33 High school equivalency. The Census Bureau says that 21% of Americans aged 18 to 24 do not have a high school diploma. A vocational school wants to attract young people who may enroll in order to achieve high school equivalency. The school mails an advertising flyer to 25,000 persons between the ages of 18 and 24.

(a) If the mailing list can be considered a random **sample** of the **population**, what is the **mean** number of high school dropouts who will receive the flyer?

(b) What is the approximate **probability** that at least 5000 dropouts will receive the flyer?

Show Answer

13.34 Survey demographics. According to the Census Bureau, 13% of American adults (age 18 and over) are Hispanic. An opinion poll plans to contact an SRS of 1200 adults.

(a) What is the **mean** number of Hispanics in such samples? What is the standard deviation?

(b) According to the 68–95–99.7 rule, what range will include the counts of Hispanics in 95% of all such samples?

(c) How large a **sample** is required to make the **mean** number of Hispanics at least 200?

13.35 Multiple-choice tests. Here is a simple **probability model** for multiple-choice tests. Suppose that each student has **probability** p of correctly answering a question chosen at random from a universe of possible questions. (A strong student has a higher p than a weak student.) Answers to different questions are independent.

(a) Jodi is a good student for whom $p = 0.75$. Use the **Normal approximation** to find the **probability** that Jodi scores between 70% and 80% on a 100-question test.

(b) If the test contains 250 questions, what is the **probability** that Jodi will score between 70% and 80%? You see that Jodi's score on the longer test is more likely to be close to her "true score."

Show Answer

13.36 Is this coin balanced? While he was a prisoner of war during World War II, John Kerrich tossed a coin 10,000 times. He got 5067 heads. If the coin is perfectly balanced, the **probability** of a head is 0.5. Is there reason to think that Kerrich's coin was not balanced? To answer this question, find the **probability** that tossing a balanced coin 10,000 times would give a count of heads at least this far from 5000 (that is, at least 5067 heads or no more than 4933 heads.)

13.37 Binomial variation. Never forget that **probability** describes only what happens in the long run. Example 13.5 concerns the count of bad CDs in inspection **samples** of size 10. The count has the **binomial distribution** with $n = 10$ and $p = 0.1$. The ***Probability*** applet simulates inspecting a lot of CDs if you set the **probability** of heads to 0.1, toss 10 times, and let each head stand for a bad CD.

(a) The **mean** number of bad CDs in a **sample** is 1. Click "Toss" and "Reset" repeatedly to simulate 20 **samples**. How many bad CDs did you find in each sample? How close to the **mean** 1 is the average number of bad CDs in these samples?

(b) Example 13.5 shows that the **probability** of exactly 1 bad CD is 0.3874. How close to the **probability** is the proportion of the 20 lots that have exactly 1 bad CD?

Whooping cough. *Whooping cough (pertussis) is a highly contagious bacterial infection that was a major cause of childhood deaths before the development of vaccines. About 80% of unvaccinated children who are exposed to whooping cough will develop the infection, as opposed to only about 5% of vaccinated children. Exercises 13.38 to 13.41 are based on this information.*

13.38 Vaccination at work. A group of 20 children at a nursery school are exposed to whooping cough by playing with an infected child.

(a) If all 20 have been vaccinated, what is the **mean** number of new infections? What is the **probability** that no more than 2 of the 20 children develop infections?

(b) If none of the 20 have been vaccinated, what is the **mean** number of new infections? What is the **probability** that 18 or more of the 20 children develop infections?

13.39 A whooping cough outbreak. In 2007, Bob Jones University ended its fall semester a week early because of a whooping cough outbreak; 158 students were isolated and another 1200 given antibiotics as a precaution.[7] Authorities react strongly to whooping cough outbreaks because the disease is so contagious. Because the effect of childhood vaccination often wears off by late adolescence, treat the Bob Jones students as if they were unvaccinated. It appears that about 1400 students were exposed. What is the **probability** that at least 75% of these students develop infections if not treated? (Fortunately, whooping cough is much less serious after infancy.)

Show Answer

13.40 A mixed group: means. A group of 20 children at a nursery school are exposed to whooping cough by playing with an infected child. Of these children 17 have been vaccinated and 3 have not.

(a) What is the **distribution** of the number of new infections among the 17 vaccinated children? What is the **mean** number of new infections?

(b) What is the **distribution** of the number of new infections among the 3 unvaccinated children? What is the **mean** number of new infections?

(c) Add your **means** from parts (a) and (b). This is the **mean** number of new infections among all 20 exposed children.

13.41 A mixed group: probabilities. We would like to find the **probability** that exactly 2 of the 20 exposed children in the previous exercise develop whooping cough.

(a) One way to get 2 infections is to get 1 among the 17 vaccinated children and 1 among the 3 unvaccinated children. Find the **probability** of exactly 1 infection among the 17 vaccinated children. Find the **probability** of exactly 1 infection among the 3 unvaccinated children. These **events** are independent: what is the **probability** of exactly 1 infection in each group?

(b) Write down all the ways in which 2 infections can be divided between the two groups of children. Follow the pattern of part (a) to find the **probability** of each of these possibilities. Add all of your results (including the result of part (a)) to obtain the **probability** of exactly 2 infections among the 20 children.

Show Answer

13.42 Estimating π from random numbers. Kenyon College student Eric Newman used basic geometry to evaluate software random number generators as part of a summer research project. He generated 2000 independent random points (X, Y) in the unit square. (That is, X and Y are independent random numbers between 0 and

1, each having the density function illustrated in **Figure 10.4** (**page 274**). The **probability** that (X, Y) falls in any region within the unit square is the area of the region.)[8]

(a) Sketch the unit square, the region of possible values for the point (X, Y).

(b) The set of points (X, Y) where $X^2 + Y^2 < 1$ describes a circle of radius 1. Add this circle to your sketch in part (a) and label the intersection of the two regions A.

(c) Let T be the total number of the 2000 points that fall into the region A. T follows a **binomial distribution**. Identify n and p. (*Hint:* Recall that the area of a circle is πr^2.)

(d) What are the **mean** and **standard deviation** of T?

(e) Explain how Eric used a random number generator and the facts above to estimate π.

13.43 The continuity correction. One reason why the **Normal approximation** may fail to give accurate estimates of binomial **probabilities** is that the **binomial distributions** are discrete and the **Normal distributions** are continuous. That is, counts take only whole number values but Normal **variables** can take any value. We can improve the **Normal approximation** by treating each whole number count as if it occupied the interval from 0.5 below the number to 0.5 above the number. For example, approximate a binomial **probability** $P(X \geq 10)$ by finding the Normal **probability** $P(X \geq 9.5)$. Be careful: binomial $P(X > 10)$ is approximated by Normal $P(X \geq 10.5)$.

We saw in Exercise 13.30 that Tiger Woods hits the fairway in 60% of his drives. We will assume that his drives are independent and that each has **probability** 0.6 of hitting the fairway. Tiger drives 25 times. The exact binomial **probability** that he hits 15 or more fairways is 0.5858.

(a) Show that this setting satisfies the rule of thumb for use of the **Normal approximation** (just barely).

(b) What is the **Normal approximation** to $P(X \geq 15)$?

(c) What is the **Normal approximation** using the continuity correction? That's a lot closer to the true binomial **probability**.

Show Answer

CHAPTER 13 EXERCISES