

1. Take derivatives to zero to get the minimum.

$$\langle \text{MSE} \rangle \quad h(x') = \min \sum_{i=1}^n (x - x')^2$$

$$h(x') = 0$$

$$\Rightarrow \sum_{i=1}^n (x^2 - 2xx' + x'^2) = 0$$

$$\sum_{i=1}^n (-2x + 2x') = 0$$

$$\cancel{\sum_{i=1}^n} x = \cancel{\sum_{i=1}^n} x'$$

$$\sum_{i=1}^n x = nx'$$

$$\Rightarrow x' = \frac{\sum_{i=1}^n x}{n} \text{ (average of all } x) = 163.5 \text{ cm}$$

$$\langle \text{MSPE} \rangle \quad h(x') = \min \sum_{i=1}^n \left(\frac{x - x'}{x} \right)^2 = 0$$

$$h(x') = \sum_{i=1}^n \left(\frac{x^2 - 2xx' + x'^2}{x^2} \right) = \sum_{i=1}^n (1 - 2x'x^{-1} + x'^2x^{-2})$$

$$\Rightarrow h(x')' = \sum_{i=1}^n (-2x^{-1} + 2x'x^{-2}) = 0$$

$$\sum_{i=1}^n (x'x^{-2}) = \sum_{i=1}^n x^{-1}$$

$$x' = \sum_{i=1}^n \frac{1}{x} \div \sum_{i=1}^n \frac{1}{x^2} \text{ (sigma can't combine with product and division)}$$

$$\div 155.42 \text{ cm}$$