1 Linearisation

$$\ddot{\alpha} = \frac{3}{2l}\left(g\sin(\alpha) - \ddot{x}_w\cos(\alpha)\right) \tag{1}$$

$$f_{\text{lin}} = f(x_0) + \frac{\partial f}{\partial x}(x_0)\cdot(x - x_0) + \underbrace{r(x)}_{\approx 0} \tag{2}$$

$$\begin{aligned}\ddot{\alpha} &= \frac{3}{2l}\left[g\sin(x_0) - \ddot{x}_w\cos(x_0) + (g\cos(x_0) + \ddot{x}_w\sin(x_0))\cdot(\alpha - x_0)\right] \Big| x_0 = 0 \\ &= \frac{3}{2l}\left[g\sin(0) - \ddot{x}_w\cos(0) + (g\cos(0) + \ddot{x}_w\sin(0))\cdot(\alpha - 0)\right] \\ &= \frac{3}{2l}\left(g\alpha - \ddot{x}_w\right)\end{aligned} \tag{3}$$

$$\ddot{x}_w = -\frac{1}{T_1}\dot{x}_w + \frac{1}{T_1}u$$

$$\ddot{\alpha} = \frac{3}{2l}\left(g\alpha - \ddot{x}_w\right) \tag{4}$$

$$\Rightarrow \ddot{\alpha} = \frac{3}{2l}\left(g\alpha + \frac{1}{T_1}\dot{x}_w - \frac{1}{T_1}u\right) \tag{5}$$

$$\tag{6}$$

$$\begin{pmatrix}\dot{x}_w\\ \ddot{x}_w\\ \dot{\alpha}\\ \ddot{\alpha}\end{pmatrix} = \begin{pmatrix}0 & 1 & 0 & 0\\ 0 & -\frac{1}{T_1} & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{3}{2lT_1} & \frac{3g}{2l} & 0\end{pmatrix}\begin{pmatrix}x_w\\ \dot{x}_w\\ \alpha\\ \dot{\alpha}\end{pmatrix} + \begin{pmatrix}0\\ \frac{1}{T_1}\\ 0\\ -\frac{1}{T_1}\end{pmatrix}u \tag{7}$$

$$\begin{pmatrix}\dot{x}_w\\ \dot{\alpha}\end{pmatrix} = \begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x_w\\ \dot{x}_w\\ \alpha\\ \dot{\alpha}\end{pmatrix} \tag{8}$$

2 Characteristics

please refer Matlab:
$\Rightarrow$ controllable
$\Rightarrow$ not observable

3 Control

3.1 Soar up part

For $|\alpha| > \frac{\pi}{4}$

$$\begin{aligned} \dot{\alpha} > 0 \qquad & \ddot{x}_w < 0 \\ \dot{\alpha} < 0 \qquad & \ddot{x}_w > 0 \\ \Rightarrow \quad & k(\dot{\alpha}) = u \\ \Rightarrow \quad F = \begin{pmatrix} 0 & 0 & 0 & k \end{pmatrix} & \end{aligned} \tag{9}$$

3.2 Balance part

For $|\alpha| < \frac{\pi}{4}$

F-Matrix with Riccati

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad r = 1 \tag{10}$$

$$F = \begin{pmatrix} -1 & 3.1652 & 5.8983 & 1.3573 \end{pmatrix} \tag{11}$$

F-Matrix with pole placement

$$p = \begin{pmatrix} -10 & -11 & -12 & -13 \end{pmatrix} \tag{12}$$

$$F = \begin{pmatrix} -23.0316 & 2.0397 & 6.4678 & 1.2227 \end{pmatrix} \tag{13}$$