Mathematical Basics in Electrical Capacitance Tomography Differential Sensor

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Index Terms—Electrical capacitance tomography, differential sensor, dynamic range, capacitance measuring circuit, optimization.

I. Method

A. Cylindrical capacitor

The capacitance of a cylindrical capacitor is described by:

$$C_{cc} = \frac{2\pi h \varepsilon_0}{\ln(b/a)} \tag{1}$$

where ε_0 is the dielectric permittivity of air, h is the axial length of active screen, a and b are the radius of measuring electrodes and active screen, respectively.

Therefore, it can be assumed that

$$\begin{aligned} C_{d1} &= C_{d2} = \cdots = C_{di} && (2) \\ C_{cc} &= \sum_{i=1}^{n_E} C_{di} && (3) \end{aligned}$$

where C_{di} is the capacitance between the i^{th} measuring electrode and the active screen.

In consideration of the gaps between measuring electrodes and radial screens and the edge effect caused by finite h, C_{di} can be estimated by:

$$C_{di} = k_c \frac{C_{cc}}{n_E} = \frac{2k\pi h \varepsilon_0}{\ln(b/a) n_E} \tag{4}$$

where k_c is a correction factor related to sensor geometry, n_E is the number of measuring electrodes.

B. Circuit output

Therefore, the output voltage can be described by:

$$V_o(t) = -\frac{j\omega R_f (C_m - kC_{d2})}{j\omega C_f R_f + 1} V_i(t) \tag{5}$$

where ω is the applied angular frequency, R_f and C_f are the feedback resistor and capacitor.

By introducing $k_0 = j\omega R_f/(j\omega C_f R_f + 1)$, Eq(5) can be simplified into:

$$V_o(t) = -k_0 (C_m - kC_{d2}) V_i(t) \tag{6}$$

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C. Noise analysis

In consideration that

$$\begin{aligned} |j\omega R_{s3}(C_m + C_d + C_{s3} + C_{s4})| &\ll 1, \\ |j\omega R_{s4}(C_m + C_d + C_{s3} + C_{s4})| &\ll 1, \\ |j\omega R_f C_f| &\gg 1 \end{aligned}$$

the equations for noise gains can be properly simplified.

The total RMS noise can be expressed by

$$E_t = \sqrt{\sum_{i=1}^{q} E_n^2(i)} [V_{RMS}] \tag{7}$$

where $E_n(i)$ represents the i^{th} RMS noise source, i.e. the passive elements and the intrinsic noise sources in the op-amp.

Each noise source generates a noise density at the inverting or non-inverting input of the op-amp, which can be calculated by:

$$E_n = \sqrt{\int_0^{\omega_c} |A_n(\omega)|^2 e_n^2(\omega) d\omega} [V_{RMS}] \tag{8}$$

where $A_n(\omega)$ is the noise gain, ω_c is the cut-off frequency of the following conditioning circuit.

D. Sensitivity distribution

In ECT, the sensitivity distribution of electrode pair (i,j) $\mathbf{S}_{i,j}(k)$ is defined as:

$$\mathbf{S}_{i,j}(k) = \mu(k) \frac{C_{i,j}(k) - C_{i,j}^l}{\Delta C_{i,j} \Delta\varepsilon} \tag{9}$$

where

$$\begin{aligned} \Delta C_{i,j} &= C_{i,j}^h - C_{i,j}^l && (10) \\ \Delta\varepsilon &= \varepsilon^h - \varepsilon^l && (11) \end{aligned}$$

and where $C_{i,j}(k)$ is the capacitance when the k^{th} element has the high dielectric constant ε^h and all the other elements have low dielectric constant ε^l. And $C_{i,j}^h$ and $C_{i,j}^l$ are the capacitances when the pipe is filled with high and low permittivity materials, respectively. $\Delta C_{i,j}$ is, therefore, the full scale capacitance range.