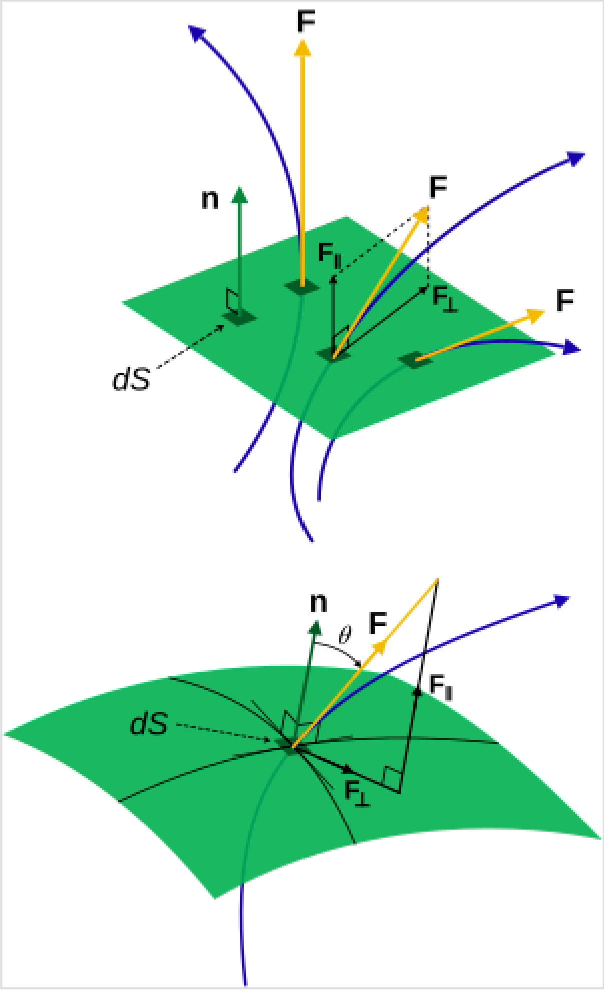
List of equations in fluid mechanics

This article summarizes equations in the theory of fluid mechanics.

Definitions



Flux **F** through a surface, d**S** is the differential vector area element, **n** is the unit normal to the surface. **Left:** No flux passes in the surface, the maximum amount flows normal to the surface. **Right:** The reduction in flux passing through a surface can be visualized by reduction in **F** or d**S** equivalently (resolved into components, θ is angle to normal **n**). **F**•d**S** is the component of flux passing through the surface, multiplied by the area of the surface (see dot product). For this reason flux represents physically a flow *per unit area*.

Here $\hat{\mathbf{t}}$ is a unit vector in the direction of the flow/current/flux.

Quantity (common name/s)	(Common) symbol/s	Defining equation	SI units	Dimension
Flow velocity vector field	**u**	$\mathbf{u} = \mathbf{u}\left(\mathbf{r}, t\right)$	m s^{-1}	[L][T]$^{-1}$
Velocity pseudovector field	**ω**	$\boldsymbol{\omega} = \nabla \times \mathbf{v}$	s^{-1}	[T]$^{-1}$
Volume velocity, volume flux	φ_V (no standard symbol)	$\phi_V = \int_S \mathbf{u} \cdot \mathrm{d}\mathbf{A}$	m^3 s^{-1}	[L]^3 [T]$^{-1}$
Mass current per unit volume	*s* (no standard symbol)	$s = \mathrm{d}\rho / \mathrm{d}t$	kg m^{-3} s^{-1}	[M] [L]$^{-3}$ [T]$^{-1}$
Mass current, mass flow rate	I_m	$I_\mathrm{m} = \mathrm{d}m / \mathrm{d}t$	kg s^{-1}	[M][T]$^{-1}$
Mass current density	$\mathbf{j}_\mathrm{m}$	$I_\mathrm{m} = \iint \mathbf{j}_\mathrm{m} \cdot \mathrm{d}\mathbf{S}$	kg m^{-2} s^{-1}	[M][L]$^{-2}$[T]$^{-1}$
Momentum current	I_p	$I_\mathrm{p} = \mathrm{d}\left\|\mathbf{p}\right\| / \mathrm{d}t$	kg m s^{-2}	[M][L][T]$^{-2}$
Momentum current density	$\mathbf{j}_\mathrm{p}$	$I_\mathrm{p} = \iint \mathbf{j}_\mathrm{p} \cdot \mathrm{d}\mathbf{S}$	kg m s^{-2}	[M][L][T]$^{-2}$

Equations

Physical situation	Nomenclature	Equations
Fluid statics, pressure gradient	**r** = Position $\rho = \rho(\mathbf{r})$ = Fluid density at gravitational equipotential containing **r** $\mathbf{g} = \mathbf{g}(\mathbf{r})$ = Gravitational field strength at point **r** ∇P = Pressure gradient	$\nabla P = \rho \mathbf{g}$
Buoyancy equations	ρ_f = Mass density of the fluid V_imm = Immersed volume of body in fluid $\mathbf{F}_\mathrm{b}$ = Buoyant force $\mathbf{F}_\mathrm{g}$ = Gravitational force $\mathbf{W}_\mathrm{app}$ = Apparent weight of immersed body **W** = Actual weight of immersed body	Buoyant force $\mathbf{F}_\mathrm{b} = -\rho_f V_\mathrm{imm} \mathbf{g} = -\mathbf{F}_\mathrm{g}$ Apparent weight $\mathbf{W}_\mathrm{app} = \mathbf{W} - \mathbf{F}_\mathrm{b}$
Bernoulli's equation	p_constant is the total pressure at a point on a streamline	$p + \rho u^2/2 + \rho g y = p_\mathrm{constant}$
Euler equations	ρ = fluid mass density **u** is the flow velocity vector *E* = total volume energy density *U* = internal energy per unit mass of fluid *p* = pressure ⊗ denotes the tensor product	$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$

		$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes (\rho \mathbf{u})) + \nabla p = 0$ $\frac{\partial E}{\partial t} + \nabla \cdot (\mathbf{u}\,(E + p)) = 0$ $E = \rho \left(U + \frac{1}{2} \mathbf{u}^2 \right)$
Convective acceleration		$\mathbf{a} = (\mathbf{u} \cdot \nabla)\, \mathbf{u}$
Navier–Stokes equations	$\mathbf{T}_\mathrm{D}$ = Deviatoric stress tensor $\mathbf{f}$ = volume density of the body forces acting on the fluid ∇ here is the del operator.	$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot \mathbf{T}_\mathrm{D} + \mathbf{f}$

See also

- Defining equation (physical chemistry)
- List of electromagnetism equations
- List of equations in classical mechanics
- List of equations in gravitation
- List of equations in nuclear and particle physics
- List of equations in quantum mechanics
- List of photonics equations
- List of relativistic equations
- Table of thermodynamic equations

Sources

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Further reading

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