Ejemplos de uso de Qiskit y Braket

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CONICET

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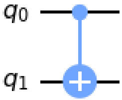
Outline

$$Cnot = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{1}$$



Notación: $|0\rangle \otimes |0\rangle := |00\rangle$

Base del espacio de dos qubits ($\mathbb{C}^2 \otimes \mathbb{C}^2$): $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

$$Cnot = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{2}$$

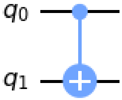


Table de verdad del Cnot (en la base computacional)

Input 1	**Input 2**	**Output 1**	**Output 2**
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

Cnot combinado con Hadamard

Aplico primero Hadamard al primer qubit:

$$(H|0\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

Luego, aplico Cnot (que combina a los dos qubits):

$$Cnot\frac{1}{\sqrt{2}}(|00\rangle+|10\rangle) = \frac{1}{\sqrt{2}}(Cnot|00\rangle+Cnot|10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \neq |\psi_1\rangle|\psi_2\rangle \implies \text{No separable} = \text{¡Entrelazado!}$$

Cnot combinado con Hadamard y X

Aplicamos primero X a los dos qubits:

$$(X|0\rangle)(X|0\rangle) = |11\rangle$$

Aplico primero Hadamard al primer qubit:

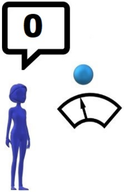
$$(H|1\rangle)|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|1\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle)$$

Luego, aplico Cnot (que combina a los dos qubits):

$$Cnot\frac{1}{\sqrt{2}}(|01\rangle - |11\rangle) = \frac{1}{\sqrt{2}}(Cnot|01\rangle - Cnot|11\rangle) = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

$$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \neq |\psi_1\rangle|\psi_2\rangle \implies \text{No separable} = \text{¡Entrelazado!}$$

Laboratorios rojo y azul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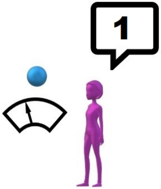
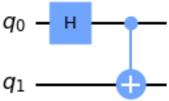

1


1
0


0
0



Si preparamos al sistema en el estado:

$$Cnot(H|0\rangle|0\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

(en la base computacional) sólo vamos a observar 00 y 11.

Si preparamos al sistema en el estado:

$$|Singlete\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$$

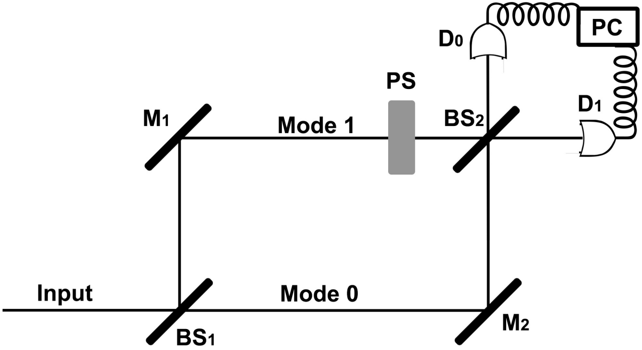
(en la base computacional) sólo vamos a observar 01 y 10.

Ejemplos

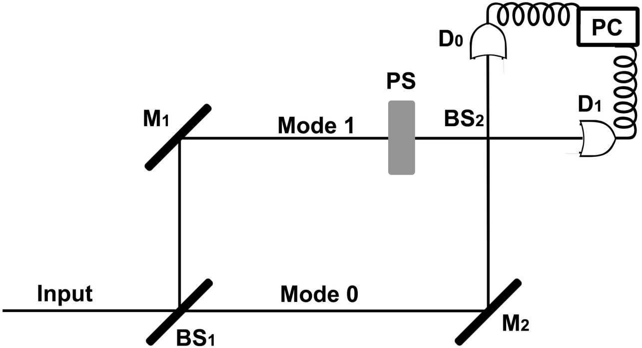
Ahora vemos ejemplos de cómo programar circuitos:

- Qiskit
- aws
  Amazon Braket

Mach-Zender: medición de interferencia



Mach-Zender: medición de camino



Brazo de abajo (Modo **0**):

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$

Brazo de arriba (Modo **1**):

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

Los dos separadores de haz se representan por la matriz:

$$BS_1 = BS_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \tag{5}$$

y el phase shifter viene dado por:

$$PS = \begin{pmatrix} 1 & 0 \\ 0 & \exp i\Delta \end{pmatrix} \tag{6}$$

A la salida del primer divisor de haz:

$$|\psi_1\rangle = BS_1|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{i}{\sqrt{2}}\end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \tag{7}$$

Luego, opera el phase shifter (cambiador de fase):

$$|\psi_2\rangle = PS|\psi_1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Delta} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + ie^{i\Delta}|1\rangle) \qquad (8)$$

Finalmente, aplico el segundo divisor de haz:

$$|\psi_3\rangle = BS_2|\psi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{ie^{i\Delta}}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{1}{2}(1-e^{i\Delta})\\ \frac{i}{2}(1+e^{i\Delta})\end{pmatrix} = \tag{9}$$

$$\frac{1}{2}((1-e^{i\Delta})|0\rangle + i(1+e^{i\Delta})|1\rangle) \tag{10}$$

El estado final queda:

$$|\psi_3\rangle = \frac{1}{2}((1 - e^{i\Delta})|0\rangle + i(1 + e^{i\Delta})|1\rangle) \tag{11}$$

$$|\psi_3\rangle = \frac{e^{i\frac{\Delta}{2}}}{2}((e^{-i\frac{\Delta}{2}} - e^{i\frac{\Delta}{2}})|0\rangle + i(e^{-i\frac{\Delta}{2}} + e^{i\frac{\Delta}{2}})|1\rangle) \tag{12}$$

$$|\psi_3\rangle = \frac{e^{i\frac{\Delta}{2}}}{2}((e^{-i\frac{\Delta}{2}} - e^{i\frac{\Delta}{2}})|0\rangle + i(e^{-i\frac{\Delta}{2}} + e^{i\frac{\Delta}{2}})|1\rangle) \tag{13}$$

$$|\psi_3\rangle = ie^{i\frac{\Delta}{2}}((-\sin(\frac{\Delta}{2}))|0\rangle + (\cos(\frac{\Delta}{2}))|1\rangle) \tag{14}$$

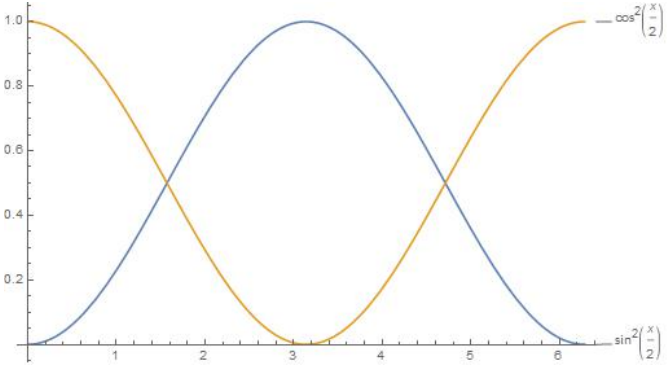
Uso que $e^{i\frac{\Delta}{2}} = \cos(\frac{\Delta}{2}) + i\sin(\frac{\Delta}{2})$:
$e^{-i\frac{\Delta}{2}} - e^{i\frac{\Delta}{2}} = -2i\sin(\frac{\Delta}{2})$
$e^{-i\frac{\Delta}{2}} + e^{i\frac{\Delta}{2}} = 2\cos(\frac{\Delta}{2})$

Por ende, las probabilidades de detección en D_1 y D_2 vienen dadas por:

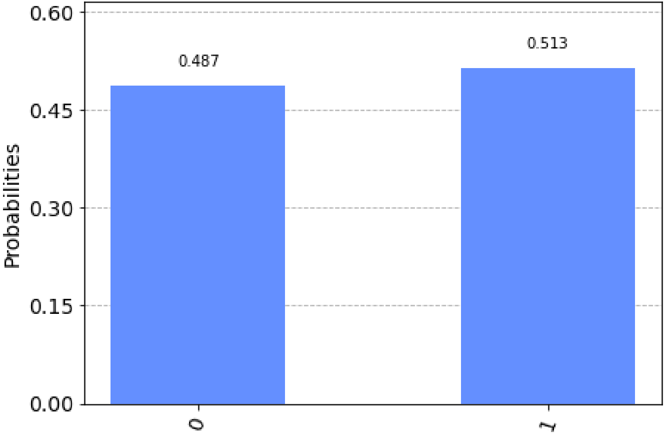
$$P_0 = |\langle 0|\psi_3\rangle|^2 = \sin^2(\frac{\Delta}{2}) \tag{15}$$

$$P_1 = |\langle 1|\psi_3\rangle|^2 = \cos^2(\frac{\Delta}{2}) \tag{16}$$

Gráfico de las cuentas de fotones en función de la fase



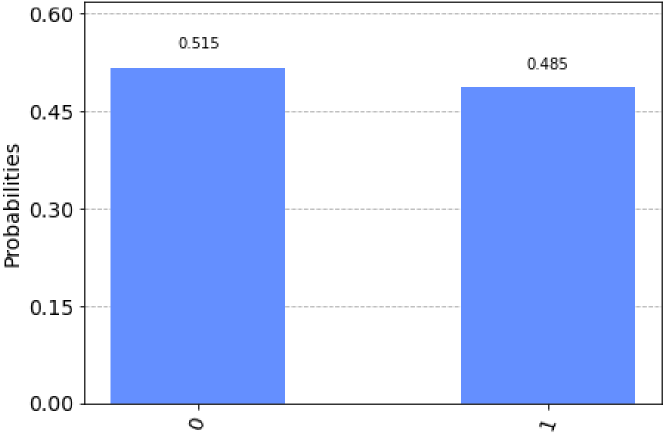
Photon counts: '0': 487, '1': 513



$$|\psi_3\rangle = ie^{i\frac{\pi}{4}}(-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) \tag{17}$$

Cuentas de fotones para 1000 corridas ($\Delta = \frac{\pi}{2}$)

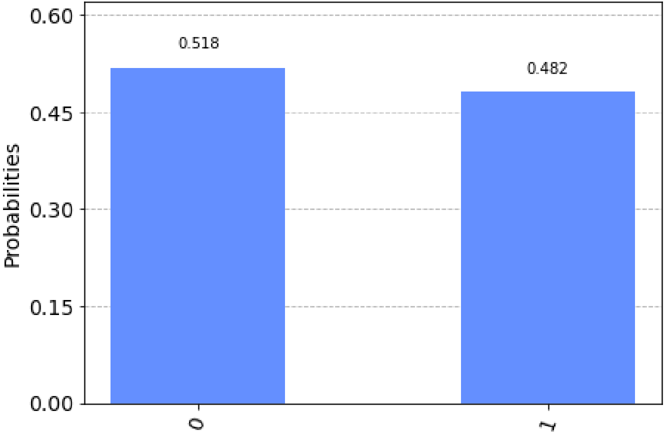
Photon counts: '0': 515, '1': 485



$$|\psi_3\rangle = ie^{i\frac{\pi}{4}}(-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) \tag{18}$$

Cuentas de fotones para 1000 corridas ($\Delta = \frac{\pi}{2}$)

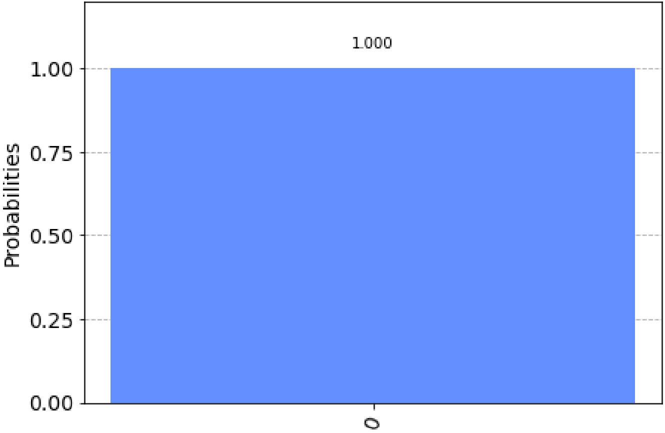
Photon counts: '0': 518, '1': 482



$$|\psi_3\rangle = ie^{i\frac{\pi}{4}}(-\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) \tag{19}$$

Cuentas de fotones para 1000 corridas ($\Delta = \pi$)

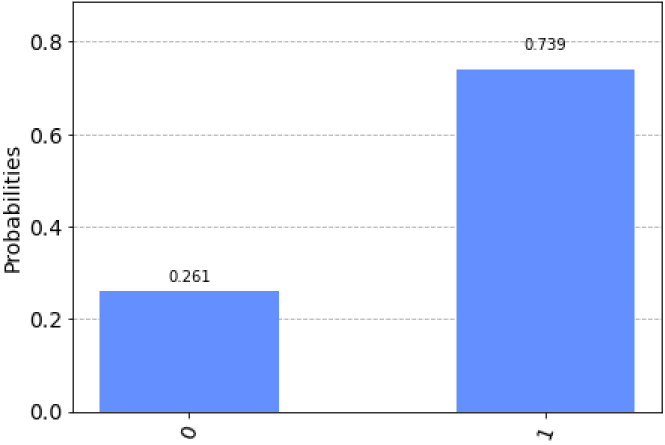
Photon counts: '0': 1000



$$|\psi_3\rangle = -ie^{i\frac{\pi}{2}}|0\rangle \tag{20}$$

Cuentas de fotones para 1000 corridas ($\Delta = \frac{\pi}{3}$)

Photon counts: '0': 261, '1': 739



$$|\psi_3\rangle = ie^{i\frac{\pi}{6}}((-\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle) \tag{21}$$

Si, por el contrario, hubiésemos implementado el contexto de detección de caminos, sólo tendríamos $|\psi_2\rangle$, y las probabilidades vendrían dadas por:

$$P_0 = |\langle 0|\psi_2\rangle|^2 = \frac{1}{2} \tag{22}$$

$$P_1 = |\langle 1|\psi_2\rangle|^2 = \frac{1}{2} \tag{23}$$