CS 480/680
Introduction to Machine Learning

Lecture 10
Decision Trees

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Interpretability is a concern when human life is on the line

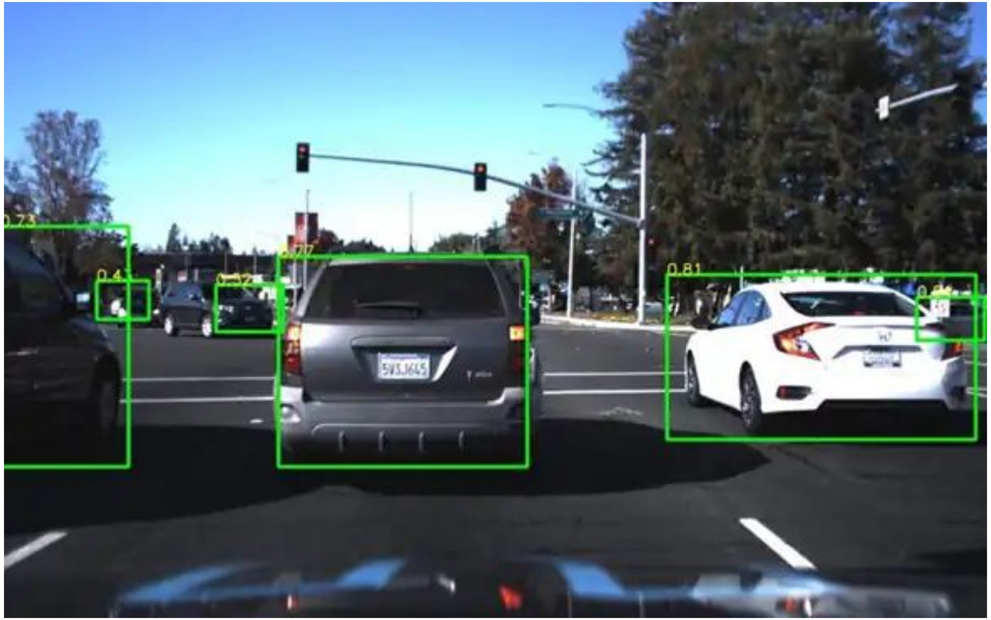
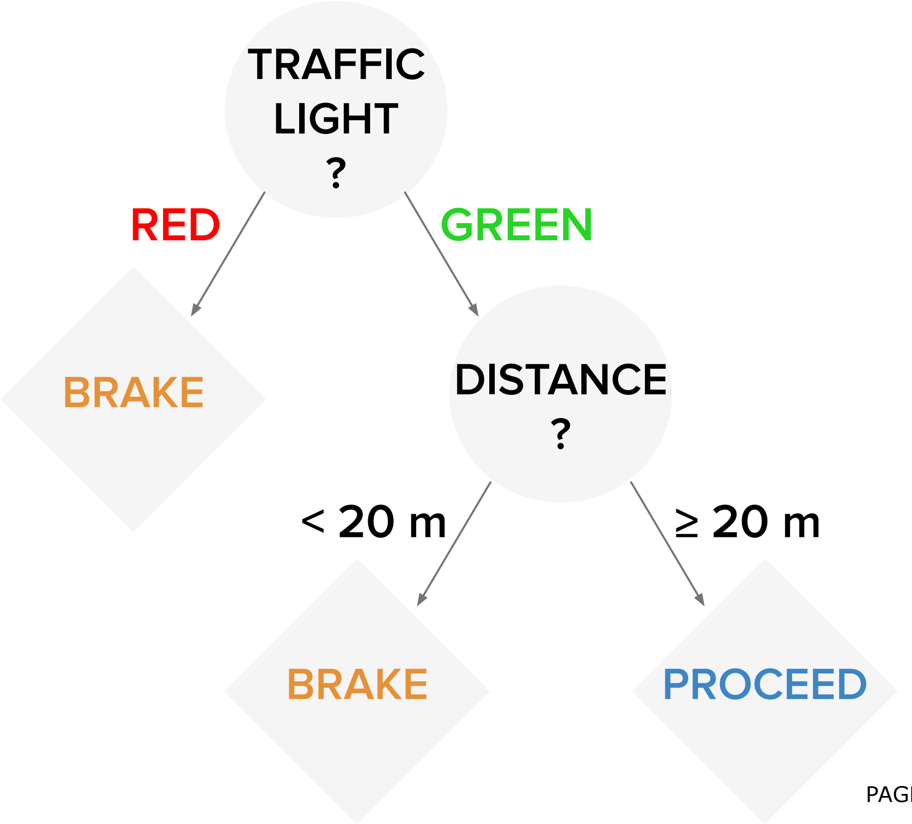
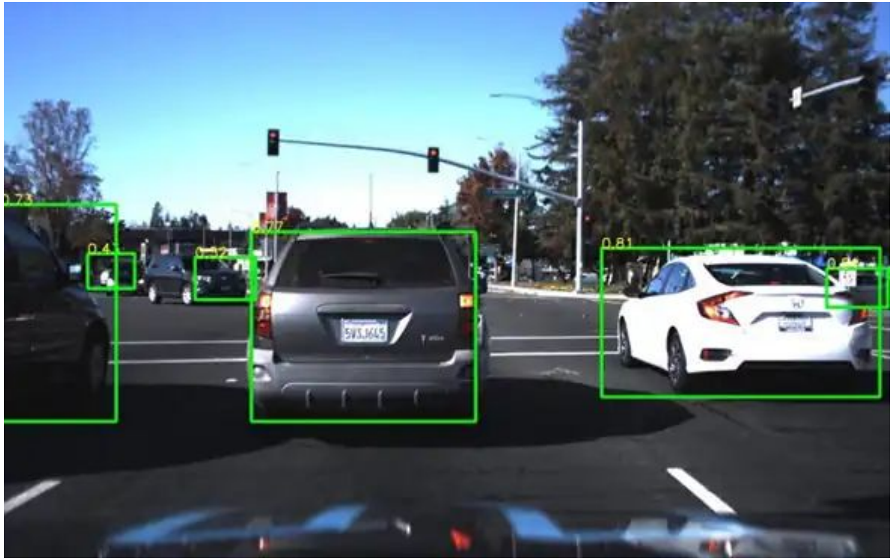


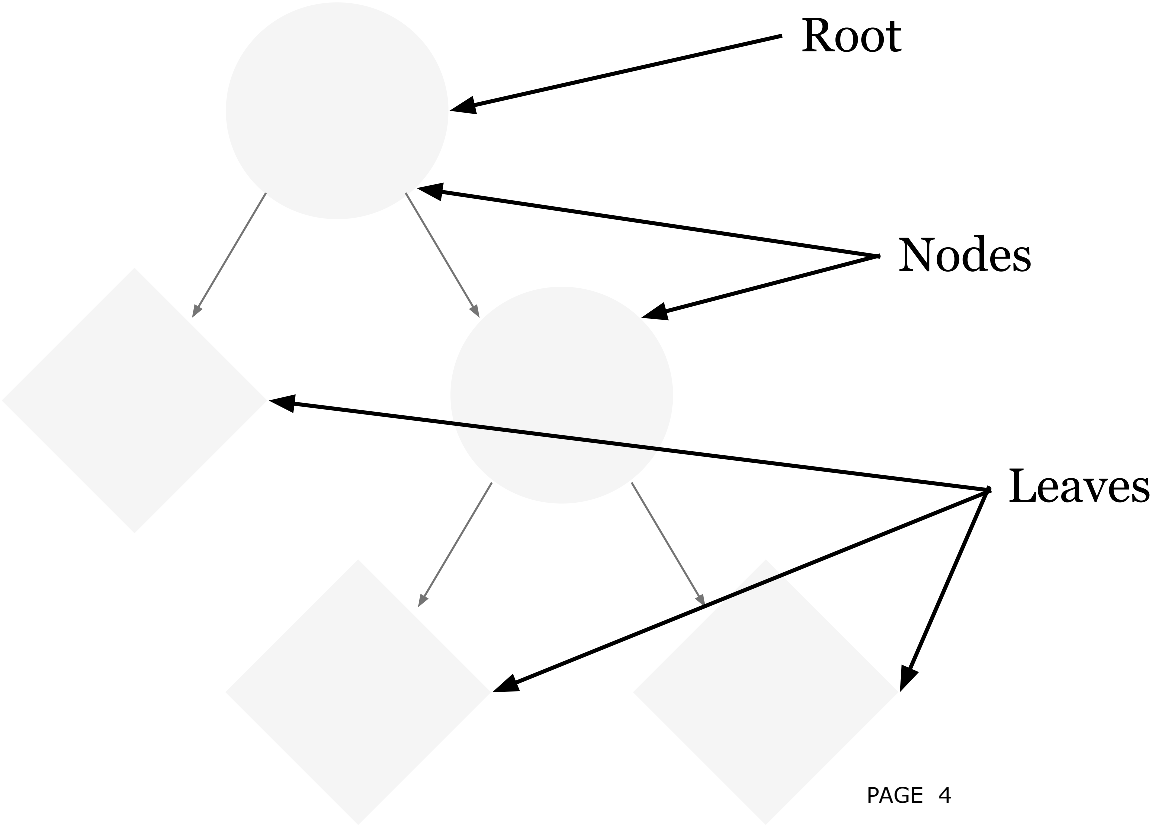
Figure: www.labellerr.com

A decision tree is a recursive partitioning model

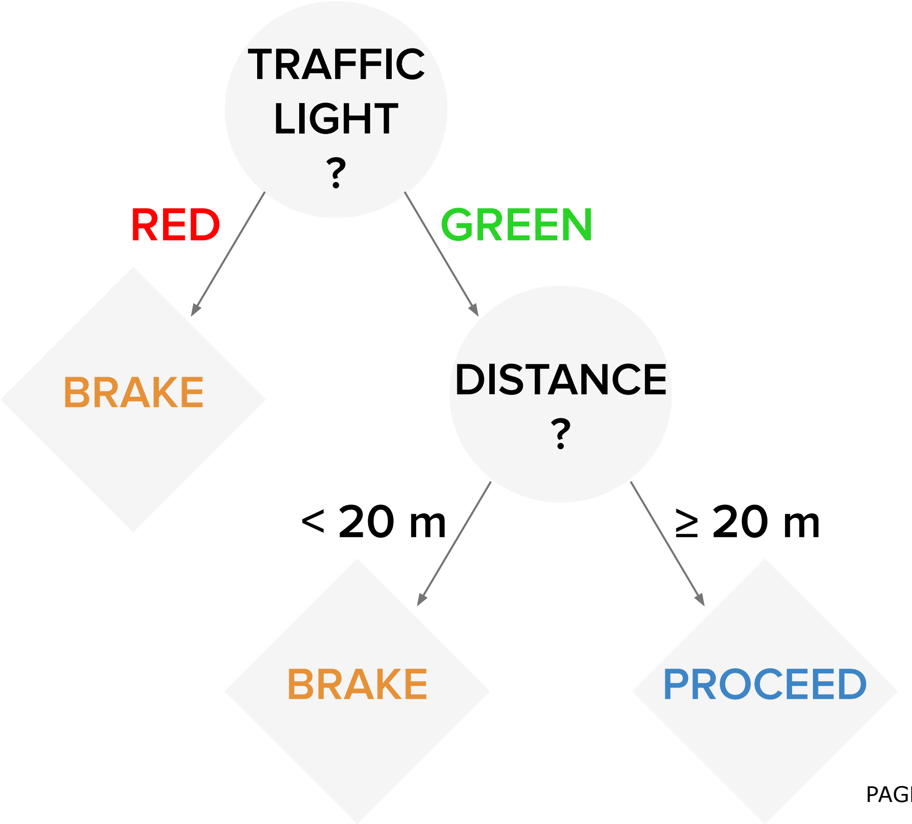


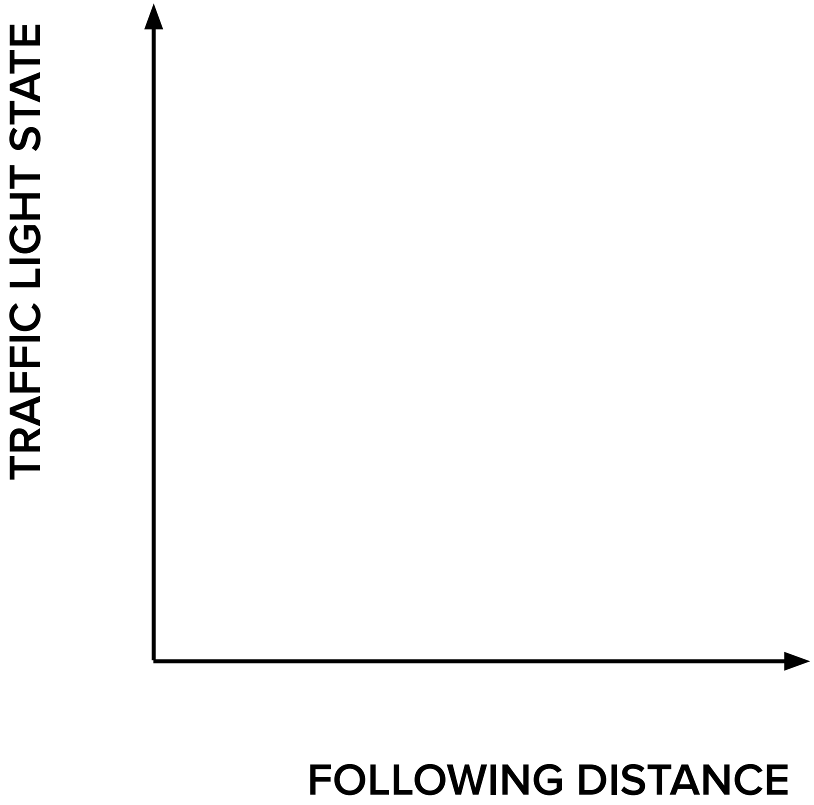


Anatomy of a tree

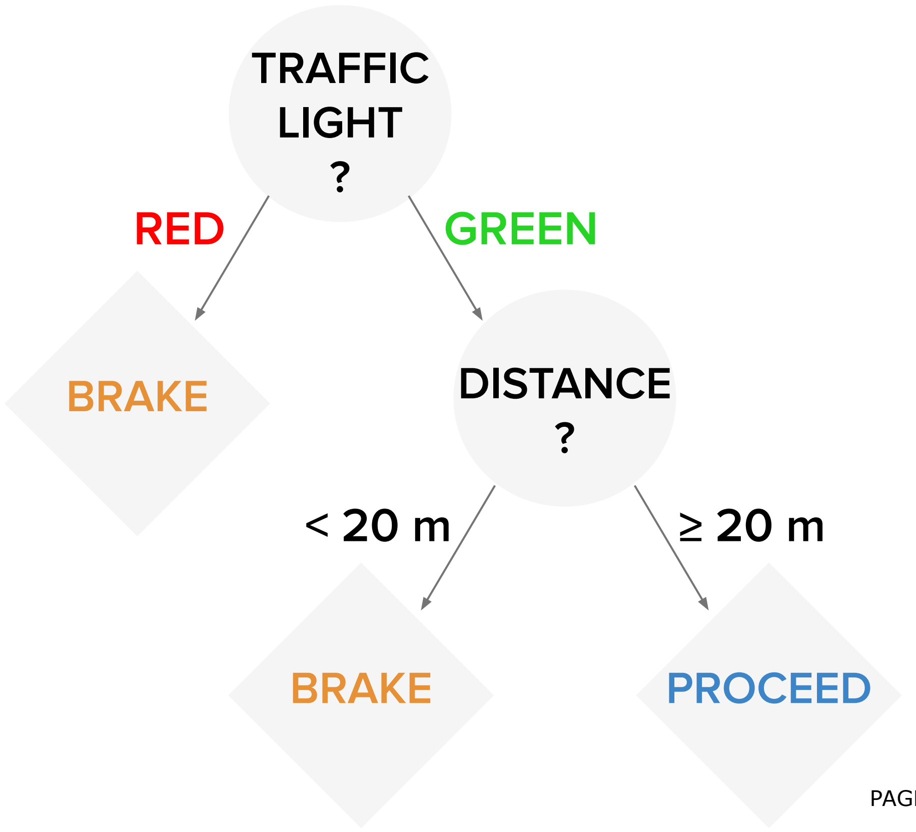


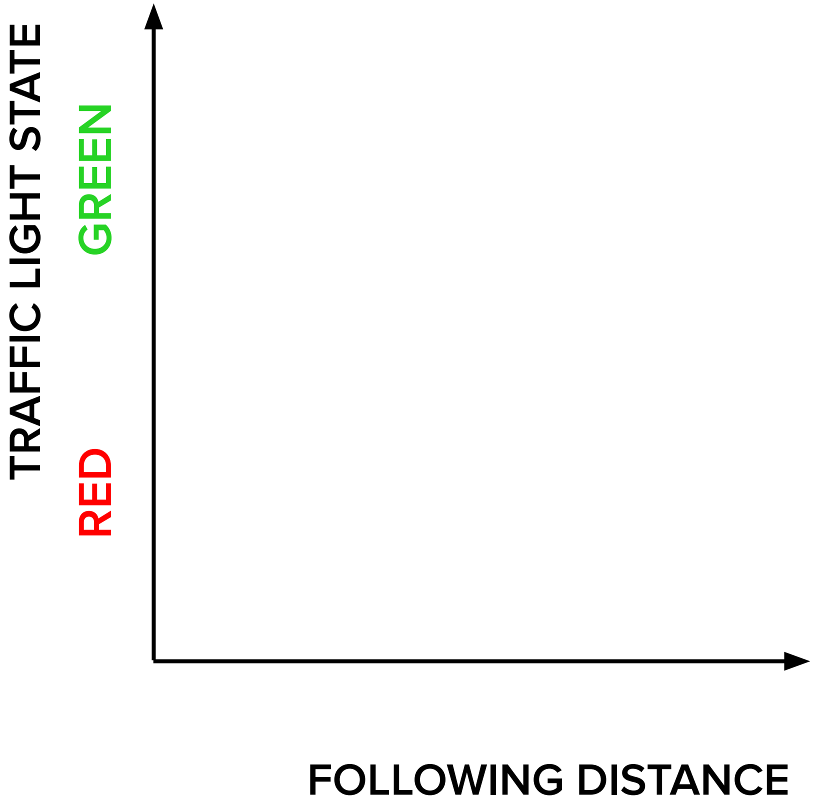
Visualizing decision trees



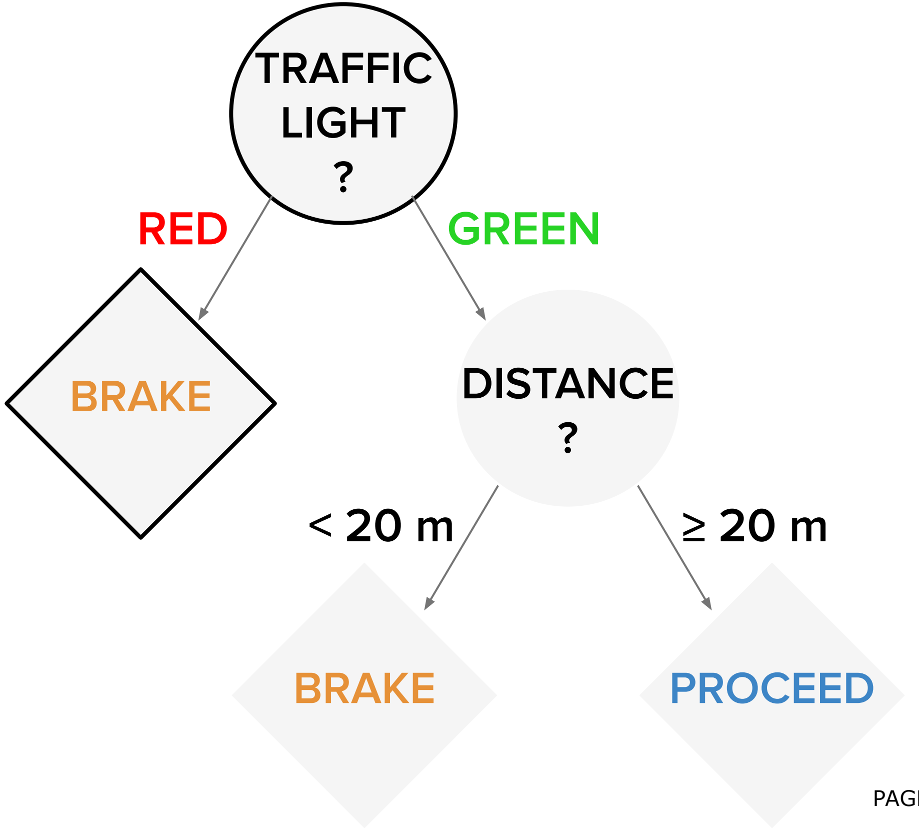


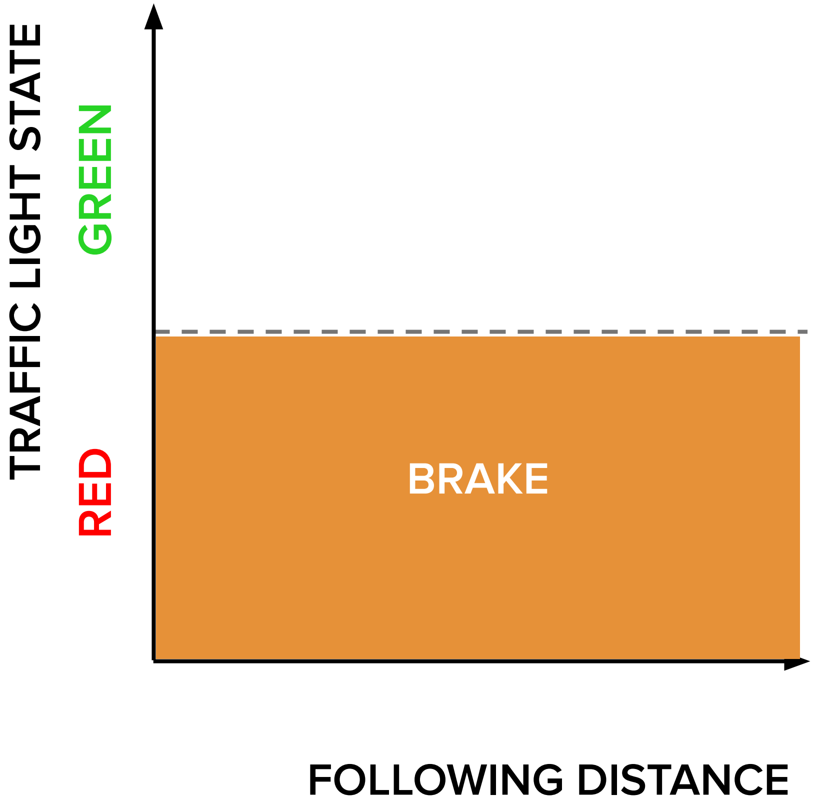
Visualizing decision trees



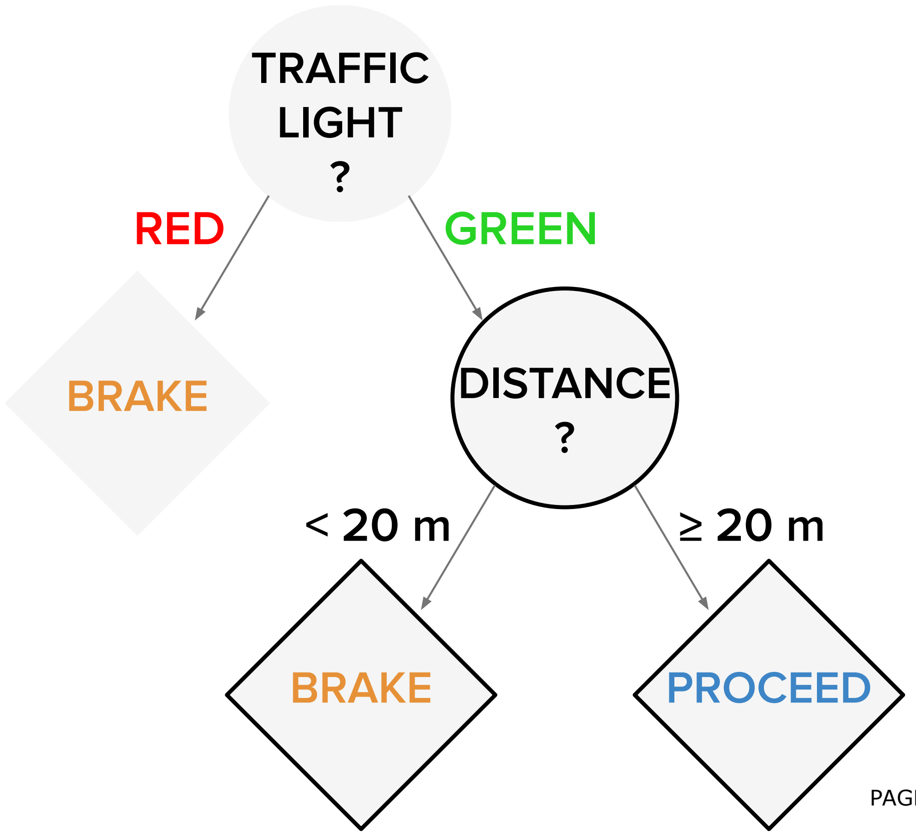


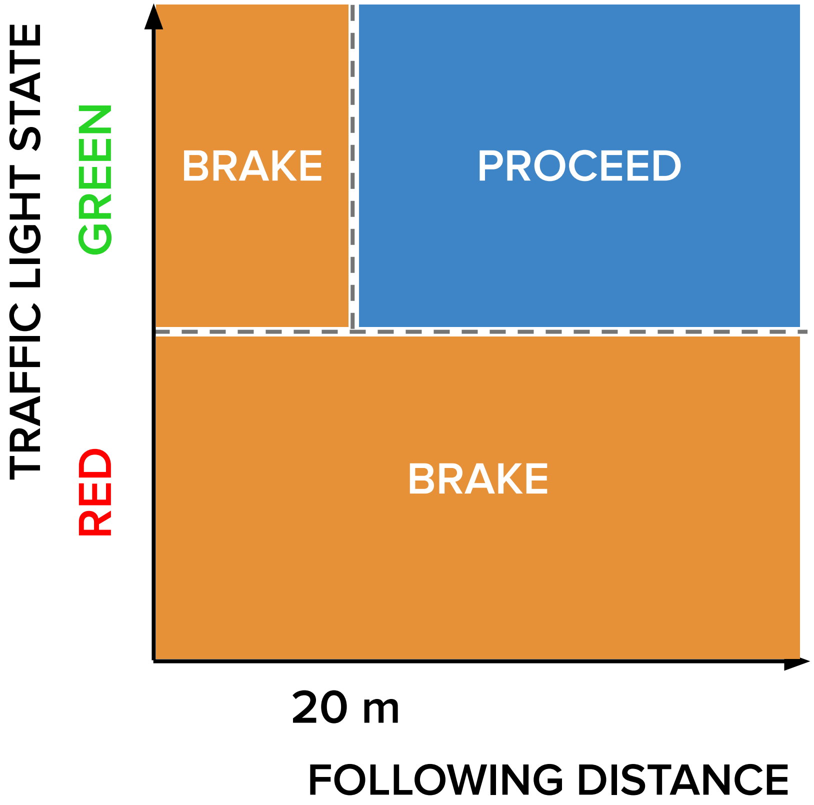
Visualizing decision trees



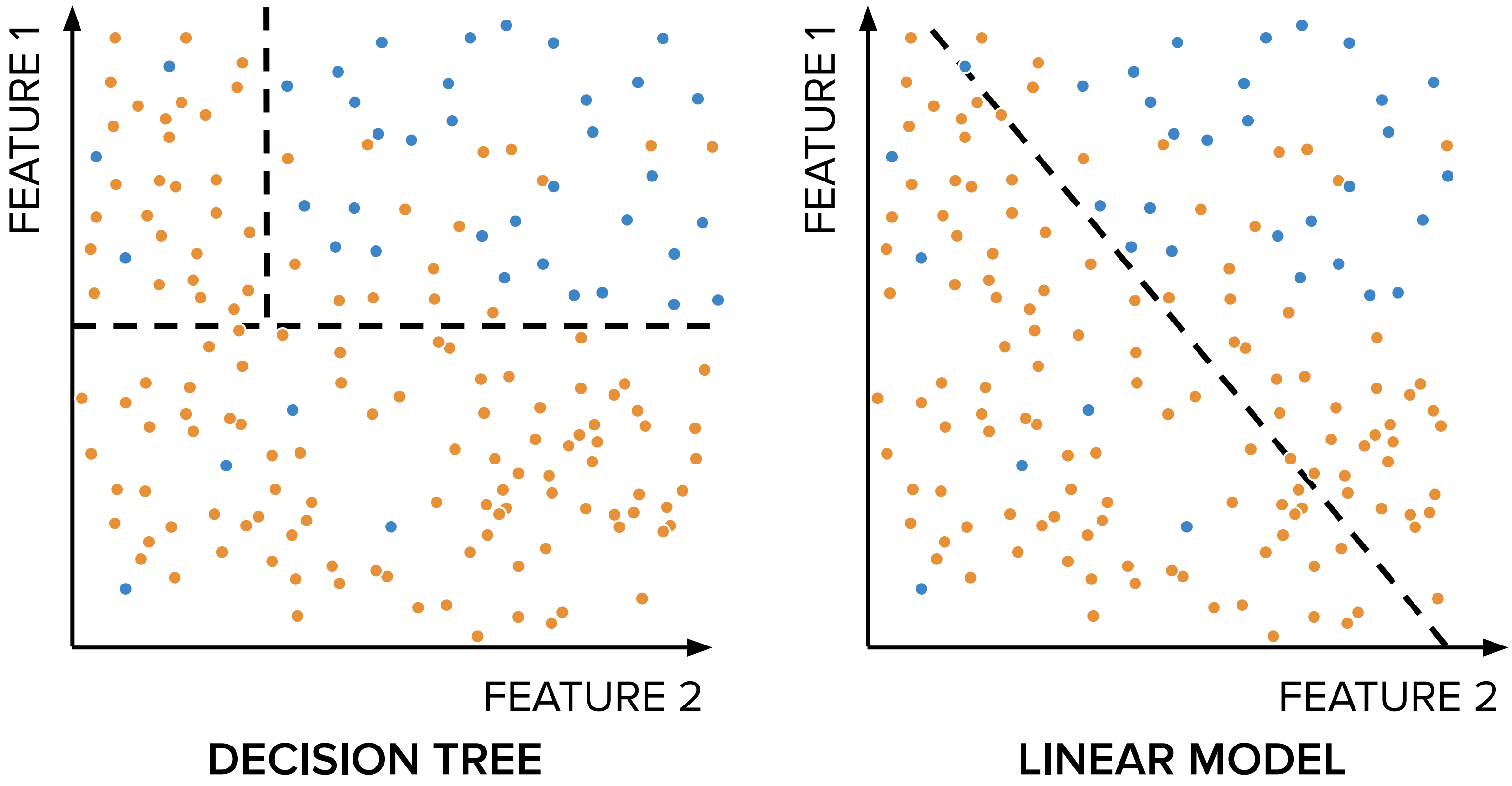


Visualizing decision trees





Decision trees can approximate certain nonlinear functions



Predictions correspond to the majority class within a region

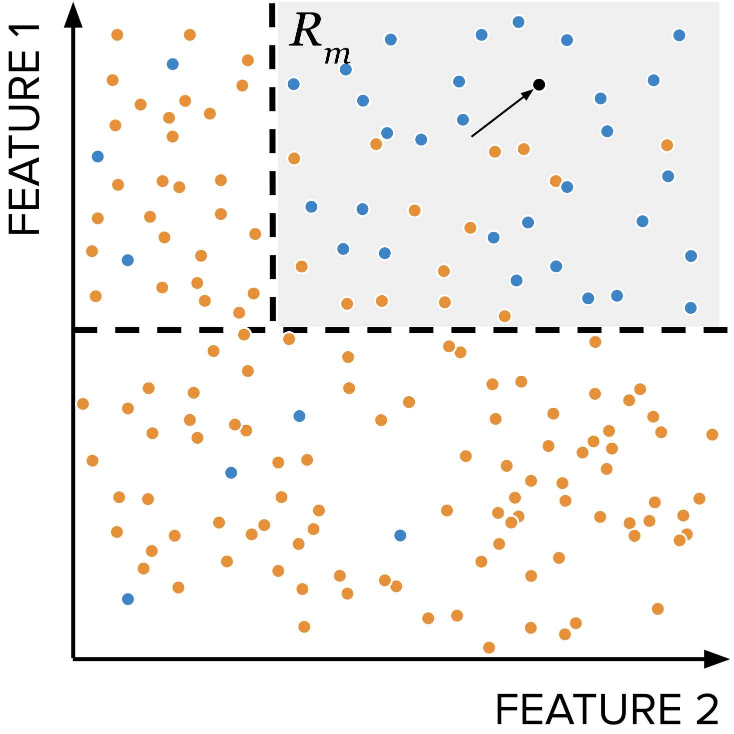
The prediction made for an observation x_i within a subregion R_m of the domain of the data is the majority class within that region:

$$\hat{y}_i = \underset{k}{\operatorname{argmax}}\, \bar{p}_{mk}$$

where $\bar{p}_{mk}$ is the empirical fraction of observations with label k within the subregion R_m:

$$\bar{p}_{mk} = \frac{1}{N_m} \sum_{x_i \in R_m} \mathbb{1}(y_i = k)$$

and N_m is the number of observations within partition R_m.



Growing a tree means defining the next node

$$(\hat{j}, \hat{t}) = \underset{j,t}{\operatorname{argmin}} |S_0| l(S_0) + |S_1| l(S_1)$$

$$\begin{aligned} l(S) &= |S_0| l(S_0) + |S_1| l(S_1) \\ &= |S_0| l\left(\{(x_i, y_i) \in S_0 : x_{ij} \leq t\}\right) \\ &+ |S_1| l\left(\{(x_i, y_i) \in S_1 : x_{ij} > t\}\right) \end{aligned}$$

Loss functions: misclassification error

$$l(S_m) = \frac{1}{N_m} \sum_{x_i \in R_m} \mathbb{1}(y_i \neq \hat{y}_i)$$
$$= 1 - \max_k \bar{p}_{mk}$$

Characterization	Example 1	Example 2
Predicted labels $\{\hat{y}_i\}$	{ 0, 0, 0, 0 }	{ 0, 0, 0, 0 }
True labels $\{y_i\}$	{ 0, 0, 0, 0 }	{ 0, 0, 1, 1 }
$\bar{p}_0$	1	0.5
l_0	0	0.5
Comments	Perfect Prediction	50% Misclassification

Other loss functions

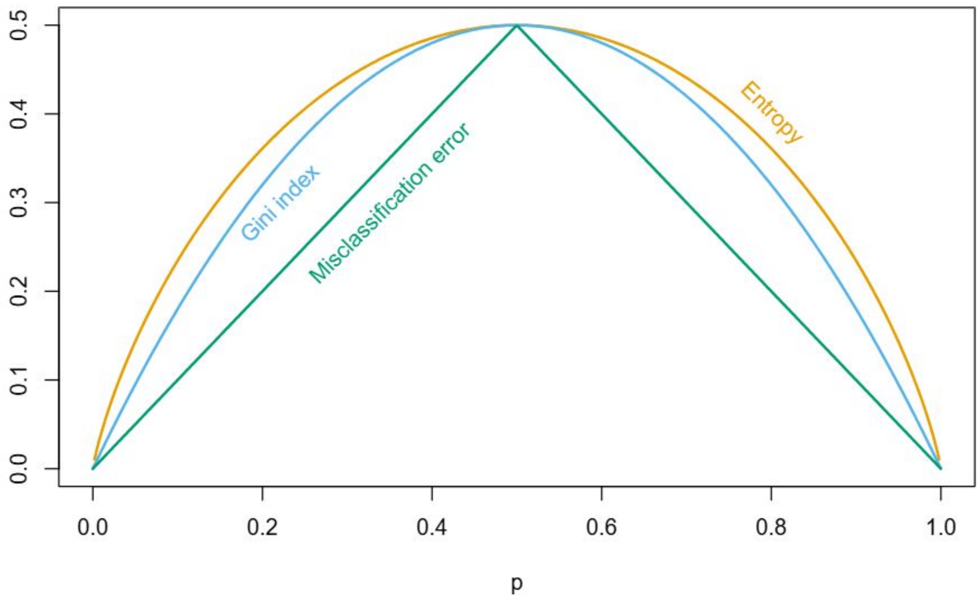
Entropy:

$$l(S_m) = -\sum_{k\in\{0,1\}} \bar{p}_{mk} \log \bar{p}_{mk}$$

Gini Index:

$$l(S_m) = \sum_{k\in\{0,1\}} \bar{p}_{mk}(1 - \bar{p}_{mk})$$

Comparing loss functions



Example: Let's learn a tree for this dataset

Traffic Light Color	Following Distance (m)	Vehicle Decision
Red	5.0	Brake
Green	5.0	Brake
Green	8.0	Brake
Green	10.0	Brake
Red	15.0	Brake
Green	20.0	Cruise
Red	30.0	Brake
Green	30.0	Cruise
Green	50.0	Cruise
Red	80.0	Cruise

Growing the tree

1. Select Gini Index as loss function

1. Define the Root Node

Split on Traffic Light Condition?

- RED: “Brake”: 0.75, "Cruise": 0.25

  Gini = 0.75 * (1 - 0.75) + 0.25 * (1 - 0.25) = 0.1875 + 0.1875 = 0.375.

- GREEN: “Brake”: 0.5, “Cruise”: 0.5

  Gini = 0.5 * (1 - 0.5) + 0.5 * (1 - 0.5) = 0.25 + 0.25 = 0.5

Total loss for split = (4/10) * 0.375 + (6/10) * 0.5 = 0.15 + 0.3 = **0.45**

Split on distance?

- <= 20 m: “Brake”: 5/6 = 0.833, “Cruise”: 1/6 = 0.167

  Gini = 0.833 * (1 - 0.833) + 0.167 * (1 - 0.167) = 0.1875 + 0.1875 = 0.278

- > 20 m: “Brake”: 1/4 = 0.25, “Cruise”: 3/4 = 0.75

  Gini = 0.25 * (1 - 0.25) + 0.75 * (1 - 0.75) = 0.1875 + 0.1875 = 0.375

Total loss for split = (6/10) * 0.278 + (4/10) * 0.375 = 0.1668 + 0.15 = **0.3168**

3. Select split on distance with a threshold of 20 m for the root node.

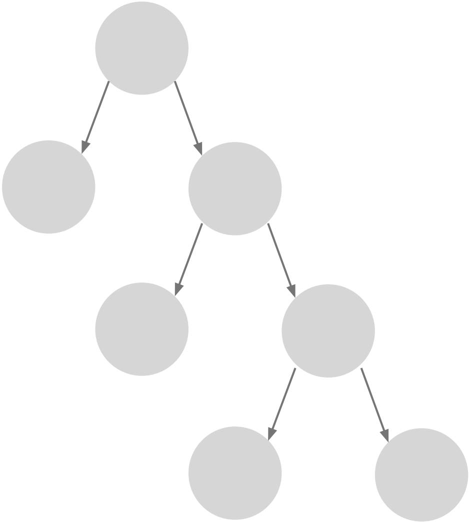
Stopping criteria

- Have achieved homogeneity in leaves
- Improvements are negligible
  - $\Delta = l(S_{OLD}) - (|S_0| \; l(S_0) + |S_1| \; l(S_1)) < \delta$
- Leaves are sparse
  - There are
- The tree has grown to a certain depth (height?)
  - Decision stump: One feature, one threshold
- The algorithm has run for some amount of time

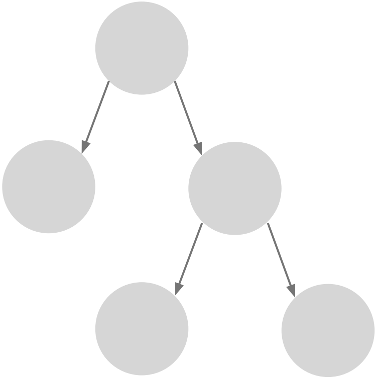
Pruning a tree

- Grow the tree fully, then regularize using hyperparameter α

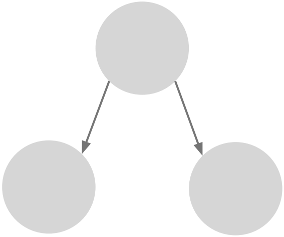
$$min \sum_v l_v(S) + \alpha N_v$$



$N_v = 4$



$N_v = 3$



$N_v = 2$

Now that we're at the end of the lecture, you should be able to…

- ★ Identify the components and structure of a decision tree, including **nodes, leaves, partitions**, and **thresholds**.
- ★ Implement a decision tree model by applying **recursive partitioning techniques**.
- ★ Differentiate between commonly-used loss functions and impurity measures (**entropy**, **Gini index**, and **misclassification error**).
- ★ Recognize when a decision tree can be used in **practical applications**.
- ★ Recommend strategies to **improve robustness**.

Errata

- On slide 16, the loss for the split on distance with a threshold of 30 m was miscalculated, and would actually have produced identical loss to a split on traffic light condition. The slides now consider the case of a split on 20 m, which produces lower loss than the split on traffic light condition.