Applied Cryptography

thgoebel@ethz.ch

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Credits: images are generally taken from the lecture slides.

1 Symmetric Cryptography

One-time pad Plaintext p, key k such that $|p| = |k|$. Ciphertext $c = p \oplus k$.

If k u.a.r. and only used once then the OTP is **perfectly secure**, i.e. $\Pr[P = p|C = c] = \Pr[P = p]$.

Note: keys can re-occur (as a result of random sampling) but they must not be re-used (i.e. the adversary must not be aware that the same key is used).

Issues: same lengths, key distribution, single use.

1.1 Block Ciphers

Block cipher A block cipher with key length k and block size n consists of two efficiently computable permutations[1]:

$$E : \{0,1\}^k \times \{0,1\}^n \mapsto \{0,1\}^n \quad D : \{0,1\}^k \times \{0,1\}^n \mapsto \{0,1\}^n$$

such that for all keys K D_K is the inverse of E_K (where we write E_K short for $E(K, \cdot)$).

Security notions Known plaintext attack, chosen plaintext attack, chosen ciphertext attack. Exhaustive key search on (P, C) pairs – no attack should be better, else we throw the cipher away.

Pseudo-randomness

- Adversary $\mathcal{A}$ interacts either with block cipher (E_K, D_K) or a truly random permutation (Π, Π^{-1}).
- A block cipher is called a **pseudo-random permutation PRP** if no efficient[2] $\mathcal{A}$ can tell the difference between E_K and Π (no access to the inverse).
- A block cipher is called a **strong-PRP** if no efficient $\mathcal{A}$ can tell the difference between (E_K, D_K) and (Π, Π^{-1}).

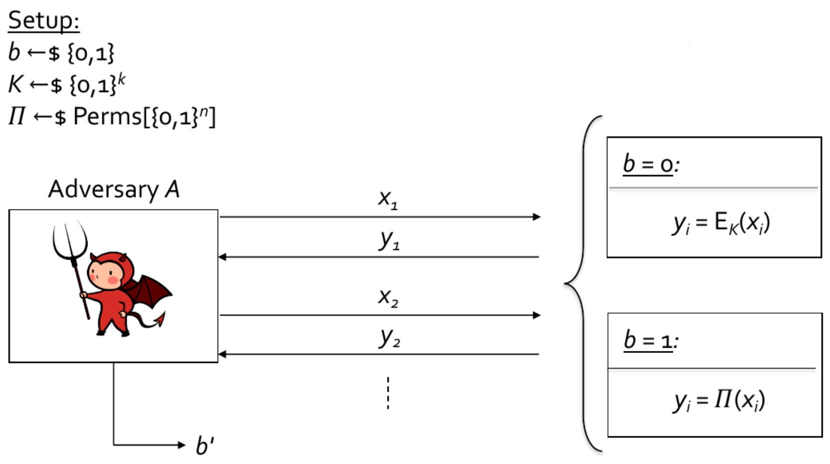


Figure 1: PRP game

[1]Encipher and decipher
[2]Quantified by runtime + number of oracle queries.

The advantage is defined as:

$$\mathbf{Adv}_E^{PRP}(\mathcal{A}) = 2 \cdot \left| \Pr[\text{Game } \mathbf{PRP}(\mathcal{A}, E) \Rightarrow \text{true}] - \frac{1}{2} \right|$$

where the probability is over the randomness of $b, K, \Pi, \mathcal{A}$. Note that:

$$\Pr[\text{Game } \mathbf{PRP}(\mathcal{A}, E) \Rightarrow \text{true}] = \Pr[b' = b]$$

Also see the Advantage Rewriting Lemma (1.2).

Constructing block ciphers In general: keyed round function that is repeated many times.

- Feistel cipher: halved blocks crossing back and forth, e.g. DES
- Substitution-permutation network: confusion + diffusion, e.g. AES

Electronic Code Book (ECB) mode Same plaintext always maps to the same ciphertext (deterministic). Thus serious leakage, don't use.

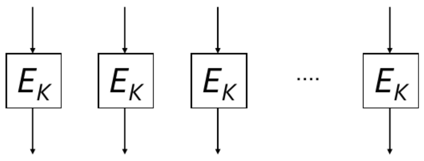


Figure 2: ECB mode

Cipher Block Chaining (CBC) mode Use u.a.r. IV/previous ciphertext block to randomise encryption.

A bit flip in C_i completely scrambles/randomises P_i and flips the same bit in P_{i+1}.

Caveats: non-random IV, padding oracle attack, ciphertext block collisions (after using the same key for $2^{n/2}$ blocks by the birthday bound).

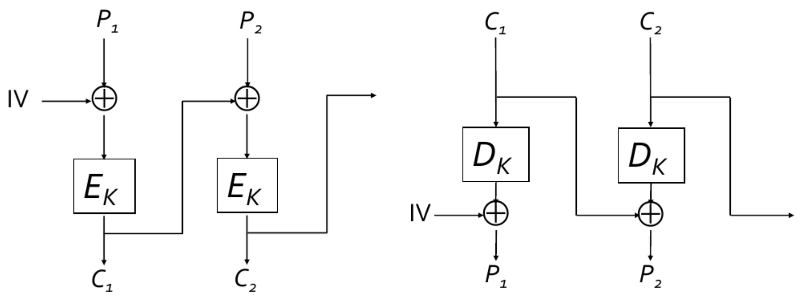


Figure 3: CBC mode (left: encipher, right: decipher)

Counter (CTR) mode Incrementing counter is encrypted with block cipher to produce a pseudo-random value to xor the plaintext block with.

Effectively a stream cipher producing OTP keys. E_K does not even need to be invertible. No padding needed, can just truncate the last block. A bit flip it C_i flips the same bit in P_i.

Caveats: counter must not repeat/wrap around (else xor of ciphertexts = xor of plaintexts).

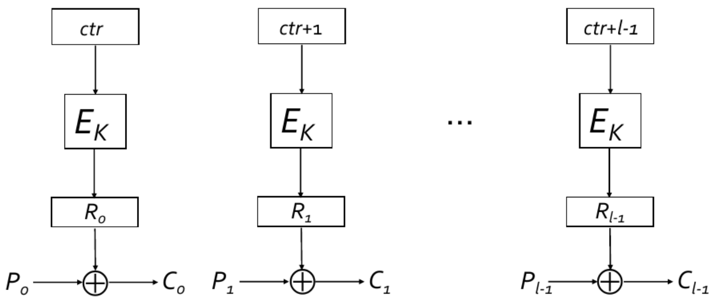


Figure 4: CTR mode

1.2 Symmetric Encryption

Symmetric Encryption Scheme is a triple $SE = (KGen, Enc, Dec)$. We have key space $\mathcal{K} = \{0,1\}^k$, message space $\mathcal{M} = \{0,1\}^{*}$[3] and ciphertext space $\mathcal{C} = \{0,1\}^*$. For correctness, we have $Dec_K(Enc_K(m)) = m$.

IND-CPA Security Informally: computational version of perfect security – an efficient adversary cannot compute anything useful from a ciphertext (e.g. hide every bit of the plaintext). Equivalent to *semantic security.*

Formally: For any efficient adversary $\mathcal{A}$, given the encryption of one of two equal-length messages of its choice, $\mathcal{A}$ is unable to distinguish which one of the two messages was encrypted.

In the security game, $\mathcal{A}$ gets access to a *Left-or-Right encryption oracle.* The advantage of $\mathcal{A}$ is:

$$\mathbf{Adv}_{SE}^{IND-CPA}(\mathcal{A}) = 2 \cdot \left| \Pr[\text{Game } \mathbf{IND\text{-}CPA}(\mathcal{A}, SE) \Rightarrow \text{true}] - \frac{1}{2} \right|$$

Notes: Deterministic schemes **cannot** be IND-CPA secure (why?[4]). CBC and CTR mode (if used properly) can be proven to be IND-CPA secure (assuming that Enc is a PRP-secure block cipher).

Caveats: No integrity. Says nothing about messages of non-equal length. No chosen ciphertexts.

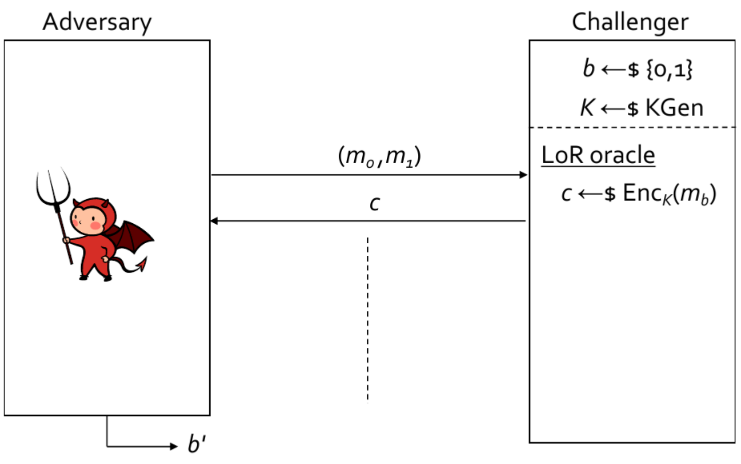


Figure 5: IND-CPA game

[3]In reality we might have a maximum plaintext length.
[4]The adversary can first query (m_0, m_0) to learn c_0, then query (m_0, m_1) and thus win with high probability.

Advantage Rewriting Lemma Let b be a uniformly random bit and b' the output of some algorithm. Then:

$$\begin{aligned} 2\left|\Pr[b' = b] - \frac{1}{2}\right| &= \Big|\Pr[b' = 1|b = 1] - \Pr[b' = 1|b = 0]\Big| \\ &= \Big|\Pr[b' = 0|b = 0] - \Pr[b' = 0|b = 1]\Big| \end{aligned}$$

Difference Lemma Let Z, W_1, W_2 be events. If

$$(W_1 \wedge \neg Z) \text{ occurs if and only if } (W_2 \wedge \neg Z) \text{ occurs}$$

then

$$\Big|\Pr[W_2] - \Pr[W_1]\Big| \leq \Pr[Z]$$

In practice: Z is a bad event that rarely happens, W_1, W_2 are when $\mathcal{A}$ wins in security games G_1, G_2. Useful for *game hopping* proofs.

PRP-PRF Switching Lemma Let E be a block cipher. Then for any algorithm $\mathcal{A}$ making q queries:

$$\Big|\mathbf{Adv}_E^{PRP}(\mathcal{A}) - \mathbf{Adv}_E^{PRF}(\mathcal{A})\Big| \leq \frac{q^2}{2^{n+1}}$$

1.3 Attacks

Padding Added before encryption to expand plaintext to a multiple of the block size, i.e.
$pad(\cdot) : \{0,1\}^* \mapsto \{\{0,1\}^n\}^*$. Must be expanding (why?) and efficiently computable. May be either randomised or deterministic.
E.g. TLS padding: appends $t+1$ bytes of value t.

Problem: adversary can detect padding errors (explicit error messages, logs, timing differences).

Padding Oracle Given a ciphertext C, returns whether the padding of the decryption is valid or invalid. Leaks 1 bit of information. Kind of a chosen ciphertext attack, thus not covered by IND-CPA security! Main problem: no ciphertext integrity.

In practice: $\mathcal{A}$ needs 128 oracle queries on average to recover one plaintext byte. Repeat for all bytes and all blocks for full plaintext recovery. Additional practical details to consider. For TLS attacks, see Lucky 13 and POODLE.

Predictable IVs Random IVs are not just theoretically relevant for IND-CPA security of CBC mode. Consider the following steps (see Figure 7):

1. $\mathcal{A}$ queries LoR oracle with (P_0, P_1)
2. Oracle responds with $C = C_0||C_1$ where $C_1 = E_K(P_b \oplus C_0)$
3. $\mathcal{A}$ predicts next $IV = C'_0$
4. $\mathcal{A}$ queries $P_0 \oplus C_0 \oplus C'_0$
5. $\implies b = 0 \iff P_b = P_0 \iff C_1 = C'_1 \implies$ breaks IND-CPA security

In practice: systems may use *IV chaining* (use the last ciphertext as the next IV, in order to avoid having to sample new randomness). E.g.: SSLv3, TLS 1.0, SSH in CBC mode. See also the BEAST attack on TLS.

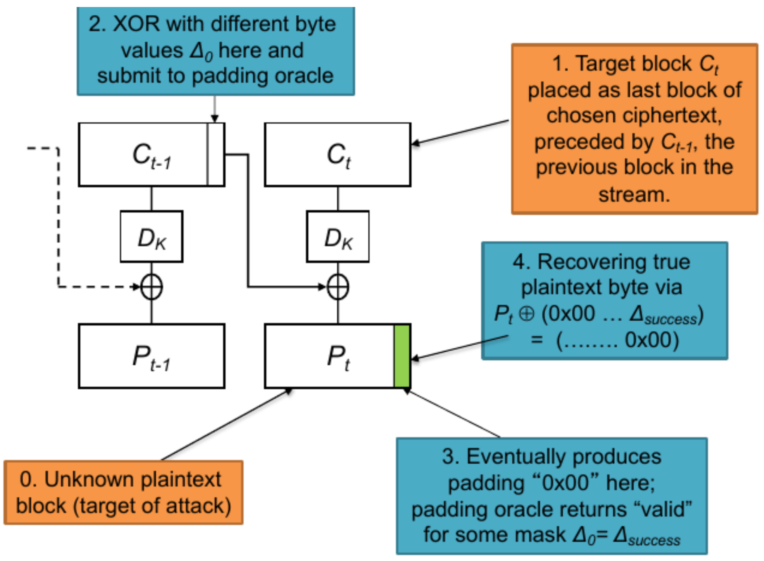

2. XOR with different byte values Δ_0 here and submit to padding oracle
1. Target block C_t placed as last block of chosen ciphertext, preceded by C_{t-1}, the previous block in the stream.
C_{t-1}
C_t
D_K
D_K
4. Recovering true plaintext byte via $P_t \oplus$ (0x00 … $\Delta_{success}$) = (…….. 0x00)
P_{t-1}
P_t
0. Unknown plaintext block (target of attack)
3. Eventually produces padding "0x00" here; padding oracle returns "valid" for some mask $\Delta_0 = \Delta_{success}$

Figure 6: Padding oracle attack on CBC mode

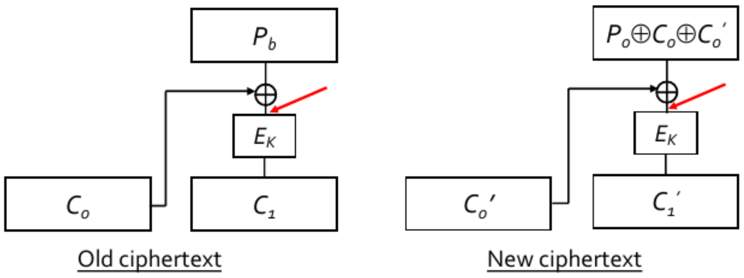


Figure 7: Attack on predictable IVs in CBC mode

1.4 Hash Functions

Cryptographic Hash Function An efficiently computable function $H : \{0,1\}^* \mapsto \{0,1\}^n$ that maps arbitrary-length inputs to fixed-length outputs ("*message digests*"). Not keyed!

Applications: MACs, signatures, key derivation, commitments.

Random oracle model Model under which hash functions are assumed to produced outputs that are uniformly random distributed. I.e. a hash function could be modelled by a random oracle. Very strong assumption!

Birthday paradox When drawing elements at random from a set of size s then after $\sqrt{s}$ trials we expect a collision with 39% probability. We quickly get to 99% with an additional constant factor.

Security goals Primary goals:

- *Pre-image resistance* (one-wayness): Given h, it is infeasible to find an m such that $H(m) = h$.
- *Second pre-image resistance*: Given m_1, it is infeasible to find an m_2 such that $H(m_1) = H(m_2)$.
- *Collision resistance*: It is infeasible to find any $m_1 \neq m_2$ such that $H(m_1) = H(m_2)$.

Secondary goals:

- *Near-collision resistance*: It is infeasible to find any $m_1 \neq m_2$ such that $H(m_1) \approx H(m_2)$ (e.g. hashes agree on most bits).
- *Partial pre-image resistance 1*: Given $H(m)$, it is infeasible to recover any partial information about m.
- *Partial pre-image resistance 2*: Given a target string t with $|t| = l$ it is infeasible to find an m such $H(m) = t||x$ (faster than with 2^l brute-force hash evaluations).

CR adversary:
Cannot quantify security over <u>all</u> efficient $\mathcal{A}$ (collisions exist, $\mathcal{A}$ can hardcode one). Instead, define "(t, ε)-CR adversaries", running in time t and with $\mathbf{Adv}_H^{CR}(\mathcal{A}) = \varepsilon$. Build reductions from there.

Relationships:

- Collision resistance $\implies$ second pre-image resistance
- Maybe: Collision resistance $\implies$ pre-image resistance – depending on how you define pre-image resistance (sampling from the domain or from the range?)

Generic attacks (in the ROM):
(Second) pre-image resistance: 2^n evaluations.
Collision resistance: $2^{n/2}$ evaluations! (by the birthday paradox)
$\implies$ e.g. SHA-1 with 160-bit outputs only achieves 80-bit security

Merkle-Damgård iterated hashing Constructs a hash function H from a *compression function* $h : \{0,1\}^k \times \{0,1\}^n \mapsto \{0,1\}^n$. Used e.g. in MD5, SHA-1, SHA-2.

Steps: pad message, split into k-bit chunks, repeatedly apply h to the chunks and IV/chaining values (see Figure 8).

Padding scheme:

$$m' = pad(m) = m||10^t||[len(m)]_k$$

where $[len(m)]_k$ is the k-bit encoding of the message length[5] and $0 \leq t < k$ is minimal.

Theorem: If h is collision resistant, then so is H.[6]

Length extension attack: Given $H(m)$ (but not m!) once can compute a valid hash $y = H(pad(m)||m'')$. This is problematic e.g. when construction MACs or KDFs from a hash function.

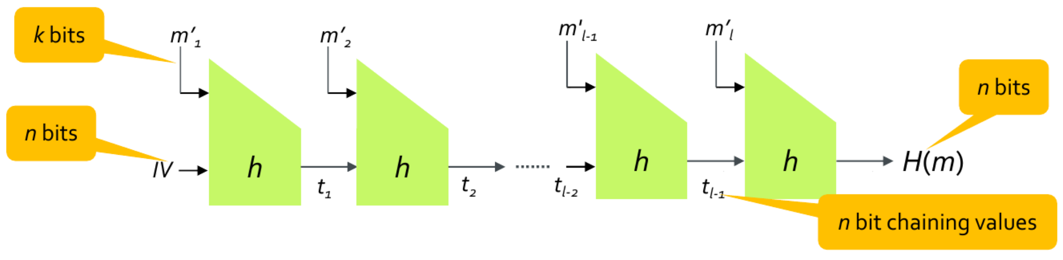


Figure 8: Merkle-Damgård transform

[5]This limits the maximum length of a message that can be hashed to $2^k - 1$.

[6]Note that the choice of padding scheme matters for the proof! Also note that the two IVs must be the same for a *full collision*. The much weaker form of two colliding messages for different IVs is called a *freestart collision*.

Constructing compression functions from block ciphers E.g. Davies-Meyer, Matyas-Meyer-Oseas, Miyaguchi-Preneel constructions. Use message as the key (need fast rekeying!) and (some variation of) chaining value as “plaintext” input.

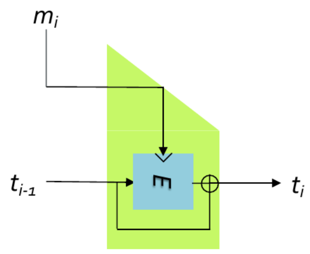
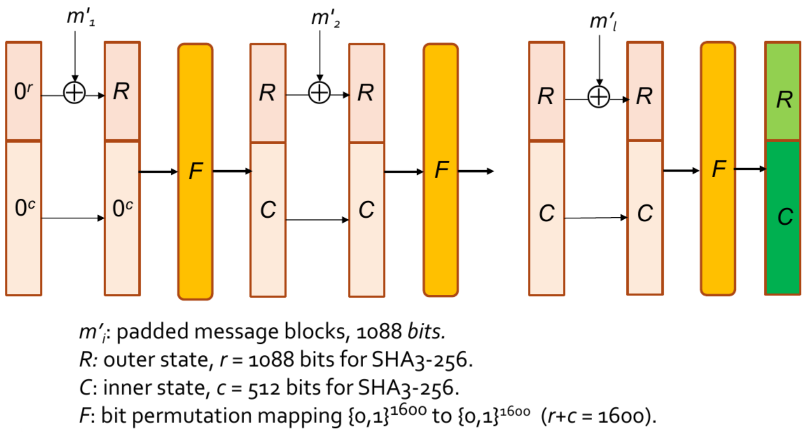


Figure 9: Davies-Meyer construction

Sponge construction Different design approach. Centered around 2 phases: absorbing + squeezing. Key ideas: giant bit permutation F, not inputting/outputting entire state. Variable length output.

E.g. used in SHA-3/Keccak: SHA-3$(m) = out_1||out_2||out_3||\ldots$.



m'_i: padded message blocks, 1088 *bits*.
R: outer state, r = 1088 bits for SHA3-256.
C: inner state, c = 512 bits for SHA3-256.
F: bit permutation mapping $\{0,1\}^{1600}$ to $\{0,1\}^{1600}$ (r+c = 1600).

Figure 10: Sponge construction: absorbing

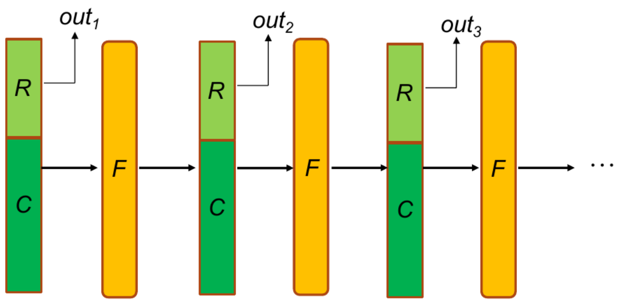


Figure 11: Sponge construction: squeezing

1.5 Message Authentication Codes MACs

Motivation Provide *integrity* and *data origin authentication*, i.e. no attacker should be able to modify or forge messages (unforgeability).

They do not provide confidentiality or secure against message deletion, replay, reordering, or reflection (if the key is used in both directions).

Definition (MAC scheme) A MAC scheme with key length k and tag length t consists of three efficient algorithms:

$$\begin{aligned} KGen: &\quad \{\} \mapsto \{0,1\}^k \\ Tag: &\quad \{0,1\}^k \times \{0,1\}^* \mapsto \{0,1\}^t \\ Vfy: &\quad \{0,1\}^k \times \{0,1\}^* \times \{0,1\}^t \mapsto \{0,1\} \end{aligned}$$

For correctness[7], we require that

$$\forall K, m \;.\; Vfy(K, M, \tau) = 1 \text{ where } \tau = Tag(K, m)$$

Vfy and Tag may be randomised. Note that if they are deterministic and Vfy internally (re)computes $Tag(K,m) \overset{?}{=} \tau$, then the scheme has *unique tags*.

MAC Security Informally: it is hard for an attacker to forge a valid message-tag pair (m', τ').

Security game: $\mathcal{A}$ gets access to a tag oracle and a verify oracle.

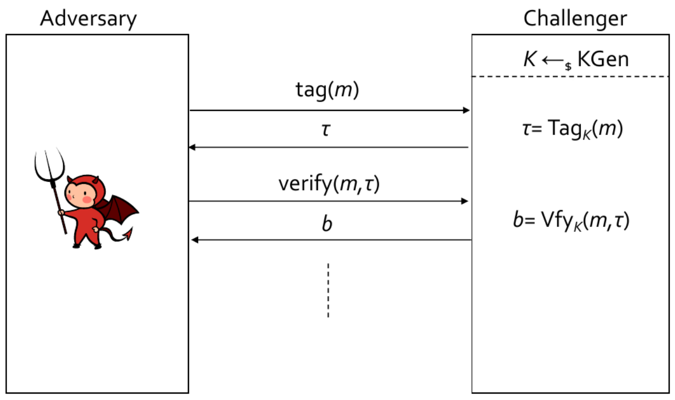


Figure 12: MAC game

Weak unforgeability under chosen message attack (WUF-CMA) $\mathcal{A}$ wins if it submitted some (m^*, τ^*) to the verify oracle such that $Vfy(m^*, \tau^*) = 1$ and no query $Tag(m^*)$ was made.

Strong unforgeability under chosen message attack (SUF-CMA) $\mathcal{A}$ wins if it submitted some (m^*, τ^*) to the verify oracle such that $Vfy(m^*, \tau^*) = 1$ and no query $Tag(m^*)$ with response τ^* was made.

Note that:

- A SUF-CMA adversary can also win by coming up with a new tag for an old message. We make the adversary "stronger" by relaxing the winning condition.
- SUF-CMA $\implies$ WUF-CMA (somewhat counter-intuitively)

[7]A functionality, not a security requirement.

- For deterministic MAC schemes that build Vfy from Tag, the two are equal (why?[8]).
- More formally: $(q_t, q_v, t, \varepsilon)$-W/SUF-CMA security

Generic attacks: guess (m, τ) pairs, guess the key, few tag queries + exhaustive key search.

MACs from PRFs Construction of $MAC(F)$: Let $F : \{0,1\}^k \times \{0,1\}^n \mapsto \{0,1\}^t$ be a PRF.

$$\begin{aligned} KGen() &: \quad K \leftarrow \$ \; \{0,1\}^k \\ Tag(K,m) &: \quad \tau \leftarrow F(K,m) \\ Vfy(K,m,\tau) &: \quad \tau' \leftarrow F(K,m); \text{ return } \tau' == \tau \end{aligned}$$

Theorem: If F is a PRF then $MAC(F)$ is a SUF-CMA.

Domain extension with CR hashing Motivation: build a MAC for a larger input domain $\mathcal{X}'$ from a MAC with fixed domain $\mathcal{X}$ (to handle more than just block-sized messages).

Construction of $HtMAC$: Let $MAC = (KGen, Tag, Vfy)$ be a MAC and let $H : \mathcal{X}' \mapsto \mathcal{X}$ be a hash function.

$$\begin{aligned} Tag'(K,m) &: \quad Tag(K, H(m)) \\ Vfy'(K,m,\tau) &: \quad Vfy(K, H(m), \tau) \end{aligned}$$

Theorem: If MAC is SUF-CMA secure and H is collision resistant then $HtMAC$ is SUF-CMA.

Hash-based MAC HMAC Turn (the often fast) hash functions into a keyed primitive using a two-key nest to build a PRF (and thus a MAC):

$$F_{nest}((K_1, K_2), m) = H(K_2 || H(K_1 || m))$$

Notes: Cannot simply prepend the key (length extension attacks). Also cannot simply append the key (offline collision attacks). In practice, derive two keys from one key using an inner and outer mask. Implementations often use an initialise-update-finalise interface (which can open timing attack vectors).

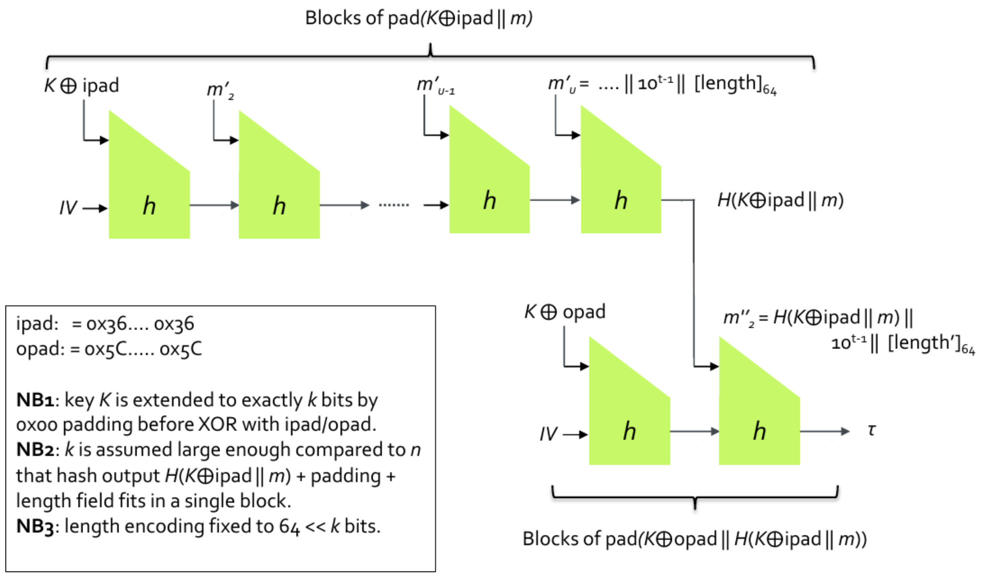
HMAC is gradually supplanted by faster designs based on universal hashing (see below), yet it is still used in key derivation scenarios (that use PRFness).

Nonce A number used once. Does neither need to be secret, nor does it need to be unpredictable. But must never ever repeat. Reasoning: it is easier to only use a number once than it is to get a good source of randomness.

Nonce-based MAC NMAC Like normal MAC, but Tag and Vfy now also take a nonce $N \in \mathcal{N}$. In the security game, we require that nonces are distinct (e.g. enforced by the tag oracle).

[8]When tags are unique then no adversary can come up with another tag for a previously queried message. However, consider the following deterministic scheme that is WUF-CMA but not SUF-CMA:

$$Tag'(K,m) = 0 || Tag(K,m) \quad Vfy'(K,m,b||\tau) = Vfy(K,m,\tau)$$


Blocks of pad(K⊕ipad || m)
K ⊕ ipad
m′₂
m′ᵥ₋₁
m′ᵥ = ... || 10ᵗ⁻¹ || [length]₆₄
IV
h
h
....
h
h
H(K⊕ipad || m)
K ⊕ opad
m″₂ = H(K⊕ipad || m) || 10ᵗ⁻¹ || [length′]₆₄
IV
h
h
τ
Blocks of pad(K⊕opad || H(K⊕ipad || m))

ipad: = 0x36.... 0x36
opad: = 0x5C..... 0x5C

NB1: key K is extended to exactly k bits by 0x00 padding before XOR with ipad/opad.
NB2: k is assumed large enough compared to n that hash output $H(K \oplus \text{ipad} \,||\, m)$ + padding + length field fits in a single block.
NB3: length encoding fixed to 64 << k bits.

Figure 13: HMAC

Keyed hash function A keyed hash function H is a deterministic algorithm that takes a key and a message and produces a *digest* $t = H(K, m)$ over the digest space $\mathcal{T}$.

Universal hashing UHF Informally: selecting a hash function at random from a family of hash functions (the universal family).

Security game: Let H be a <u>keyed</u> hash function. Then:

1. Challenger sets $K \leftarrow \$ \{0,1\}^k$
2. Adversary outputs distinct (m_0, m_1)

$\mathcal{A}$ wins if $H(K, m_0) = H(K, m_1)$ and has advantage $\mathbf{Adv}_H^{UHF}(\mathcal{A})$.

H is an *ε-bounded universal hash function* if $\mathbf{Adv}_H^{UHF}(\mathcal{A}) \leq \varepsilon$ for all $\mathcal{A}$ (even unbounded ones).

Equivalent definition: H is an ε-UHF if $\forall m_0, m_1$. $\Pr[H(K, m_0) = H(K, m_1)] \leq \varepsilon$, where the probability is over the random choice of K.

Universal hash functions from polynomials Let $\mathcal{F}$ be a finite field (e.g. integers mod p or $GF(2^n)$) and let $\mathcal{K} = \mathcal{T} = \mathcal{F}$ and $\mathcal{M} = \mathcal{F}^{\leq l}$. Then we define:

$$H_{poly}(K, (a_1, \ldots, a_v)) = K^v + a_1 K^{v-1} + a_2 K^{v-2} + \cdots + a_{v-1} K + a_v$$

That is, H_{poly} is always a degree v polynomial (even if the message vector a is zero). It can efficiently be evaluated using finite field operations and Horner's rule.

Theorem: H_{poly} is an ε-UHF for $\varepsilon = \frac{l}{|\mathcal{F}|}$.

Difference unpredictable hashing DUHF Security game: Let H be a keyed hash function and let the digest space $\mathcal{T}$ have a group operation $+$ (with inverse $-$)[9]. Then:

1. Challenger sets $K \leftarrow \$ \{0,1\}^k$
2. Adversary outputs distinct $m_0, m_1 \in \mathcal{M}$ and $\delta \in \mathcal{T}$.

$\mathcal{A}$ wins if $H(K, m_0) - H(K, m_1) = \delta$ and has advantage $\mathbf{Adv}_H^{DUHF}(\mathcal{A})$.

H is an *ε-bounded distance unpredictable hash function* (ε-DUHF) if $\mathbf{Adv}_H^{DUHF}(\mathcal{A}) \leq \varepsilon$ for all $\mathcal{A}$ (even unbounded ones).

XOR universal hashing Special case of DUHF where we have XOR as the group operation. We define H_{xpoly} as follows:

$$\begin{aligned} H_{xpoly}(K, (a_1, \dots, a_v)) &= K^{v+1} + a_1 K^v + a_2 K^{v-1} + \dots + a_{v-1} K^2 + a_v K \\ &= K \cdot H_{poly}(K, (a_1, \dots, a_v)) \end{aligned}$$

Theorem: H_{xpoly} is an ε-DUHF for $\varepsilon = \frac{l+1}{|\mathcal{F}|}$.

Carter-Wegman MACs Let H be an ε-DUHF and let F be a PRF. Then we define CW-MAC(F, H) as follows:

$$\begin{aligned} KGen() :& \quad (K_1, K_2) \leftarrow \$ \mathcal{K}_H \times \mathcal{K}_F \\ Tag((K_1, K_2), N, m) :& \quad \tau \leftarrow H(K_1, m) + F(K_2, N) \\ Vfy((K_1, K_2), N, m, \tau) :& \quad \tau' \leftarrow Tag((K_1, K_2), N, m); \text{ return } \tau' == \tau \end{aligned}$$

Theorem: If H is an ε-DUHF and F a PRF then CW-MAC(F, H) is SUF-CMA secure.

In practice: e.g. GMAC (used in AES-GCM) instantiates F with AES and H with H_{xpoly} over $\mathcal{F} = GF(2^{128})$.

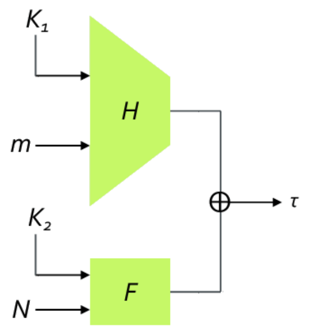


Figure 14: Carter-Wegman MAC tag algorithm

[9]E.g. addition mod n in $\mathbb{Z}_n$ or XOR in $\{0,1\}^n$.

1.6 Authenticated Encryption

Integrity of ciphertexts INT-CTXT Informally: adversary cannot forge new ciphertexts.

Security game: $\mathcal{A}$ can submit any m to an encryption oracle to obtain $c = Enc_K(m)$. At the end $\mathcal{A}$ submits one c^* to a *try* oracle.

$\mathcal{A}$ wins if 1) c^* is distinct from all previously seen c and 2) c^* decrypts to a $m^* \neq \bot$.

$\mathcal{A}$ has advantage $\mathbf{Adv}_{SE}^{INT-CTXT}(\mathcal{A})$.

Integrity of plaintexts INT-PTXT Informally: adversary cannot force a new plaintext to be accepted by the receiver.

Same as INT-CTXT but with an additional winning requirement:
$\mathcal{A}$ wins if ... 3) m^* is distinct from all previous queries m.

INT-CTXT $\implies$ INT-PTXT[10]

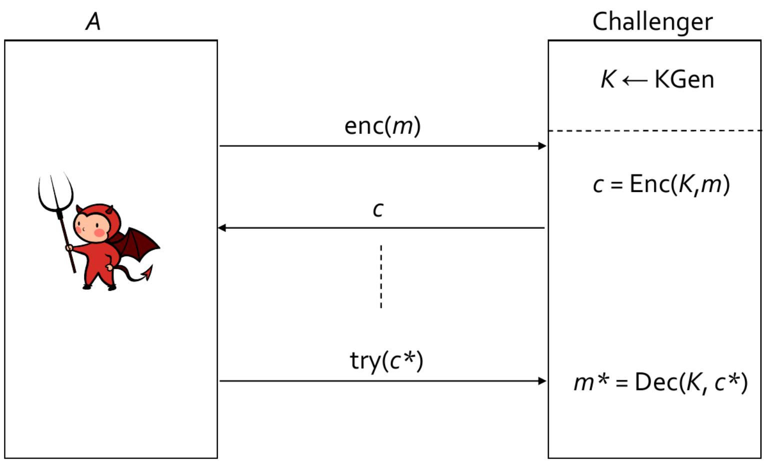


Figure 15: INT-CTXT + INT-PTXT game

IND-CCA Security (Indistinguishable under chosen ciphertext attack)

Same as IND-CPA (LoR oracle), with an additional decryption oracle.[11]

Authenticated Encryption AE An SE is said to be *AE secure* if it is IND-CPA secure and INT-CTXT secure.

AE $\implies$ IND-CCA[12]

Encrypt-and-MAC E&M Compute $c \leftarrow Enc_{KE}(m), \tau \leftarrow Tag_{KM}(m)$ and output $c' = c||\tau$.

NOT secure in general: If MAC is deterministic (e.g. a PRF) then trivial IND-CPA attack applies.

E.g. in SSH (in a stateful variant).

[10]Since due to SE correctness a new plaintext implies a new ciphertext.
[11]Trivially, the adversary is not allowed to submit ciphertexts received from the LoR oracle to the decryption oracle.
[12]Informal proof by case distinction: Either at some point the IND-CCA adversary queries a c that does not decrypt to $\bot$ (breaking INT-CTXT) or never, in which case the decryption oracle is useless (breaking IND-CPA).

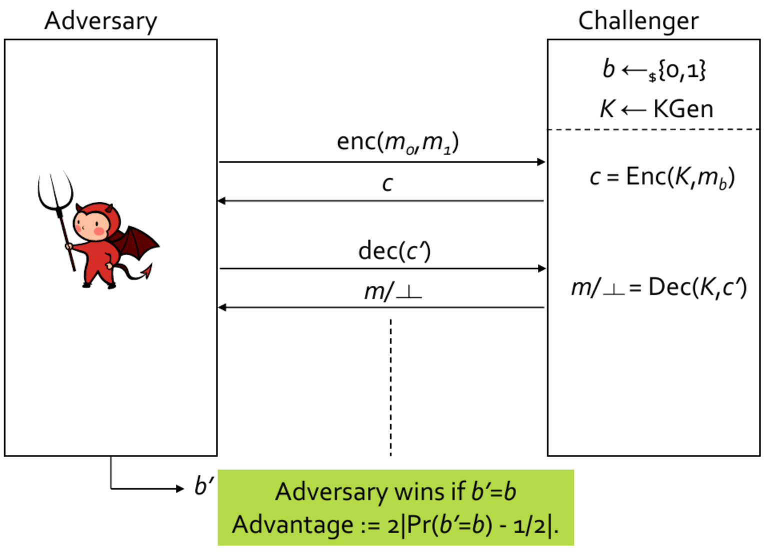

Adversary
Challenger
$b \leftarrow_{\$} \{0,1\}$
$K \leftarrow$ KGen
enc(m_0,m_1)
c
c = Enc(K,m_b)
dec(c')
$m/\perp$
$m/\perp$ = Dec(K,c')
b'
Adversary wins if b'=b
Advantage := 2|Pr(b'=b) - 1/2|.

Figure 16: IND-CCA game

Mac-then-Encrypt MtE Compute $\tau \leftarrow Tag_{KM}(m)$ and output $c \leftarrow Enc_{KE}(m||\tau)$.

Can be made secure under specific instantiations, but unsafe in general – AVOID!

E.g. in TLS prior to 1.3.

Encrypt-then-Mac EtM Compute $c \leftarrow Enc_{KE}(m), \tau \leftarrow Tag_{KM}(c)$ and output $c' = c||\tau$.

AE secure under the assumption that SE is IND-CPA and MAC is $\underline{\text{S}}$UF-CMA.

E.g. in IPSec ESP.

AE with Associated Data AEAD Informally: integrity-protect some data, provide confidentiality for the rest. E.g. AES-GCM, ChaCha20-Poly1305.

Both *Enc* and *Dec* also take the associated data AD as an additional input argument – but the ciphertext does not "contain" AD (instead AD is sent alongside, like the nonce N).
But: AD is not an output of the AEAD scheme (only c and τ are)!

IND-CPA game: adversary sends $(N, AD, (m_0, m_1))$ to LoR oracle.

INT-CTXT game: adversary sends (N, AD, m) to encryption oracle and (N^*, AD^*, c^*) to the try oracle.

EtM for AEAD:

$$c \leftarrow Enc_{KE}(m), \tau \leftarrow Tag_{KM}(len(AD)||AD||c);\ \text{output } c' = c||\tau$$

Note that tagging over the length is required to avoid mis-parsing between AD and c.

In practice: *Enc* and *Tag* may be nonce-based. If nonces implicitly use a counter, then the adversary cannot reorder or delete messages (and of course can't insert either due to INT-CTXT).

AES-GCM (Galois Counter Mode) EtM with AES in CTR mode and a CW-MAC (H_{xpoly} over $GF(2^{128})$). Single key, MAC key is derived: $K_M = AES(K, 0^{128})$. Nonces are usually 96 bit and reuse results in catastrophic failure (MAC key recovery), thus not *nonce-misuse resistant*. Counters <u>for *Enc*</u> are of the form $N||ctr$. Almost as fast as CTR mode, since UHF in MAC is fast.

2 Asymmetric Cryptography

2.1 TODO

TODO

3 Advanced Cryptography

3.1 Data at rest: Searchable Encryption

Searchable Encryption consists of three protocols:

1. Setup: Client generates an *encrypted database + encrypted search index*[13], uploads them to the server.
2. Search: Client generates a *search token*, server uses it to process the search, returns the result.
3. Update: Client generates an *update token*, server uses it to update the encrypted database and encrypted search index, returns success/failure.

Active field of research, many open problems (e.g. leakage analysis + prevention).

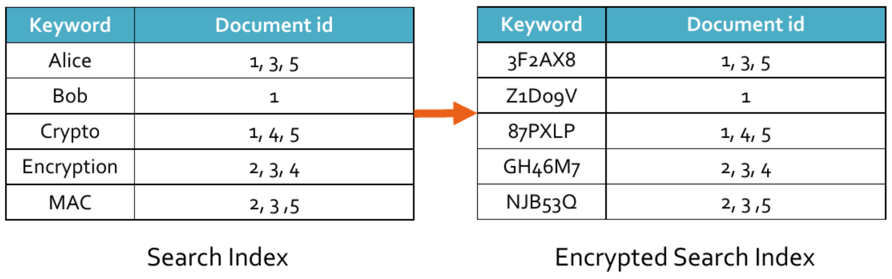
Goals

- Security: Confidentiality (of data and queries) against an *honest-but-curious* server.[14]
- Efficiency: Storage, computational, bandwidth requirements.
- Functionality: Supported query types.

First construction Idea: randomly choose a PRF F_K and replace each keyword w in the index by $F_K(w)$. Also encrypt each document under some symmetric key K_0.

Leakage from setup $\mathcal{L}_{Setup}$: total number of documents and keywords, keyword frequency, co-occurences of keywords.

Leakage from searches $\mathcal{L}_{Search}$: result patterns, query patterns, result intersection between queries.



Keyword	Document id
Alice	1, 3, 5
Bob	1
Crypto	1, 4, 5
Encryption	2, 3, 4
MAC	2, 3 ,5

Search Index

Keyword	Document id
3F2AX8	1, 3, 5
Z1D09V	1
87PXLP	1, 4, 5
GH46M7	2, 3, 4
NJB53Q	2, 3 ,5

Encrypted Search Index

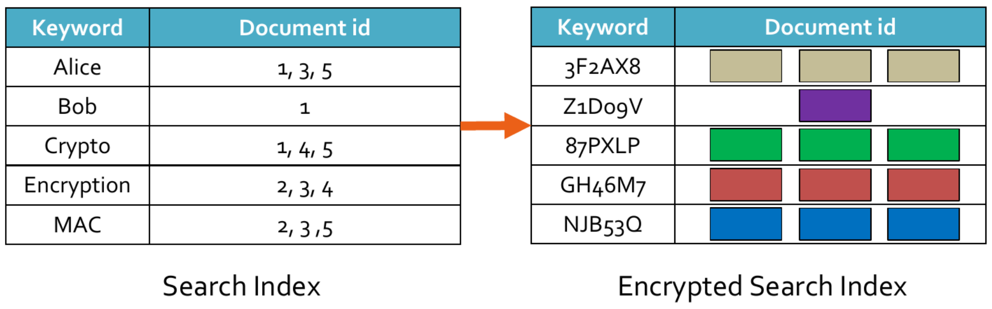
Figure 17: First construction for searchable encryption

Second construction Idea: randomly choose a PRF F_K. For each keyword w calculate $K_1||K_2 = F_K(w)$. Replace each keyword with K_1 (as before). In addition: associate each document id for w with a counter cnt and replace the id with $id \oplus F_{K_2}(cnt)$.

This hides the co-occurences of keyword pairs (PRF security).

[13] A search index is a mapping from document ids to keywords, and/or vice versa. For more details see the lecture *Information Retrieval*.

[14] Compare this security model against a fully malicious server.



Keyword	Document id
Alice	1, 3, 5
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Crypto	1, 4, 5
Encryption	2, 3, 4
MAC	2, 3 ,5

Keyword	Document id
3F2AX8	▨ ▨ ▨
Z1D09V	🟪
87PXLP	🟩 🟩 🟩
GH46M7	🟥 🟥 🟥
NJB53Q	🟦 🟦 🟦

Search Index
Encrypted Search Index

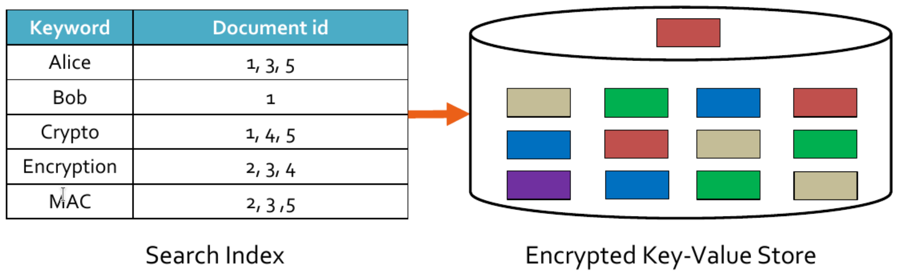
Figure 18: Second construction for searchable encryption

Third construction Idea: ... (as before) ...
In addition: associate each document id for w with a counter cnt and replace the id with $id \oplus F_{K_2}(cnt)$. Then store this document id value in a key-value store (dictionary) under "key" $F_{K_1}(cnt)$.

Search: Send $(K_1, K_2, |cnt_{querystring}|)$ to the server. Server recomputes the $F_{K_1}(cnt)$ to find the values in the dictionary. Then decrypts them to document ids using K_2. Looks up the encrypted documents and returns them.

Leakage from setup $\mathcal{L}_{Setup}$:[15] total number of documents.

Leakage from searches $\mathcal{L}_{Search}$: (same as before) result patterns, query patterns, result intersection between queries.



Keyword	Document id
Alice	1, 3, 5
Bob	1
Crypto	1, 4, 5
Encryption	2, 3, 4
MAC	2, 3 ,5

Figure 19: Third construction for searchable encryption (setup)

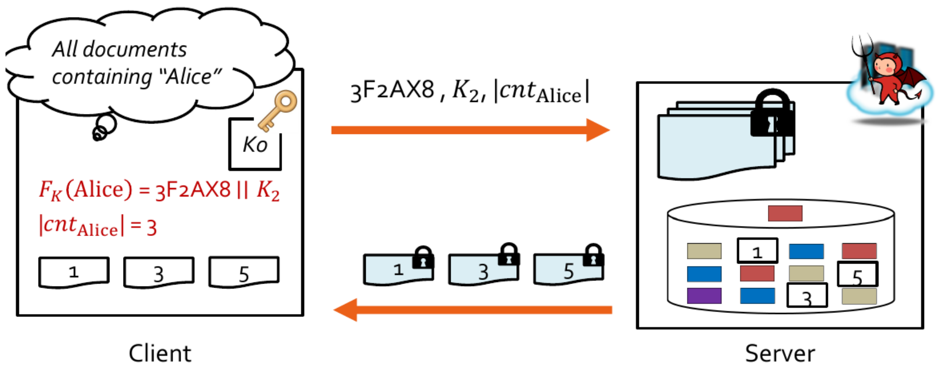

All documents containing "Alice"
Ko
F_K(Alice) = 3F2AX8 || K_2
$|cnt_{Alice}|$ = 3
1
3
5
3F2AX8 , K_2, $|cnt_{Alice}|$
1
3
5
Client
Server

Figure 20: Third construction for searchable encryption (search)

Defining Leakage Goal: formally define the leakage $\mathcal{L} = (\mathcal{L}_{Setup}, \mathcal{L}_{Search})$ of searchable encryption schemes.

[15]This is also what an adversary learns from a single snapshot.

Security game: an adversary $\mathcal{A}$ interacts with a challenger $\mathcal{C}$ that contains either the real world or a simulator. The simulator $\mathcal{S}$ only has access to the leakage $\mathcal{L}$ (but not to the secret key). Intuitively, the adversary should gain no extra information other than the leakage.

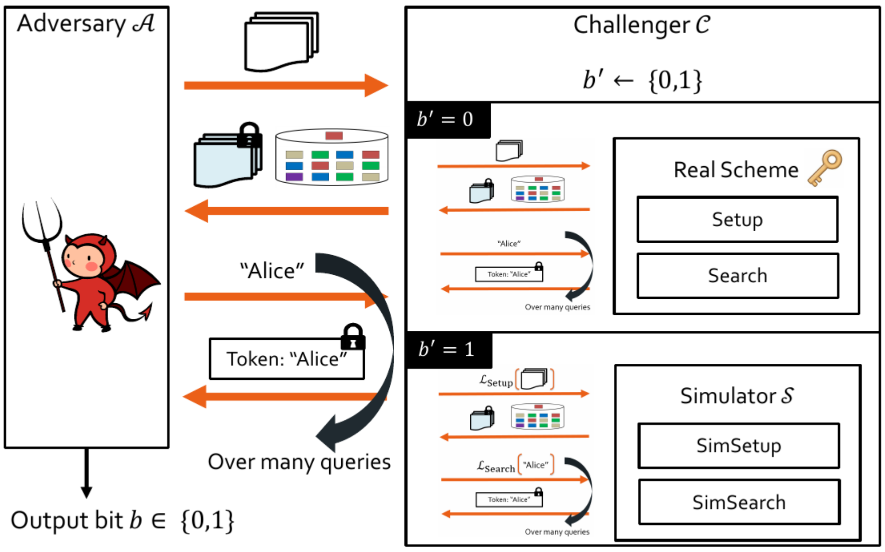


Figure 21: Searchable encryption leakage game

Analysing Leakage What can the adversary infer from the leakage? It turns out that given auxiliary information, one can e.g. perform query recovery.

3.2 Data in transit: Protocols

3.3 Data under computation: FHE