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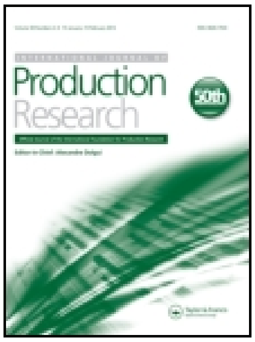

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Economic cost models of integrated APC controlled SPC charts

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In this paper, an economic cost model is proposed for processes integrating both automatic process control (APC) and statistical process control (SPC) for quality monitoring and control. Both the special cause and common cause variations are reduced by applying integrated APC and SPC. Traditionally, the integrated processes using APC and SPC are evaluated by the average run length (ARL). However, ARL may not be appropriate as a measurement of the economic design since it does not take into consideration the run length variation. Also, there are few studies that compare the cost models of such an integrated control system and the effect of cost parameters using different APC controllers. Therefore, we develop an economic cost model using non-homogenous Poisson process to describe the occurrence of an APC adjustment and develop a long run expected cost to investigate the use of different controllers in such integrated systems. Numerical examples are presented to demonstrate the applicability of the proposed model.

Keywords: automatic process control; economic design; integration; long run expected cost; non-homogenous Poisson process; statistical process control

1. Introduction

Process variability occurs due to common and unavoidable causes and due to special, assignable, and avoidable causes. The common cause variation is inherent in the process and is generally difficult to reduce by statistical process control methods. The effective approach for minimising its effect is to compensate the process by the appropriate forecast of future observations. However, if the common cause variation can be modelled as an auto-correlated process, it can be reduced by the implementation of automatic process control methods through feedback/feed-forward control schemes (Jiang and Farr 2007). On the other hand, the special causes occur at random times due to assignable reasons. The special causes make the process level shift away from the target and the process cannot be restored to its normal condition until the special cause is removed. The special cause variation can be eliminated by the implementation of statistical process control methods through identification and elimination of the root cause of the process changes. Both the common causes and the special causes are the two major sources of variation which make the process level shift away from the target.

Statistical process control (SPC) is often used to monitor process quality by the implementation of appropriate control charts such as Shewhart, CUSUM, and EWMA. As stated, SPC is effective in detecting changes due to special causes. On the other hand, the automatic process control (APC) is used to compensate for variations due to the common causes and attempts to continuously adjust the process level close to the target or set points. The adjustments of the APC process can be made using controllers with different properties. Examples of the APC controller include the minimum mean square estimate (MMSE) controller and the proportional, integral and derivative (PID) controller. APC controllers are more tactical in nature with a primary focus on continuous adjustments of the process so that its level is maintained at the set points or set target level. Although SPC and APC have a common objective (process variability reduction), they have been considered as disjoint activities in most of the studies. This is mainly due to the fact that SPC is viewed as a statistical based tool whereas APC is viewed as an engineering tool (Elsayed *et al*. 1995). The SPC and APC approaches are complementary to each other. The SPC is useful in monitoring long-term process performance and removing causes of process disturbances. The APC is more useful for short-term adjustments and in compensating for process disturbances (Elsayed *et al*. 1995, Tsung and Shi 1999, Tsung and Tsui 2003). The integration of SPC and APC has been known to be better than either of them alone

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to control the processes (Tsung *et al.* 1998, Tsung and Shi 1999, Jiang and Tsui 2000, Schippers 2001, Jiang *et al.* 2002, Duffuaa *et al.* 2004, Tsung *et al.* 2006, Kandananond 2007, Yang and Sheu 2007, Park and Reynolds Jr 2008, Park 2010). In this paper, we develop an economic cost model for the integration of SPC and APC and investigate the effect of the process parameters on the long term expected cost.

Many researchers have studied economic cost models to develop the economic design of SPC only (Lorenzen and Vance 1986, Elsayed and Chen 1994, Bai and Lee 1998, Chou *et al.* 2002, Ben-Daya and Duffuaa 2003, Chen 2004, Bassetto and Penz 2009), APC only (Kackar 1985, Phadke and Dehnad 1988, Wang and Yue 2001) and integration of SPC and APC (Jiang and Tsui 2000, Schippers 2001, Chou *et al.* 2002, Duffuaa *et al.* 2004, Kandananond 2007, Yang and Sheu 2007, Park and Reynolds Jr 2008). Elsayed and Chen (1994) propose an economic model for $\bar{X}$ control chart using the quadratic loss function. Their model is based on Duncan's (1956) economic design of the $\bar{X}$ charts. Models for the economic design of the integration of SPC and APC have been limited but it has gained significant interest in recent years. Although, traditionally the average run length (ARL) has been used as a measurement of performance, ARL may not be appropriate as a performance measure for the design since it does not take into consideration the run length variation. Therefore, in this paper we use and develop long run expected cost (LREC) as an alternative.

In the process industries, quality observations are often auto-correlated and continually drifting away from the target. The inherent wandering of the process arises from naturally occurring phenomena, or from unknown causes, thus cannot be removed economically or technologically. To control and investigate these processes, stationary time series models have been widely used as disturbance models in the integration of SPC and APC such as ARMA model (Tsung *et al.* 1998, Tsung and Shi 1999, Jiang and Tsui 2000, Tsung and Tsui 2003, Jiang 2004), ARIMA model (Tsung *et al.* 1998, Park and Reynolds Jr 2008) and IMA model (Tsung and Shi 1999, Wang and Yue 2001, Park 2007). IMA is a special form of ARIMA model. Among them and following Vander Wiel *et al.* (1992) and Jiang (2004) we use stable ARMA(1,1) model to represent the stationary behaviour of a wide variety of processes encountered in industry.

There are many controllers for APC adjustments such as adaptive, feedback and feed-forward based on the adjustment time and set points. Moreover, the controller's response varies from quick to slow (gradual) depending on the production process characteristics and the controller's parameters. In the quick response mode, the controller compensates for a disturbance only when a large disturbance is detected. Whereas gradual response is used in processes that experience gradual drifts. Also based on their types, controllers can be classified as MMSE, EWMA, PID, etc. Further, when an adjustment is made at each monitoring point, it is referred to as repeated adjustment and when an adjustment is made after observing an output deviation, it is referred to as feedback adjustment.

In this paper, the efficiency and robustness of discrete PI schemes are studied. Moreover, we compare the performances of the PI schemes with that of MMSE schemes. We also develop an economic cost model that compares the long run expected cost of both MMSE and PI controllers. We investigate the conditions when a controller is preferred to others.

In summary, the objectives of this paper are to develop long run expected cost models for MMSE and PI controllers, compare their performance and investigate the sensitivity of the proposed models to process parameters. As a novel approach, a non-homogeneous Poisson process (NHPP) is used to model the non-constant occurrence rate of an APC adjustment. Also, compared with the stationary Poisson process, the NHPP does not require the condition of stationary increments. Therefore, it makes the model more realistic and flexible to present the occurrence of APC adjustments.

The remainder of this paper is organised as follows. In Section 2, the research problem and the structures of SPC and APC are described. In Section 3, different APC controllers are investigated and compared. In Section 4, the economic cost model is developed using several APC controllers. The numerical examples with sensitivity analysis are given in Section 5. We discuss several aspects and applicability of the developed model in Section 6. Finally, concluding remarks are given in Section 7.

2. Model consideration

2.1 *Problem description*

Cost models for the integration of SPC and APC have attracted the attention of industrial practitioners. The goal of SPC is to monitor and detect process variability so that the special causes of the process shifting are investigated, whereas the goal of APC is to adjust the process parameters using appropriate controllers. The tasks of process monitoring and deviation detection as well as process adjustments represent a total quality control process. Recent implementations of the integrated SPC and APC processes resulted in significant reduction in the process output variability.

We develop an economic model for these integrated processes. As suggested by Box and Kramer (1992), it is possible to reduce both the special cause and common cause variations by applying SPC methods to monitor the output of an APC-controlled process. In practice, when an APC control scheme is applied to reduce the systematic variation, it also unintentionally compensates for the special cause process shift at the same time. This makes it difficult to apply standard SPC methods to detect the process shift. However, Box and Kramer (1992) point out that it is important to identify this type of process shift so that the engineer can understand and eliminate the root cause and thus improve the long term performance of the process. Thus, in this paper we consider APC cycles separately in the SPC cycle.

To determine appropriate SPC charts for monitoring APC controlled processes, common criteria are needed to evaluate the performance of the SPC methods. The ARL is the expected value of the run length distribution and it is a common criterion for evaluating the performance of a control chart. However, when the process mean is dynamic, as Lin and Adams (1996) point out, the ARL may not be complete as it does not take into consideration the run length variation. Besides the ARL measure, other measures such as the average time to signal (Luo *et al.* 2009, Sparks *et al.* 2010), the average quality cost (Jiang and Tsui 2000) and long run expected unit time (Park and Reynolds Jr 2008, Park 2010) are used in performance evaluation. Motivated by the importance of early signalling of process shifts, Lin and Adams (1996) propose the cumulative distribution function of the run length as an alternative to the ARL. Other researchers (Elsayed and Chen 1994, Montgomery *et al.* 1994, Ben-Daya and Duffuaa 2003, Duffuaa *et al.* 2004) consider the quadratic loss function. Jiang and Tsui (2000) extend the quadratic loss idea and develop an economic loss based criterion, the average quality cost, to evaluate the performance of SPC charts. Some researchers (Lorenzen and Vance 1986, Elsayed and Chen 1994, Jiang and Tsui 2000, Wang and Yue 2001, Park 2007, Park and Reynolds Jr 2008) use the expected cost per cycle to evaluate the performance of the integrated SPC and APC quality control system. Jiang and Tsui (2002) develop and compare SPC monitoring of APC controlled processes. The LREC derived in this paper is an alternative to these measures since it considers the run length variation as well as the dynamic nature of the output mean shift pattern.

The efficiency of the PI controllers compared to MMSE controllers is investigated by Jiang and Tsui (2002) and Jiang (2004). Under the ARMA (1,1) disturbance model, Tsung *et al.* (1998) discuss the optimal design of PI controllers and show that PI controllers can have approximately the same performance as the MMSE controllers in reducing process variation.

2.2 *Assumptions*

- Two or more APC adjustments do not occur at the same time.
- Special causes are perfectly identified and no process adjustments are made for false signals.
- One special cause is considered in a production cycle.
- The occurrence of APC adjustment follows an NHPP with rate $\lambda(t)$.
- The occurrence of special causes follows a geometric distribution with a probability p.

2.3 *Structure of the integration of SPC and APC*

The structure of the integration of SPC and APC is considered in this section. We consider that there are several APC adjustments in one SPC adjustment cycle which is a production cycle. Figure 1 describes a cycle of the APC

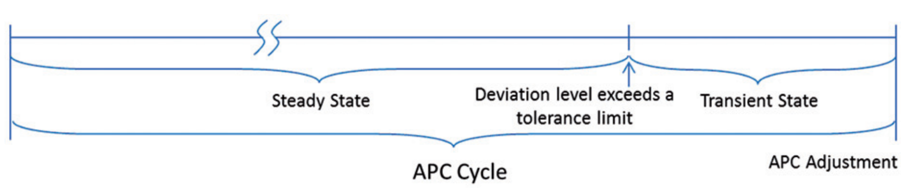


Figure 1. APC cycle.

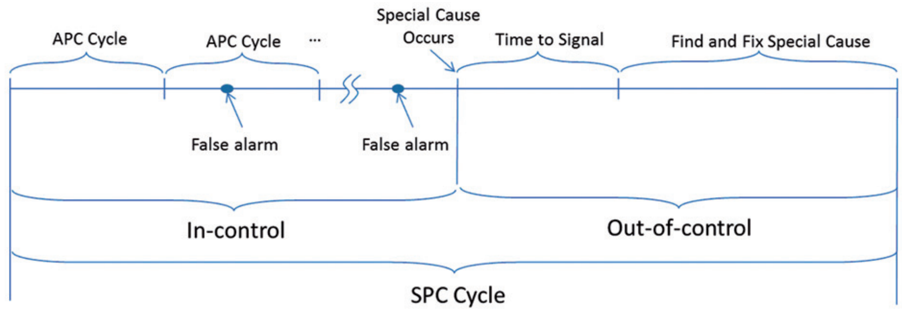


Figure 2. SPC and APC cycle.

including the steady state and the transient state. The steady state period is defined as the time from the beginning of the interval to the time when the deviation level from common cause exceeds a tolerance limit. The transient state period is the period from the time when the deviation level from common cause exceeds a tolerance limit to the time when the APC controller is used.

Similarly, the cycle of integration of SPC and APC is shown in Figure 2. There are two periods: IC and OC. The IC period is defined as a period from the beginning of the interval to the time when special cause occurs. The OC period is the interval from the time when the special cause occurs to the time when it is removed. The OC period consists of time to detect and remove the special cause.

As described in Figure 2, several APC cycles could exist in an SPC cycle. However we consider only one SPC cycle in a production cycle for simplicity. The total cost of a production cycle c_{Total} consists of two parts: the IC cost c_{in} and the OC cost c_{out} and is expressed as:

$$c_{Total} = c_{in} + c_{out} = c_{APC} + c_{SPC} \tag{1}$$

where c_{APC} is a cost for APC and c_{SPC} is a cost for SPC in a production cycle, respectively.

The cost associated with a production cycle consists of four categories:

(i) False signal cost per unit, c_F.
(ii) Diagnosis cost associated with identifying special causes from out-of-control signals, c_M.
(iii) Scrapping and reworking cost, c_A.
(iv) Disturbance cost associated with production of nonconforming items in the SPC cycle, c_D.

In next section, we investigate the APC controllers.

3. Controllers for automatic process control

3.1 *MMSE controller*

The MMSE controller is an application of the one-step-ahead forecasting statistic to compensate for disturbance with the objective of minimising the mean squared error of the output (Jiang and Tsui 2000, Tsung and Tsui 2003). It is found that MMSE controllers show better performance when the process parameters are estimated perfectly. However, it is difficult to estimate the parameters perfectly in the real world. We consider a process under discrete feedback control and let the process output e_t be given by

$$e_t = Z_t + D_t \tag{2}$$

where Z_t is the output from the process dynamics and D_t is the process disturbance. The ARMA (1,1) model is popular in modelling the noise disturbance and is expressed as (Vander Wiel *et al.* 1992)

$$D_t = \phi D_{t-1} + a_t - \theta a_{t-1}, \tag{3}$$

where parameters ϕ and θ are the autoregressive and moving average coefficients, respectively. The process output e_t can be described by $D_t + \gamma(B)X_{t-1}$, where X_t is the control action employed with transfer function $\gamma(B)$. The system is also assumed to have a one unit time delay, i.e. $\gamma(B) = -1$ and $e_t = D_t - X_{t-1}$. The quality target is assumed to be zero, without loss of generality. Box *et al*. (1994, 2009) state that the optimal control action can be derived in the form of the MMSE as

$$X_t = \frac{\phi - \theta}{1 - \phi B} a_t \tag{4}$$

where θ is the moving average coefficient, ϕ is the autoregressive coefficient, and a_t represents a white noise with mean 0 and standard deviation σ_a, and B is the backward shift operator defined by $Be_t = e_{t-1}$. For the ARMA (1,1) disturbance model, the MMSE controller is defined as (Box *et al*. 1994)

$$X_t = \phi X_{t-1} + (\phi - \theta)e_t \tag{5}$$

and the expected values of the process output e_t and the control action X_t are given by

$$E(e_t) = \frac{\left(1 - \phi + \phi\theta^t - \theta^{t+1}\right)}{1 - \theta}\eta, \quad E(X_t) = \frac{(\phi - \theta)\left(1 - \theta^{t+1}\right)}{1 - \theta}\eta, \tag{6}$$

respectively (Hu and Roan 1996). A step mean shift η is given by

$$e_t = D_t + \gamma(B)X_{t-1} + \eta_t, \tag{7}$$

where $\eta_t = 0,$ for $t < 0,$ and $\eta_t = \eta,$ for $t \geq 0.$

The transient and steady state mean shift patterns of the process output e_t and the control action X_t are given by

$$\mu_T^e = \eta, \quad \mu_S^e = \frac{1 - \phi}{1 - \theta}\eta, \quad \mu_T^X = (\phi - \theta)\eta, \quad \mu_S^X = \frac{\phi - \theta}{1 - \theta}\eta, \tag{8}$$

respectively (Jiang and Tsui 2002) and the standard deviations of the process output and the control action are given by

$$\sigma_e = \sigma_a, \quad \sigma_X = \frac{|\phi - \theta|}{\sqrt{1 - \phi^2}}\sigma_a, \tag{9}$$

respectively (Jiang and Tsui 2002).

3.2 *PI Controller*

Proportional-integral (PI) controller is the most widely used device in the process control industry (Tsung and Shi 1999, Tsung *et al*. 1999, Jiang *et al*. 2002, Tsung *et al*. 2006, Jiang and Farr 2007). Let X_t denote the control action at time t. Discrete PI control schemes are of the form

$$X_t = k_P e_t + k_I \frac{1}{1 - B} e_t, \tag{10}$$

where B is the backward shift operator defined by $Be_t = e_{t-1}$ and k_P and k_I are constants that determine the amount of proportional and integral control actions, respectively. For a P controller ($k_I = 0$), the control action is the output scaled by k_P, i.e. $X_t = k_P e_t$. Therefore, the performance of the output chart and the control action chart is expected to be the same. The transient and steady state mean shift patterns of the process output e_t and the control action X_t are given by

$$\mu_T^e = \eta, \quad \mu_S^e = \frac{1}{1 + k_P}\eta, \quad \mu_T^X = k_P\eta, \quad \mu_S^X = \frac{k_P}{1 + k_P}\eta, \tag{11}$$

respectively (Jiang and Tsui 2002) and their standard deviations are

$$\sigma_e = \sqrt{\frac{1 - k_P\phi + k_P\theta + \theta\phi k_P^2 - \theta\phi - \theta\phi^2 k_P}{(1 + k_P\phi)\left(1 - k_P^2\right)(1 - \phi^2)}}\sigma_a, \quad \sigma_X = k_P\sigma_e, \tag{12}$$

respectively where k_P is constant that determines the amount of proportional control actions.

For a general PI controller, monitoring the output and the control action can perform very differently when a mean shift occurs. Let μ_t^e be the mean shift pattern of the process output e_t. When a step mean shift η occurs at time 0, the mean shift patterns of the process output and the control action are

$$\begin{aligned} \mu_t^e &= (1 - k_P - k_I)\mu_{t-1}^e + k_P\mu_{t-2}^e, \\ \mu_t^X &= \mu_{t-1}^X + (k_P + k_I)\mu_t^e - k_P\mu_{t-1}^e, \end{aligned} \tag{13}$$

where $\mu_{t-1}^e = 0,\ \mu_0^e = \eta$ and $\mu_{-1}^X = 0$. The transient and steady state mean shifts patterns of the process output and the control action are given by $\mu_T^e = \eta,\ \mu_S^e = 0,$ and $\mu_T^X = (k_P + k_I)\eta,\ \mu_S^X = \eta,$ respectively.

4. An economic cost model for the integration of APC and SPC

4.1 *Cost analysis of APC using MMSE controller*

Taguchi (1990) proposes a quality loss function to estimate the cost due to the deviation from the target of the product characteristic. The expected loss per unit is proportional to the squared deviation from the target value and the process standard deviation. Using the expected value and standard deviation of the process output at time t during the steady state period and the transient state period given in Equations (8) and (9) respectively, the expected loss per unit in the steady-state period, $E(L_S)$ can be expressed in terms of the quadratic loss function as

$$E(L_S) = \left(\frac{c_A}{\Delta^2}\right)\left(\left(\frac{1-\phi}{1-\theta}\eta\right)^2 + \sigma_a^2\right) \tag{14}$$

where Δ is in-process acceptable deviation and c_A is the reworking or scrapping cost per unit of production. We assume that the number of adjustments follows an NHPP with rate $\lambda(t)$. The average time to signal for an adjustment is $E(Y)$. If R_P is the production rate, $E(Y)$ is the expected length of APC cycle and T is the transient state period then the expected number of items produced during the steady state period, $E(N_S)$, is given by $R_P \cdot (E(Y) - T)$.

We now consider the number of adjustments for the SPC monitoring chart. If the APC adjustment at time t follows an NHPP with intensity function $\lambda(t)$ and a mean value function, $g(t) = \int_0^t \lambda(s)\mathrm{d}s,$ where t represents the current time, it could represent the IC period of the SPC. Let each inter-adjustment interval of the APC be $Y_i,\ i = 1, 2, \ldots.$ If the first APC adjustment time is larger than t, then the number of adjustments in t is zero. Using the mean function, the probability that the first adjustment time is larger than t is given by

$$P\{Y_1 > t\} = P(N(t) = 0) = e^{-g(t)}.$$

Similarly, if $n \geq 2,$ then we obtain the probability that the time until the nth APC adjustment, S_n is larger than t.

$$P\{Y_1 + Y_2 + Y_3 + \cdots + Y_n > t\} = P(N(t) \leq n-1) = P\{S_n > t\} = \sum_{i=0}^{n-1} \frac{e^{-g(t)}[g(t)]^i}{i!} \tag{15}$$

The probability density function of the number of adjustments is given by

$$P\{N(t) = n\} = \frac{e^{-\int_0^t \lambda(s)\mathrm{d}s}\left[\int_0^t \lambda(s)\mathrm{d}s\right]^n}{n!} \tag{16}$$

Also, the expected length time of the nth adjustment interval can be derived by:

$$E(Y_n) = \sum_{i=0}^{n-1} \int_0^\infty \frac{e^{-\int_0^t \lambda(s)\mathrm{d}s}\left(\int_0^t \lambda(s)\mathrm{d}s\right)^i}{i!}\,\mathrm{d}t - \sum_{i=0}^{n-2} \int_0^\infty \frac{e^{-\int_0^t \lambda(s)\mathrm{d}s}\left(\int_0^t \lambda(s)\mathrm{d}s\right)^i}{i!}\,\mathrm{d}t \tag{17}$$

Consider the simple case when $\lambda(s) = s$, then,

$$\begin{aligned}E(Y_n) &= E(Y_1 + Y_2 + Y_3 + \cdots + Y_n) - E(Y_1 + Y_2 + Y_3 + \cdots + Y_{n-1}) \\ &= \sum_{i=0}^{n-1}\left(\frac{(2i-1)!!}{(2i)!!}\right)\left(\frac{\sqrt{2\pi}}{2}\right) - \sum_{i=0}^{n-2}\left(\frac{(2i-1)!!}{(2i)!!}\right)\left(\frac{\sqrt{2\pi}}{2}\right) \\ &= \left(\frac{(2n-3)!!}{(2n-2)!!}\right)\left(\frac{\sqrt{2\pi}}{2}\right)\end{aligned} \tag{18}$$

where

$$n!! = \begin{cases} n\cdot(n-2)\cdot(n-4)\cdot(n-6)\cdot\cdots\cdot 4\cdot 2 & \text{if } n \text{ is even} \\ n\cdot(n-2)\cdot(n-4)\cdot(n-6)\cdot\cdots\cdot 3\cdot 1 & \text{if } n \text{ is odd.}\end{cases}$$

As $n \to \infty$, we obtain

$$E(Y) = \lim_{n\to\infty} E(Y_n) = \lim_{n\to\infty}\left(\frac{(2n-3)!!}{(2n-2)!!}\right)\left(\frac{\sqrt{2\pi}}{2}\right) = \frac{\sqrt{2\pi}}{2}. \tag{19}$$

Using the steady state interval until the occurrence of the adjustment, the expected cost during the steady state is given by

$$E(C_S) = E(L_S)\cdot E(N_S) = \left(\frac{c_A}{\Delta^2}\right)\left(\left(\frac{1-\phi}{1-\theta}\eta\right)^2 + \sigma_a^2\right)\cdot R_P\cdot(E(Y) - T). \tag{20}$$

As shown in Equation (20), the expected cost during the steady state period depends on the expected values and variance of the control action and the frequency of the shift occurrence.

The average process output during the transient state period is given by $\mu_T^e = \eta$. The expected number of items produced during the transient period is $E(N_T) = R_P \cdot T$, and the expected cost during the transient period can be derived as

$$\begin{aligned}E(C_T) &= E(L_T)\cdot E(N_T) \\ &= \frac{c_A}{\Delta^2}\left[(\eta)^2 + \sigma_a^2\right]\cdot R_P\cdot T\end{aligned} \tag{21}$$

The expected APC cost per adjustment is given by

$$\begin{aligned}E(C_{APC}) &= E(C_S) + E(C_T) \\ &= \frac{R_P\cdot c_A}{\Delta^2}\left(\sigma_a^2\cdot E(Y) + \eta^2\left(\left(\frac{1-\phi}{1-\theta}\right)^2(E(Y) - T) + T\right)\right)\end{aligned} \tag{22}$$

where $E(Y)$ is given by Equation (19).

4.2 *Cost analysis of APC using pure P controller and PI controller*

Steady state covers the period before the deviation limit exceeds a tolerance limit. Using the expected value and variance of the control action at time t during the steady state period given in Equations (11) and (12), the expected loss per unit during the steady-state period when we use the P controller can be expressed as

$$E(L_S) = \left(\frac{c_A}{\Delta^2}\right)\left(\left(\frac{\eta}{1+k_P}\right)^2 + (\sigma_e)^2\right) \tag{23}$$

As a result, the expected cost during the steady state is given by

$$E(C_S) = \left(\frac{c_A}{\Delta^2}\right)\left(\left(\frac{\eta}{1+k_P}\right)^2 + (\sigma_e)^2\right)R_P\cdot(E(Y) - T) \tag{24}$$

where

$$\sigma_e = \sqrt{\frac{1 - k_P\phi + k_P\theta + \theta\phi k_P^2 - \theta\phi - \theta\phi^2 k_P}{(1 + k_P\phi)\left(1 - k_P^2\right)(1 - \phi^2)}}\sigma_a.$$

Similar to the MMSE controller, the expected number of items produced during the transient period is $E(N_A) = R_P \cdot T$ and the expected cost during the transient period is given by

$$\begin{aligned} E(C_T) &= E(L_T) \cdot E(N_T) \\ &= \frac{c_A}{\Delta^2}\left[(\eta)^2 + (\sigma_e)^2\right] R_P \cdot T. \end{aligned} \tag{25}$$

When the P controller is used, the expected APC cost per adjustment is given by

$$\begin{aligned} E(C_{APC}) &= E(C_S) + E(C_T) \\ &= \frac{R_P \cdot c_A}{\Delta^2}\left(\eta^2 \cdot \left(\frac{E(Y) - T}{(1 + k_P)^2} + T\right) + \sigma_e^2 E(Y)\right). \end{aligned} \tag{26}$$

The expected loss per unit during the steady-state period when we use the PI controller can be expressed as

$$E(L_S) = \left(\frac{c_A}{\Delta^2}\right)\left(\sigma_e^2\right) \tag{27}$$

As a result, the expected cost during the steady state is given by

$$E(C_S) = \left(\frac{c_A}{\Delta^2}\right)\left(\sigma_e^2\right) \cdot R_P \cdot (E(Y) - T) \tag{28}$$

where

$$\sigma_e^2 = 2\int_0^\pi \frac{\left|1 - \phi e^{-i\lambda}\right|^2}{\left|1 - \phi e^{-i\lambda} + (1 - \theta e^{-i\lambda})e^{-i\lambda}\right|^2} f_D(\lambda)\mathrm{d}\lambda,$$

$f_D(\lambda)$ is defined as the ARMA(1,1) disturbance model. The variance of process output for PI controller can be obtained using Fourier transformation (Jiang 2004, Brockwell and Davis 2009). Similar to the P controller, the expected number of items produced during the transient period is $E(N_A) = R_P \cdot T$ and it is assumed that the variance of process output during the transient state period is the same as that during the steady state period. Then, the expected cost during the transient period is given by

$$\begin{aligned} E(C_T) &= E(L_T) \cdot E(N_T) \\ &= \frac{c_A}{\Delta^2}\left[(\eta)^2 + \sigma_e^2\right] R_P \cdot T. \end{aligned} \tag{29}$$

When the PI controller is used, the expected APC cost per adjustment is given by

$$\begin{aligned} E(C_{APC}) &= E(C_S) + E(C_T) \\ &= \frac{R_P \cdot c_A}{\Delta^2}\left(\eta^2 \cdot T + \sigma_e^2 \cdot E(Y)\right). \end{aligned} \tag{30}$$

where σ_e^2 is given in Equation (28).

4.3 *Cost analysis of an EWMA chart in IC and OC periods*

If x_i is a count, $i = 1, 2, \ldots,$ then the basic EWMA recursion remains unchanged:

$$z_i = \lambda x_i + (1 - \lambda)z_{i-1} \tag{31}$$

with $z_0 = \mu_0$ the in-control or target count rate. The control limits are as follows:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \tag{32}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \tag{33}$$

where L is the control limit and i is the sample number or time. The expected sum of squared observed deviations per unit in the period can be expressed in terms of the quadratic loss function as

$$E(S_D) = \left(\frac{c_A}{\Delta^2}\right)\left((\mu_0)^2 + \frac{\sigma^2\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\right) \tag{34}$$

In the SPC monitoring charts, we consider the false alarm cost for SPC, because the false alarms incur an additional cost to search for the special causes when none exist. Let N_F be the number of false alarms and N_A be the number of adjustments during the IC period. τ is defined as a random time when special cause occurs. If a special cause occurs at time τ, then the expected number of APC adjustments during the IC period $E(N_A)$ is given by

$$\begin{aligned} E(N_A) &= \sum_{n=1}^{\infty} n \cdot P\{N(\tau) = n\} \\ &= \sum_{n=1}^{\infty} \frac{e^{-\int_0^{\tau}\lambda(s)\mathrm{d}s}\left[\int_0^{\tau}\lambda(s)\mathrm{d}s\right]^n}{(n-1)!} \end{aligned} \tag{35}$$

Let $E(S)$ be the expected number of samples taken while in control and h is the sampling period and the IC time is distributed as a negative exponential random variable with mean $1/\lambda_1$. The expected number of false alarms $E(N_F)$ is given by (Lorenzen and Vance 1986):

$$E(N_F) = \frac{E(S)}{1/\alpha} = \frac{\alpha \cdot e^{-\lambda_1 \cdot h}}{1-e^{-\lambda_1 \cdot h}} \tag{36}$$

where α is the probability of type I error.

Let c_F, c_D and c_M be false signal cost, disturbance cost and diagnosis cost, respectively. The expected SPC cost is given by

$$\begin{aligned} E(C_{SPC}) &= c_F \cdot E(N_F) + c_D \cdot E(S_D) + c_M(E(\tau) + ARL_1) \\ &= c_F \cdot \frac{\alpha \cdot e^{-\lambda_1 \cdot h}}{1-e^{-\lambda_1 \cdot h}} + c_D \cdot \left(\frac{c_A}{\Delta^2}\right)\left((\mu_0)^2 + \frac{\lambda\mu_0}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\right) + c_M(E(\tau) + ARL_1). \end{aligned} \tag{37}$$

In statistical design the efficiency of the control scheme is completely evaluated in terms of the ARL. The run length of the IC process is the number of observations taken until a signal assuming that no special cause occurs. The run length of the OC process is the number of observations taken until a signal assuming that a special cause has already occurred from the beginning of the process. The production process is composed of only two states, the IC and OC states, and continues alternately. The duration of the IC state follows a geometric distribution. That is, special causes are assumed to occur with probability p. We consider one production cycle which would end by finding and fixing the special cause for SPC, while there can be several APC adjustments in the same production cycle.

The economic design is based on the design of a monitoring scheme cost that may produce the minimum expected cost per unit time. We assume that a rectifying action followed by a true signal resets the process conditions to the same conditions as when the process is in control, thus these series of cycles correspond to a renewal process. The proposed economic cost model is then used to design and to determine the parameters of the control chart by minimising the expected cost per unit time defined as

$$
\begin{aligned}
LREC &= \frac{\text{Expected Cost}}{\text{Expected Duration}} \\
&= \frac{E(N_A)E(C_{APC}) + E(C_{SPC})}{E(\tau) + ARL_1} \\
&= \frac{E(N_A)E(C_{APC}) + c_F \cdot \alpha \cdot E(S) + c_D \cdot E(S_D) + c_M(E(\tau) + ARL_1)}{E(\tau) + ARL_1} \\
&= \frac{1}{\frac{1}{\lambda_1} + ARL_1} \left\{ \begin{array}{l} \left(\sum_{n=1}^{\infty} \frac{e^{-\int_0^\tau \lambda(s)\mathrm{d}s} \left[\int_0^\tau \lambda(s)\mathrm{d}s \right]^n}{(n-1)!} \right) E(C_{APC}) + c_F \cdot \frac{\alpha \cdot e^{-\lambda_1 \cdot h}}{1 - e^{-\lambda_1 \cdot h}} \\ + c_D \cdot \left(\frac{c_A}{\Delta^2} \right) \left((\mu_0)^2 + \frac{\lambda \mu_0}{2-\lambda} \left[1 - (1-\lambda)^{2i} \right] \right) \end{array} \right\} + c_M
\end{aligned}
$$

when MMSE controller is used,

$$E(C_{APC}) = \frac{R_P \cdot c_A}{\Delta^2} \left(\sigma_a^2 \cdot E(Y) + \eta^2 \left(\left(\frac{1-\phi}{1-\theta} \right)^2 (E(Y) - T) + T \right) \right),$$

when P controller is used,

$$E(C_{APC}) = \frac{R_P \cdot c_A}{\Delta^2} \left(\eta^2 \cdot \left(\frac{E(Y) - T}{(1+k_P)^2} + T \right) + \frac{1 - k_P\phi + k_P\theta + \theta\phi k_P^2 - \theta\phi - \theta\phi^2 k_P}{(1+k_P\phi)\left(1-k_P^2\right)(1-\phi^2)} \sigma_a^2 E(Y) \right),$$

and when PI controller is used,

$$E(C_{APC}) = \frac{R_P \cdot c_A}{\Delta^2} \left(\eta^2 \cdot T + 2 \int_0^\pi \frac{\left|1 - \phi e^{-i\lambda}\right|^2}{\left|1 - \phi e^{-i\lambda} + (1 - \theta e^{-i\lambda})e^{-i\lambda}\right|^2} f_D(\lambda)\mathrm{d}\lambda \cdot E(Y) \right). \tag{38}$$

5. An illustrative example with sensitivity analysis

In this section, we conduct a fractional factorial experiment to study the effects and sensitivities of the input parameters for the nominal-the-best type characteristics by following the similar setting of Elsayed and Chen (1994). Table 1 shows the input parameters used in Elsayed and Chen (1994). We use the parameters in Table 1 for the sensitivity analysis.

Additionally, we use parameters such as $\lambda_1 = 0.004$ (Montgomery *et al.* 1994). The results of LREC for various parameters ϕ, θ, k_P and k_I and different controllers are shown in Table 2 which are used in the numerical example in Jiang and Tsui (2002). Also, the optimal P control parameter k_P of the ARMA time series processes is chosen using the approach in Tsung *et al.* (1998) to minimise the mean squared error. For illustration purpose, the smoothing factor of the EWMA is $\lambda = 0.2$.

Table 2 shows, when $\eta = 0$, MMSE controllers are better than P controllers and PI controllers in terms of LREC. When considering the parameters $(\varphi, \theta) = (0.8, -0.3)$, $(0.5, -0.9)$, and $(0.2, 0.6)$, in the levels of all four factors in Table 1, LRECs with MMSE controllers are larger than those with P controllers. Further, as shift

Table 1. Input parameters.

Factors	Parameters
Reworking cost c_A	1
Production rate R_P	100
Diagnosis cost c_M	35
False signal cost c_F	50

magnitudes η increase, LRECs with MMSE controllers are larger than those with PI controllers. On the contrary, for the parameters $(\varphi, \theta) = (-0.2, -0.6)$ and $(-0.7, -0.2)$, LRECs with MMSE controllers are generally smaller than those with P controllers. The performance of the output and control action charts of an APC controlled process depends on the shift pattern, the shift magnitude, and the APC controller being employed (Jiang and Tsui 2002). The process with a positive value ϕ is positively auto-correlated and the MMSE controller automatically reduces the shift level in the output which makes detection difficult, as discussed in Hu and Roan (1996) and Jiang and Tsui (2002). If the detection is difficult, then it results in an increase of LREC. As shown in Figure 3(a)–(b), the LREC with the MMSE controller is larger than that with the P controller. Similarly when ϕ is less than zero, in which the process is negatively auto-correlated, the reverse is expected to occur since the shift would be amplified in the output as shown in Figure 3(c)–(d). These results are consistent with Jiang and Tsui (2002). Additionally, according to Jiang (2004) and Jiang and Tsui (2002), for a general PI controller, due to the I component, the mean shift of the output converges to zero, i.e. the mean shift is completely compensated in the steady-state condition, which makes it extremely difficult for the output chart to detect small shifts. Therefore, LREC with PI controller may be higher than that of MMSE controller when there are small shifts in Figure 3(a) and (b).

Now, we investigate the effects of each parameter on the LREC for the P controllers and summarise the results in Tables 3 and 4. Other than changing some parameters, we use most of the parameters as in Table 1. The LREC increases as with the increase of the reworking cost and mean shift magnitudes increase as shown in Table 3 and Figure 4(a). The LREC increases gradually when production rates and the mean shift magnitudes increase as shown in Table 4 and Figure 4(b). Similarly, we study the effects of the disturbance cost, autoregressive coefficients, moving average coefficients, the constant k_P on the LREC. The LREC increases slightly when the disturbance cost and mean shift magnitudes increase, as shown in Figure 5(a) and it also increases when the autoregressive coefficient ϕ and the mean shift magnitudes increase as shown in Figure 5(b). The lowest LREC is achieved at an autoregressive

Table 2. LREC using different controllers with parameters (φ, θ) and mean shift magnitudes η.

MMSE controller ($\times 10^6$)

	Mean shift magnitudes, η					
(φ, θ)	0	1	2	3	4	5
(0.8, −0.3)	0.0383	0.9746	3.7835	8.4649	15.0190	23.4456
(0.5, −0.9)	0.0382	1.1746	4.5840	10.2663	18.2215	28.4496
(0.2, 0.6)	0.0378	0.7196	2.7650	6.1740	10.9466	17.0828
(−0.2, −0.6)	0.0378	0.1417	0.4537	0.9736	1.7014	2.6372
(−0.7, −0.2)	0.0378	0.2471	0.8749	1.9213	3.3863	5.2698

(a) PI controller ($\times 10^6$)

			Mean shift magnitudes, η					
(φ, θ)	k_P	k_I	0	1	2	3	4	5
(0.8, −0.3)	0.3	0.7	1.9300	2.5464	4.3954	7.4772	11.7916	17.3387
(0.5, −0.9)	0.9	0.1	1.9745	2.5909	4.4399	7.5216	11.8361	17.3832
(0.2, 0.6)	−0.4	0.0	1.1117	1.3844	2.2025	3.5661	5.4751	7.9295
(−0.2, −0.6)	0.4	0.0	1.4442	1.7169	2.5350	3.8986	5.8075	8.2619
(−0.7, −0.2)	−0.4	0.0	1.1248	1.3975	2.2156	3.5792	5.4882	7.9426

(b) P controller ($\times 10^6$)

		Mean shift magnitudes, η					
(φ, θ)	k_P	0	1	2	3	4	5
(0.8, −0.3)	0.3	0.2527	0.3223	0.5312	0.8794	1.3669	1.9937
(0.5, −0.9)	0.9	0.1503	0.6200	2.0289	4.3772	7.6648	11.8917
(0.2, 0.6)	−0.4	0.2033	0.5427	1.5609	3.2578	5.6335	8.6880
(−0.2, −0.6)	0.4	0.2033	0.3191	0.6664	1.2453	2.0556	3.0976
(−0.7, −0.2)	−0.4	0.2302	0.5696	1.5877	3.2847	5.6604	8.7149

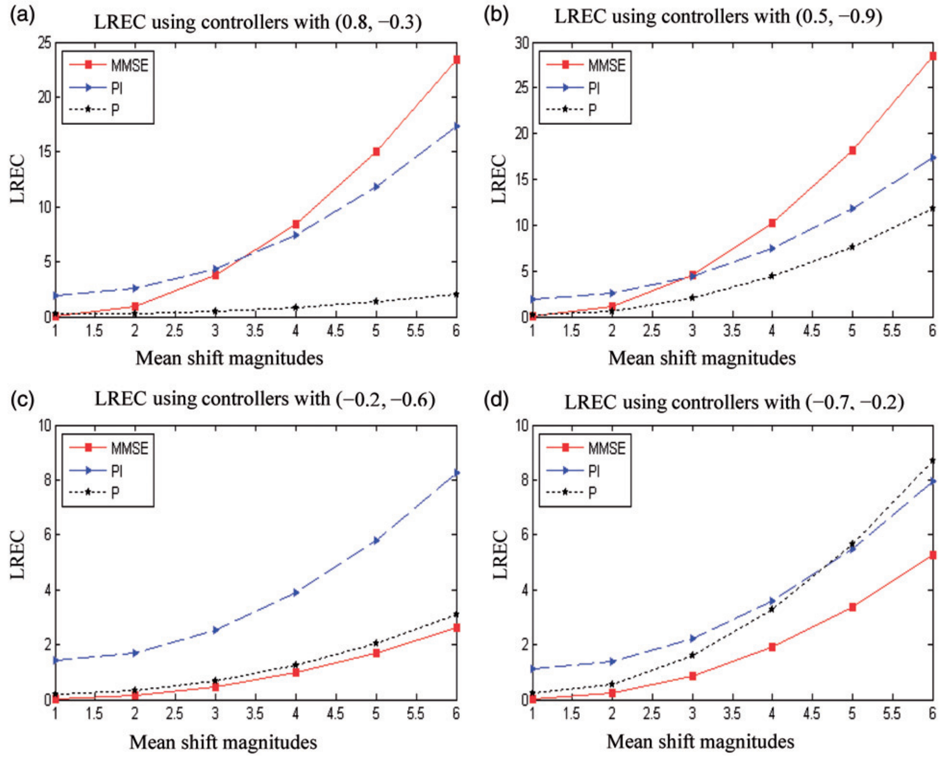


Figure 3. LREC using different controllers with various parameters (φ, θ). (a) LREC using controllers with (0.8, −0.3). (b) LREC using controllers with (0.5, −0.9). (c) LREC using controllers with (−0.2, −0.6). (d) LREC using controllers with (−0.7, −0.2).

Table 3. LREC using different reworking cost, c_A.

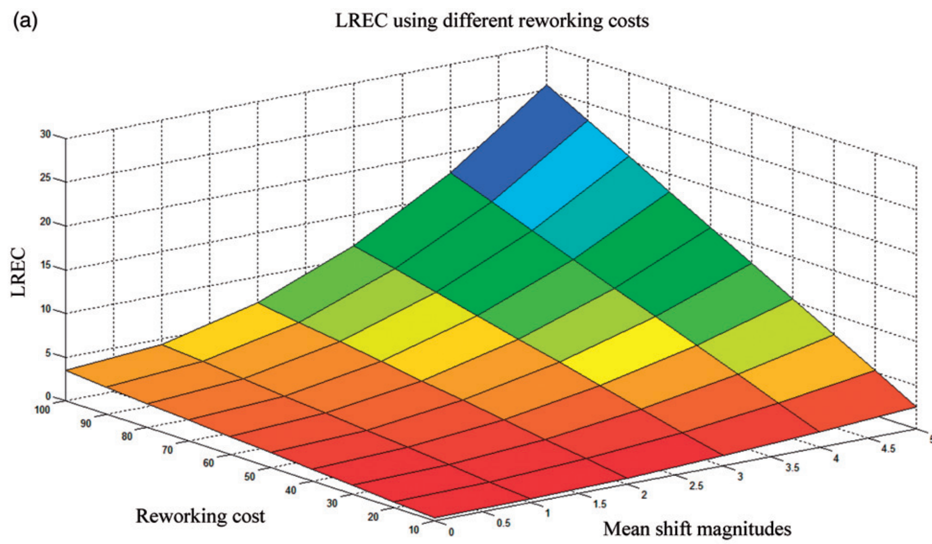
	Mean shift magnitudes, η ($\times 10^7$)					
c_A	0	1	2	3	4	5
10	0.3502	0.4388	0.7047	1.1478	1.7681	2.5656
20	0.6966	0.8738	1.4055	2.2917	3.5323	5.1274
30	1.0430	1.3088	2.1064	3.4356	5.2966	7.6892
40	1.3894	1.7439	2.8073	4.5796	7.0608	10.2510
50	1.7358	2.1789	3.5081	5.7235	8.8251	12.8128
60	2.0822	2.6139	4.2090	6.8675	10.5893	15.3746
70	2.4286	3.0489	4.9098	8.0114	12.3536	17.9364
80	2.7750	3.4839	5.6107	9.1553	14.1179	20.4982
90	3.1214	3.9189	6.3115	10.2993	15.8821	23.0600
100	3.4678	4.3539	7.0124	11.4432	17.6464	25.6218

coefficient of $\phi = 0.1$. When we use different moving average coefficients, θ, as the acceptable disturbance decreases and as the mean shift magnitude increases, the LREC increases as shown in Figure 6(a). Likewise, the LREC increases as the constant k_P, which determines proportional control actions and mean shift magnitudes, increases as shown in Figure 6(b).

The changes of the mean shift magnitudes significantly influence the LREC when the reworking cost, the production rate, or the k_P are changed. The changes of reworking cost have the most impact on the LREC. The changes of the disturbance cost, the autoregressive coefficients and the moving average coefficients have little influence on the LREC. Also, the changes of reworking cost and the constant k_P have a strong impact on the LREC when the mean shift magnitudes increase. The change of constant k_P and/or the mean shift magnitudes affect the

Table 4. LREC using different production rate, R_P.

	Mean shift magnitudes, η ($\times 10^8$)					
R_P	0	1	2	3	4	5
100	0.0385	0.0473	0.0739	0.1182	0.1802	0.2600
200	0.0731	0.0908	0.1440	0.2326	0.3567	0.5162
300	0.1077	0.1343	0.2141	0.3470	0.5331	0.7724
400	0.1424	0.1778	0.2842	0.4614	0.7095	1.0285
500	0.1770	0.2213	0.3542	0.5758	0.8859	1.2847
600	0.2117	0.2648	0.4243	0.6902	1.0624	1.5409
700	0.2463	0.3083	0.4944	0.8046	1.2388	1.7971
800	0.2809	0.3518	0.5645	0.9190	1.4152	2.0533
900	0.3156	0.3953	0.6346	1.0334	1.5916	2.3094
1000	0.3502	0.4388	0.7047	1.1478	1.7681	2.5656



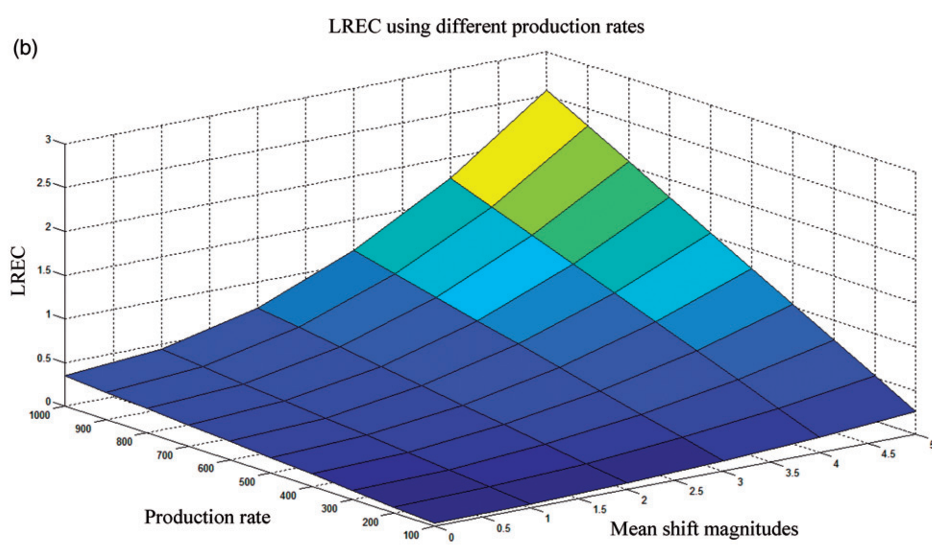


Figure 4. LREC using different reworking cost and production rate. (a) LREC using different reworking costs. (b) LREC using different production rates.

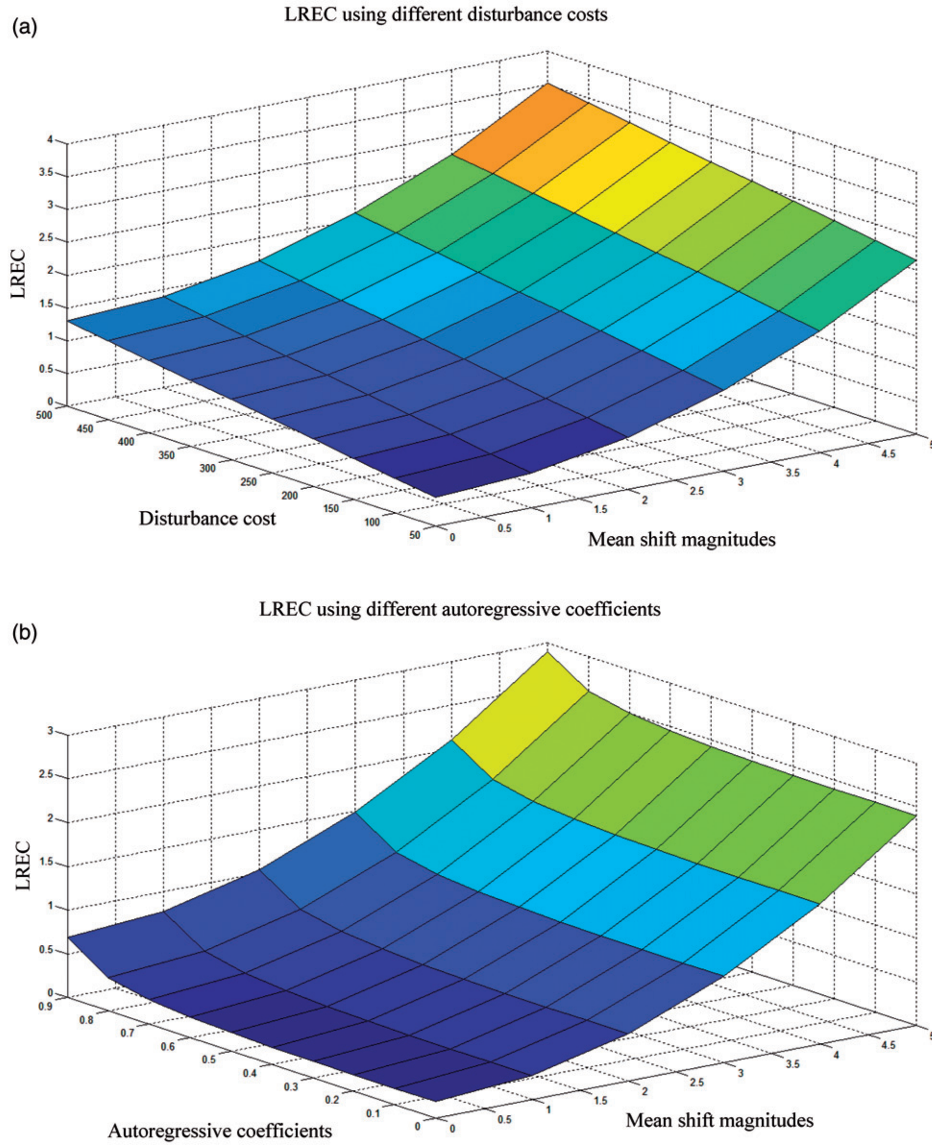


Figure 5. LREC using different disturbance cost and autoregressive coefficients. (a) LREC using different disturbance costs. (b) LREC using different autoregressive coefficients.

LREC noticeably. We can also note that the larger autoregressive coefficients and the lower moving average coefficients increase the LREC. However, the LREC increases gradually as the moving average coefficients decrease.

6. Discussion

Table 5 summarises existing economic cost models in SPC, APC and the integration of both. In some studies, decision variables such as sample interval, sample size and control limits are considered. Park and Reynolds (2008) use the expected cost per unit time (ECU) to measure the effectiveness of the economic design. Jiang and Tsui (2000) derive an economic model for applying SPC charts to monitor an MMSE controlled processes. They compare three common SPC charting methods, the Shewhart chart, the EWMA chart and the combined EWMA-Shewhart chart under the average quality cost (AQC) criterion as well as the ARL criterion. It is shown that AQC is generally consistent with ARL criterion except when the APC control action significantly compensates the process shift. Duffuaa *et al*. (2004) propose a scheme to integrate SPC, APC and Taguchi's loss function to determine whether the

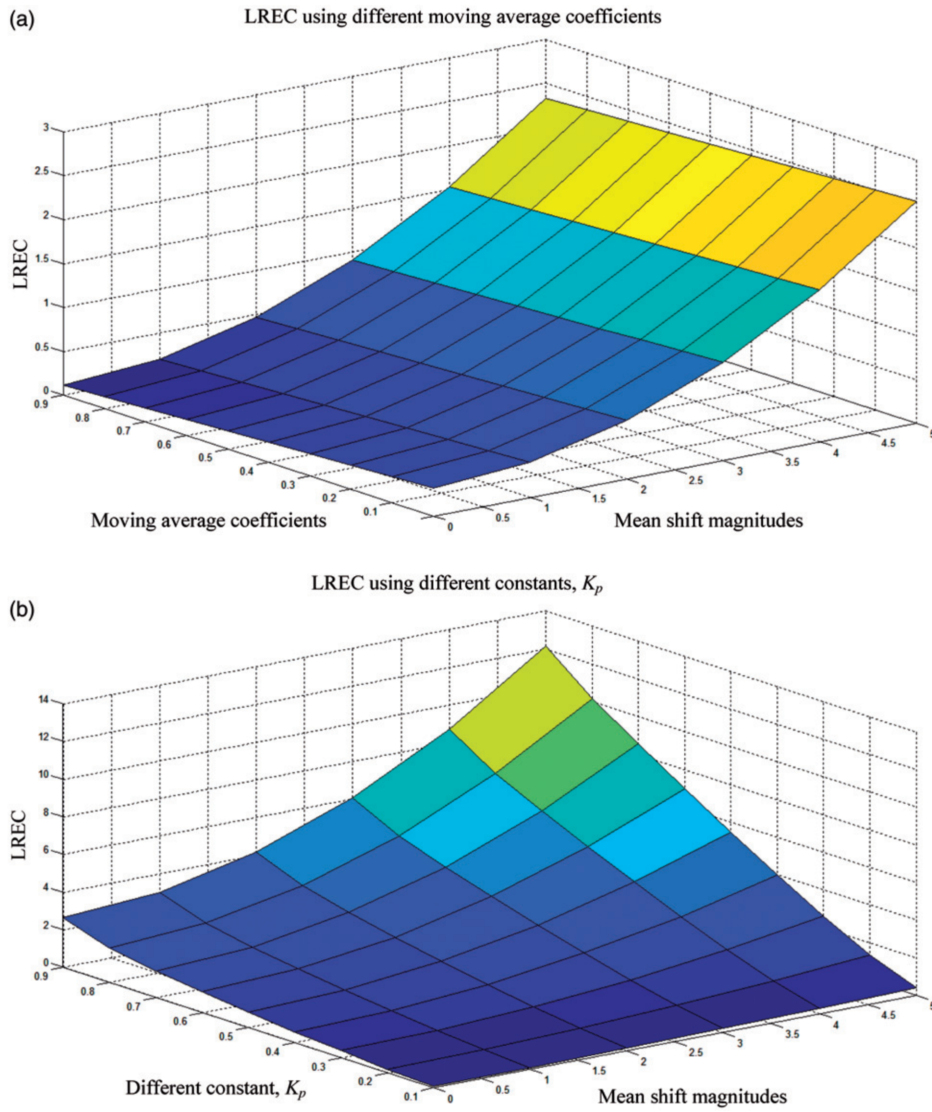


Figure 6. LREC using different moving average coefficients and constants K_p. (a) LREC using different moving average coefficients. (b) LREC using different constants, K_p.

APC should be performed or the process should be left without change. Yang and Sheu (2007) propose other statistical and economic criteria, such as the average Euclidean distance from the target vector and the AQC to evaluate the performance of the multivariate APC/multivariate SPC charts. They compare and investigate the ARL, average Euclidean distance and AQC to evaluate the performance of the multivariate APC/multivariate SPC charts.

The integration of APC and SPC is a fertile area of research that has a significant impact on industry. The APC actions for process adjustments include the MMSE controller or PID controller which minimise the output deviations from the quality target. The MMSE controller is optimal in terms of the minimisation of mean squared residual errors when the model and its parameters are exactly known. On the other hand, the PID controller is very efficient and also robust against non-stationary assumptions due to the fact that it can continuously adjust the process whenever the data are auto-correlated. Many studies have focused on these two controllers, but there is limited research for other types of controllers than MMSE and PID that could indeed provide further improvements than these traditional controllers.

The development of an economic model which incorporates process improvement as well as disturbance and process variability is another area for exploration. Economic models appear to provide a natural approach for process engineers to use in control chart design and its application (Stoumbos *et al.* 2000). Although this topic is

Table 5. Economic design of the integration of APC and SPC.

Reference	Topics	Costs considered in their studies	Measurement & tools	Models	Selected notations*
Lorensen and Vance (1986)	SPC	Nonconformity cost, C_0 & C_1 Diagnosis cost Adjustment cost Sampling and tesing cost False alarm cost, Y Repairing cost, W	Quadratic loss function Expected cost per cycle Sample size(n) Sampling interval(h) Control limits(L)	$\dfrac{\left\{\begin{array}{l}\frac{C_0}{\lambda}+C_1(-\tau+nE+h(ARL_2)+\delta_1T_1+\delta_2T_2)\\ +\frac{sY}{ARL_1}+W+(a+bn)\cdot(\frac{1}{\lambda}-\tau+nE\\ +h(ARL_2)+\delta_1T_1+\delta_2T_2)lh\end{array}\right\}}{\frac{1}{\lambda}+(1-\delta_1)sT_0/ARL_1-\tau+nE+h(ARL_2)+T_1+T_2}$	E: expected time to sample T_1: expected time to discover the cause T_2: expected time to repair the process τ: expected time of cause occurrence s: expected number of samples
Elsayed and Chen (1994)	APC and SPC	Measurement cost, $b+cn$ False alarm cost, F Cost of finding & fixing, W	Quadratic loss function Expected cost per unit Sample size(n) Sampling interval(h) Control limits(k)	$\dfrac{(b+cn)}{h}+F\cdot\dfrac{\alpha}{\lambda h\cdot E(T)}+\dfrac{W}{E(T)}+\dfrac{RA}{\Delta^2}\left\{\left[\sigma^2+(\mu-\mu_0)^2\right]+\dfrac{\left[E(T)-\frac{1}{\lambda}\right]\delta^2\sigma^2}{E(T)}\right\}$	$E(T)$: expected time length A: in plant cost to rework Δ: in process accuptable deviation μ_0: target value of the quality
Bai and Lee (1998)	SPC	False alarm cost, c_1 Cost of detecting cause, c_2 Prodcution cost, c_3 Cost of sampling and testing, c_4+c_5n	Expected cost per unit Sample size Sampling interval Control limits Varaiable & fixed interval	$\dfrac{q_0c_1S_0+c_2+c_3E(O)+(c_4+c_5n)(S_0+S_1)}{\frac{1}{\lambda}+E(O)+q_0t_1S_0+t_2}$	O: out-of-control period S_1: expected number of samples in OC S_0: expected number of samples in IC q_0: false alarm rate
Jiang and Tsui (2000)	APC and SPC	Diagnosis cost, C_D Loss cost per unit Adjustment cost, C_A	Quadratic loss function MMSE controller Average qualtiy cost	$\sigma^2+\alpha C_D+\dfrac{\sum_{T=1}^{\infty}P_T(\mu)\sum_{t=0}^{T-1}\mu_t^2-[ARL_1\cdot\alpha-1]C_D}{\frac{1}{p}+ARL_1}$	T: out of control run length α: false alarm rate
Wang and Yue (2001)	APC	Major adjustment cost, C_{major} Minor adjustment cost, C_{minor} Off target cost	Quadratic loss function Unit quality cost	$\dfrac{C_{\text{major}}}{m\cdot n}+\dfrac{C_{\text{minor}}}{m}+\dfrac{\sum_{i=0}^{n-1}k\left[\sigma_i^2+(1-\theta)^2\sigma_i^2\frac{m-1}{2}\right]}{n}$	n: number of minor adjustments m: number of units btn. two minor adj. k: proportaionlity constant
Chou *et al.* (2002)	SPC	Sampling and testing cost Fixing cost in OC Defective products cost	Multivariate QLF Test statsitics, -2-*l*-*n*-*L*	$\dfrac{a_1+n\cdot a_2}{k}+\left(a_3\sum_{i=0}^{2}\rho_i\alpha_i\right)\Big/k+a_4\sum_{i=0}^{2}\delta_i\gamma_i$	a_3: average cost of investing & correcting ρ_i: prob. that process is OC in state i α_i: prob. of process being in i at test time a_4: defective product cost γi: prob. of pro cess being in i δ_i: prob. of producing defective unit at i

(*continued*)

Table 5. Continued.

Reference	Topics	Costs considered in their studies	Measurement & tools	Models	Selected notations*
Duffuaa *et al*. (2004)	APC and SPC	Diagnosis cost, B Adjustment cost, C Loss cost per unit	Quadratic loss function No measurement	Model I; $\frac{A}{\Delta^2}\left[\frac{D^2}{3}+\frac{D^2}{\bar{u}}\left(\frac{n+1}{2}+l\right)+\sigma_m^2\right]=\frac{B}{n}+\frac{C}{\bar{u}}$ Model II; $C=\frac{A}{\Delta^2}v^2$	A: reworking/scrapping cost D: adjustment or control limit l: time lag in units Δ: tolerance of product characteristics m: target value
Chen (2004)	SPC	Extended D.S. Bai and K.T. Lee (1998)	Expected cost per unit Burr distribution Variable sampling interval	$\frac{q_0c_1S_0+c_2+c_3E(O)+(c_4+c_5n)(S_0+S_1)}{\frac{1}{\lambda}+E(O)+q_0t_1S_0+t_2}$	O: out-of-control period S_1: expected number of samples in OC S_0: expected number of samples in IC q_0: false alarm rate
Yang and Sheu (2007)	APC and SPC	Extended T. J. Lorensen and L.C. Vance 1986	Multivariate APC/EWMA Average quality cost Euclidean distance	$\frac{C_0}{\theta}+C_1+(-\varphi+nE+h\cdot ARL_1+\gamma_1T_1+\gamma_2T_2)$ $+\frac{SY}{ARL_0}+W+\left(\frac{a+bn}{h}\right)$ $\cdot\left(\frac{1}{\theta}-\varphi+nE+h\cdot ARL_1+\gamma_1T_1+\gamma_2T_2\right)$	E: expected time to sample T_1: expected time to discover the cause T_2: expected time to repair the process τ: expected time of cause occurrence s: expected number of samples
Kandananond (2007)	APC and SPC	False alarm cost, Loss cost per unit	Quadratic loss function Expected net savings	$\frac{R}{\lambda}\left\{\left(\left(\frac{A}{\Delta^2}\right)\sigma^2+\alpha F\right)+\left(\frac{A}{\Delta^2}\right)\right.$ $\times\left[\sigma^2l+\left(\delta^2(1-\theta^{T+1})^2+\sigma^2\right)T\right]p$ $\left.+\left(\frac{4\delta^2}{9}+\sigma^2\right)ARL_1\cdot q\right\}$ λ: occurrence rate of shift, l: lag period	R: product rate A: reworking/scrapping cost Δ: tolerance of product characteristics θ: moving average parameter δ: shift of size, T: adjustment period
Park and Reynolds (2008)	APC and SPC	Monitoring cost, C_M Adjustment cost, C_A Off target cost, C_T False alarm cost, C	Expected cost per unit Repeated adjustment Feedback adjustment EWMA	$C_M+C_A+C_T+\frac{C_F\cdot E(N_F)+C_T\cdot\left(\frac{E(S_1)}{\sigma_a^2}-E(T_1)\right)}{\frac{1}{p}+E(T_1)}$	N_F: number of false alarm in IC p: probability that a cause occurs T_1: OC period length S_1: sum of squared deviations
This paper	APC and SPC	Disturbance cost, c_D Diagnosis cost, c_M False alarm cost, c_F Reworking/scrapping cost, c_D	Quadratic loss function PID/MMSE controllers Compare two controllers Expected cost rate NHPP	$\frac{\left\{\left\{\left(\sum_{n=1}^{\infty}\frac{e^{-\int_0^{\tau}\lambda(s)ds}\left[\int_0^{\tau}\lambda(s)ds\right]^n}{(n-1)!}\right)E(C_{APC})+c_F\cdot\frac{\alpha\cdot e-\lambda_1\cdot h}{1-e-\lambda_1\cdot h}+c_D\cdot\left(\frac{c_A}{\Delta^2}\right)\left((\mu_0)^2+\frac{\lambda\mu_0}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\right)\right\}\right\}}{\frac{1}{\lambda_1}+ARL_1}+c_M$	R_P: product rate Δ: tolerance of product characteristics θ: moving average parameter N_A: number of adjustment in IC

controversial (Woodall and Montgomery 1999), many researchers are actively conducting the development of economic cost model/designs not only regarding SPC and APC individually but also the integration of SPC and APC as summarised in Table 5. Economic modelling of SPC brought many individuals from the engineering communities into the SPC field and fostered other collaboration of these researchers with statisticians (Woodall and Montgomery 1999). For practical economic models, the assumptions should be realistic and not too complicated for better applications. Many proposed approaches are too theoretical to apply in the industry and practical applications of such models will have a major impact in the theoretical and practical areas. One way to demonstrate their real applications is through case studies (Matos *et al*. 2008, Shanoun *et al*. 2011). Matos *et al*. (2008) integrate SPC and APC and show the case study of the integration in the pulp and paper industry. Furthermore, generally ARL is used to evaluate the performance of the SPC charts but other performance measures can be developed for practical use.

For future research, a greater synthesis of the theoretical change point and its detection and control in the APC-SPC integration models presents a challenging area of research which has significant consequences in practice. Use of new approaches for investigating the change point detection needs to be considered. For example, while procedures for change point analysis estimate specific locations of change point, the Bayesian procedure offers a probability distribution, the probability of a change point at each location in a sequence, such as prior distribution and posterior distribution (Barry and Hartigan 1993). Various SPC monitoring charts may also be integrated with a Bayesian tool for change point problems.

7. Concluding remarks

In this paper, we develop an economic model for the integrated SPC and APC controlled processes. We also propose a new measure of performance, LREC, to evaluate SPC charts for monitoring the APC controlled process. The LREC assigns a different weight proportional to the level of mean shift to each run length probability, and thus provides more economic information than the ARL. The results show that in a process with a positive value ϕ, the LREC with the MMSE controller is larger than that of the P controller since the MMSE controller automatically reduces the shift level in the output which makes detection difficult. Also, the changes of reworking cost have the most impact on the LREC. However, the changes of the disturbance cost, the autoregressive coefficients and the moving average coefficients do not significantly influence the LREC. On the other hand, changes of constant k_P and mean shift magnitudes affect the LREC significantly. It would be interesting to extend the similar concept of the integrated LREC criterion to evaluate the cost effectiveness of SPC monitoring under different controllers.

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