CS480/680: Introduction to Machine Learning

Differential Privacy

Saber Malekmohammadi

UNIVERSITY OF WATERLOO | FACULTY OF MATHEMATICS
DAVID R. CHERITON SCHOOL OF COMPUTER SCIENCE

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The Netflix Challenge

	Inside Out	Good Will Hunting	Mean Girls	Terminator	Titanic	Warrior
Tina Fey	3	1	5	1	?	1
Helen Mirren	2	?	?	2	5	1
Sylvester Stallone	1	3	1	4	2	5
Tom Hanks	?	3	1	?	4	3
George Clooney	2	2	1	3	1	4

- <user, movie, date of rating, rating>
- $\sim$1M ratings, .5M users, 20k movies

1M Prize

NETFLIX
2009
DATE 09-21-09
PAY TO THE ORDER OF BellKor's Pragmatic Chaos
$ 1,000,000 00
AMOUNT: ONE MILLION 00/100
FOR The Netflix Prize
Reed Hastings

Lawsuit

Anonymization is not Enough

ZIP Code	*Birth Date*	*Gender*	*Race*
33171	7/15/71	m	Caucasian
02657	2/18/73	f	Black
20612	3/12/75	m	Asian

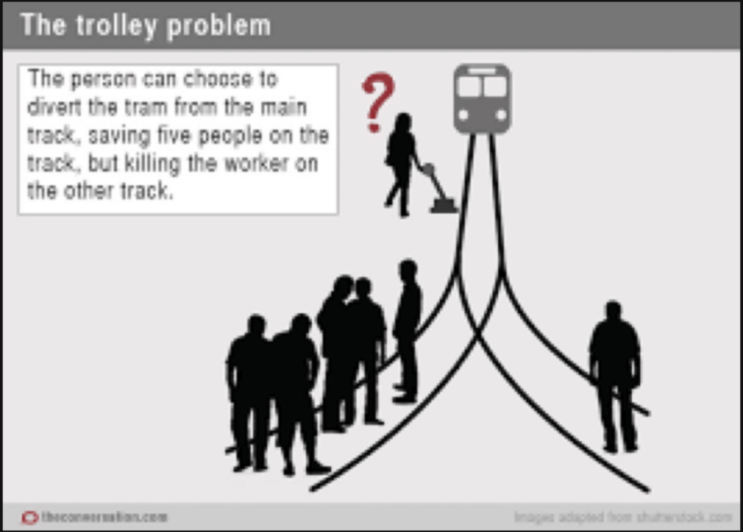
Table 2. Deidentified Data that Are Not Anonymous.

The 1997 voting list for Cambridge, Massachusetts, contains demographics on 54,805 voters. Of these, birth date, which contains the month, day, and year of birth, alone can uniquely identify the name and address of 12 percent of the voters. One can identify 29 percent of the list by just birth date and gender, 69 percent with only a birth date and a 5-digit ZIP code, and 97 percent (53,033 vot-

birth date alone	12%
birth date and gender	29%
birth date and 5-digit ZIP code	69%
birth date and full postal code	97%

Table 3. Uniqueness of Demographic Fields in Cambridge, Massachusetts, Voter List.

Just Sacrifice A Few?



Restricted Access

Example

- Consider a medical study about smoking and cancer
- Should a smoker participate?
- If yes, may lead to higher insurance premium
- But may also benefit from learning health risks
- Has the smoker's privacy been compromised?

$\boxed{\text{Participate or not, impact on the smoker is likely the same}}$

Have you cheated in any exam?

Randomized Response

- Want to estimate the percentage of cheaters
- If ask bluntly, almost certainly will under-estimate
- Toss a coin: head, answer honestly; tail, answer randomly
  - cheaters: w.p. $\frac{3}{4}$ say yes
  - non-cheaters: w.p. $\frac{1}{4}$ say yes
  - $\frac{3}{4}p + \frac{1}{4}(1-p) = \frac{1}{4} + \frac{1}{2}p =$ percentage of yes
- Plausible deniability for everyone
- What happens if we ask this question repeatedly?

S. L. Warner. "Randomised response: a survey technique for eliminating evasive answer bias". *Journal of the American Statistical Association*, vol. 60, no. 309 (1965), pp. 63–69.

Differential Privacy

- Let $\mathsf{M} : \mathcal{D} \to \mathcal{Z}$ be a randomized mechanism
- (ϵ, δ)-DP if for any $D, D' \in \mathcal{D}$ differing by one data point, for any event $E \subseteq \mathcal{Z}$,

$$\Pr[\mathsf{M}(D) \in E] \leq \exp(\epsilon) \cdot \Pr[\mathsf{M}(D') \in E] + \delta$$

  - dataset D, D' fixed; randomness from the mechanism
  - the smaller ϵ or δ is, the stricter the privacy requirement
- $(\epsilon, 0)$-DP if $\delta = 0$, a.k.a. ϵ-DP
- ϵ (roughly) bounds log odds ratio: $\boxed{\epsilon \leq 1}$ often considered "good"
- δ allows rare, possibly catastrophic event (to trade utility): often, $\boxed{\delta \ll 1/|\mathcal{D}|}$

C. Dwork and A. Roth. "The algorithmic foundations of differential privacy". *Foundations and Trends in Theoretical Computer Science*, vol. 9, no. 3-4 (2014), pp. 211–407.

Randomized Response is $(\log 3, 0)$-DP

$$\log \frac{\Pr[\mathsf{M}(D) \in E]}{\Pr[\mathsf{M}(D') \in E]} = \log \frac{\int_E p(\mathbf{x})\,\mathrm{d}\mathbf{x}}{\int_E q(\mathbf{y})\,\mathrm{d}\mathbf{y}} = \log \int_E \frac{p(\mathbf{x})}{q(\mathbf{x})} \cdot \frac{q(\mathbf{x})}{\int_E q(\mathbf{y})\,\mathrm{d}\mathbf{y}}\,\mathrm{d}\mathbf{x}$$

$$(\text{Jensen's inequality}) \leq \int_E \log\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) \cdot \frac{q(\mathbf{x})}{\int_E q(\mathbf{y})\,\mathrm{d}\mathbf{y}}\,\mathrm{d}\mathbf{x}$$

$$(\text{mean} \leq \text{max}) \leq \max_{\mathbf{x}} \log \frac{p(\mathbf{x})}{q(\mathbf{x})} \leq \epsilon$$

- Consider when D has a cheater and D' has a non-cheater:
  - $\log \frac{\Pr[\mathsf{M}(D)=\text{Yes}]}{\Pr[\mathsf{M}(D')=\text{Yes}]} = \log \frac{3/4}{1/4} = \log 3$
  - $\log \frac{\Pr[\mathsf{M}(D)=\text{No}]}{\Pr[\mathsf{M}(D')=\text{No}]} = \log \frac{1/4}{3/4} = -\log 3$

DP in Practice

- Apple: reportedly $\epsilon = 6$ in MacOS, $\epsilon = 14$ in iOS10 and $\epsilon = 2$ for health types
- Facebook: e.g., $\epsilon = 1.453$ and $\delta = 1e - 5$
- Google: e.g., ϵ up to 9
- LinkedIn: each query uses $\epsilon = 0.15$ and $\delta = 1e - 10$
- Microsoft: e.g., $\epsilon = 12$ and $\delta = 5.8e - 6$
- US Census Bureau: e.g., $\epsilon = 13.64$ and $\delta = 1e - 5$

https://desfontain.es/blog/real-world-differential-privacy.html

A Hypothesis Testing View

- Consider null hypothesis $H_0 : D$ and alternative hypothesis $H_1 : D'$
- Or simply two classes $\mathsf{Y} = 0$ vs. $\mathsf{Y} = 1$
- Treat $\hat{\mathsf{Y}} := [\![\mathsf{M}(\cdot) \in E]\!]$
  - $\Pr(\mathsf{M}(D) \in E) = \Pr(\hat{\mathsf{Y}} = 1 | \mathsf{Y} = 0)$: false positive rate; type-1 error
  - $\Pr(\mathsf{M}(D') \in E) = \Pr(\hat{\mathsf{Y}} = 1 | \mathsf{Y} = 1)$: true positive rate; power
- DP: $\text{FPR} \leq \exp(\epsilon) \cdot \text{TPR} + \delta$

J. Dong et al. "Gaussian Differential Privacy". *Journal of the Royal Statistical Society Series B: Statistical Methodology*, vol. 84, no. 1 (2022), pp. 3–37.

α Rényi-DP

$$\mathbb{D}_\alpha(\mathsf{M}(D) \| \mathsf{M}(D')) := \frac{1}{\alpha - 1} \log \mathop{\mathbb{E}}_{\mathsf{X} \sim q} \left(\frac{p(\mathsf{X})}{q(\mathsf{X})} \right)^\alpha \leq \epsilon$$

$$\text{equivalently,} \quad \mathop{\mathbb{E}}_{\mathsf{X} \sim p} e^{(\alpha-1) r(\mathsf{X})} \leq e^{(\alpha-1)\epsilon}$$

- p and q are the densities of $\mathsf{M}(D)$ and $\mathsf{M}(D')$, respectively
- Log odds ratio: $r = \log \frac{p}{q}$; a.k.a. privacy loss
- $\mathbb{D}_\alpha = \log \left[\mathbb{E}_{\mathsf{X} \sim p} (r(\mathsf{X}))^{\alpha - 1} \right]^{\frac{1}{\alpha - 1}}$ increasing w.r.t. $\alpha \geq 1$, in particular
  - $\alpha \downarrow 1 \implies \mathbb{D}_\alpha \to \mathsf{KL}$
  - $\alpha \to \infty \implies \mathbb{D}_\alpha \to \max_{\mathbf{x}} \log \frac{p(\mathbf{x})}{q(\mathbf{x})}$, used in $(\epsilon, 0)$-DP (we choose)

I. Mironov. "Rényi differential privacy". In: *IEEE 30th computer security foundations symposium*. 2017, pp. 263–275.

The Many Shades of DP

- ϵ-DP: log odds ratio r uniformly bounded by ϵ
- (ϵ, δ)-DP: roughly, with probability $1 - \delta$, we have $r \leq \epsilon$
  - anything can happen for the remaining δ probability
  - sacrificing some δ proportion for (much?) better utility
  - the smaller ϵ or δ is, the stronger the privacy guarantee
- α-DP: bounds the exponential moment of r
  - smoother transition than (ϵ, δ)-DP
  - implies (ϵ, δ)-DP by e.g. Markov's inequality
  - the bigger α or the smaller ϵ is, the stronger the privacy guarantee

Calculus for DP

- Post-processing: If M is DP, so is $\mathbf{T} \circ \mathsf{M}$ for any $\mathbf{T}$
- Parallel composition: $D = \cup_k D_k$, each M_k is DP, then $\mathsf{M}(D) := \big(\mathsf{M}_1(D_1), \ldots, \mathsf{M}_K(D_K)\big)$ is DP
- Sequential composition: $\big(\mathsf{M}(D), \mathsf{N}(D, \mathsf{M}(D))\big)$ is $(\alpha, \epsilon_N + \epsilon_M)$-RDP
  - cannot ask too many questions or run ML algorithms for too many epochs!
  - often been heavily abused in practice
- Differ by a group of k: $(k\epsilon, 0)$-DP
- Subsampling

J. Domingo-Ferrer et al. "The limits of differential privacy (and its misuse in data release and machine learning)". *Communications of the ACM*, vol. 64, no. 7 (2021), pp. 33–35.

Gaussian Mechanism

$$\mathsf{M}(D) := f(D) + \boldsymbol{\xi}, \quad \text{where} \quad \boldsymbol{\xi} \sim \mathcal{N}(\mathbf{0}, \Sigma)$$

- Sensitivity: $\Delta_2 f := \sup_{D \sim D'} \|f(D) - f(D')\|^2_{\Sigma^{-1}}$
- (α, ϵ)-RDP with $\epsilon = \frac{\alpha}{2}\Delta_2 f$
- (α, ϵ)-RDP $\implies$ $(\epsilon + \frac{1}{\alpha-1}\log\frac{1}{\delta}, \frac{\delta}{\alpha})$-DP
  - note $\alpha \to \infty \implies \mathbb{D}_\alpha \to \max_{\mathbf{x}} \log\frac{p(\mathbf{x})}{q(\mathbf{x})} \implies (\epsilon, 0)$-DP
  - to achieve $\alpha \to \infty$ with Gaussian mechanism: $\epsilon = \frac{\alpha}{2}\Delta_2 f \to \infty$

DP-SGD

Algorithm 1: Differentially private stochastic gradient descent

Input: model $\mathbf{w}$; data $\mathbf{x}_1, \ldots, \mathbf{x}_n$; noise σ, gradient bound C, batch size b

```
for t = 0, 1, ... do
    sample a random batch B_t with size b
    for i ∈ B_t do
        g_i ← ∇_w ℓ(x_i; w)                              // compute grad
        g_i ← g_i / max{1, ‖g_i‖_2 / C}                   // grad clipping
    g ← [ (1/b) Σ_{i∈B_t} g_i ] + σCξ                     // adding noise
    w ← w − η · g                                         // grad descent
    w ← P(w)                                              // projection
```

$$\mathbf{g}_i \leftarrow \nabla_{\mathbf{w}} \ell(\mathbf{x}_i; \mathbf{w})$$
$$\mathbf{g}_i \leftarrow \mathbf{g}_i / \max\{1, \|\mathbf{g}_i\|_2 / C\}$$
$$\mathbf{g} \leftarrow \left[\frac{1}{b} \sum_{i \in B_t} \mathbf{g}_i\right] + \sigma C \boldsymbol{\xi}$$
$$\mathbf{w} \leftarrow \mathbf{w} - \eta \cdot \mathbf{g}$$
$$\mathbf{w} \leftarrow \mathrm{P}(\mathbf{w})$$

M. Abadi et al. "Deep Learning with Differential Privacy". In: *Proceedings of the 2016 ACM SIGSAC Conference on Computer and Communications Security*. 2016, pp. 308–318.

Application in Generative Models

- Modern generative models are powerful, e.g., ChatGPT, DALLE-2
  - We can release the generative model as a proxy of releasing data
  - We can conduct data analysis / ML downstream tasks using generated data
- How to protect privacy when sensitive data (medical records, face images) are used in training?
- One solution: Differentially Private Generative Models - equip generative models with DP guarantees

Figure 2: Qualitative comparison under $(0.2, 10^{-5})$-DP on MNIST and Fashion MNIST

D. Jiang et al. "Functional Rényi differential privacy for generative modeling" *Advances in Neural Information Processing Systems*, vol. 36 (2023).