A3-Q3: Combatting Overfitting with Dropout and Regularization

Preliminaries

In [1]:
```
# Standard imports
import numpy as np
import torch
import matplotlib.pylab as plt
import copy

#for reproducibility purposes
torch.manual_seed(2025)
np.random.seed(2025)
```

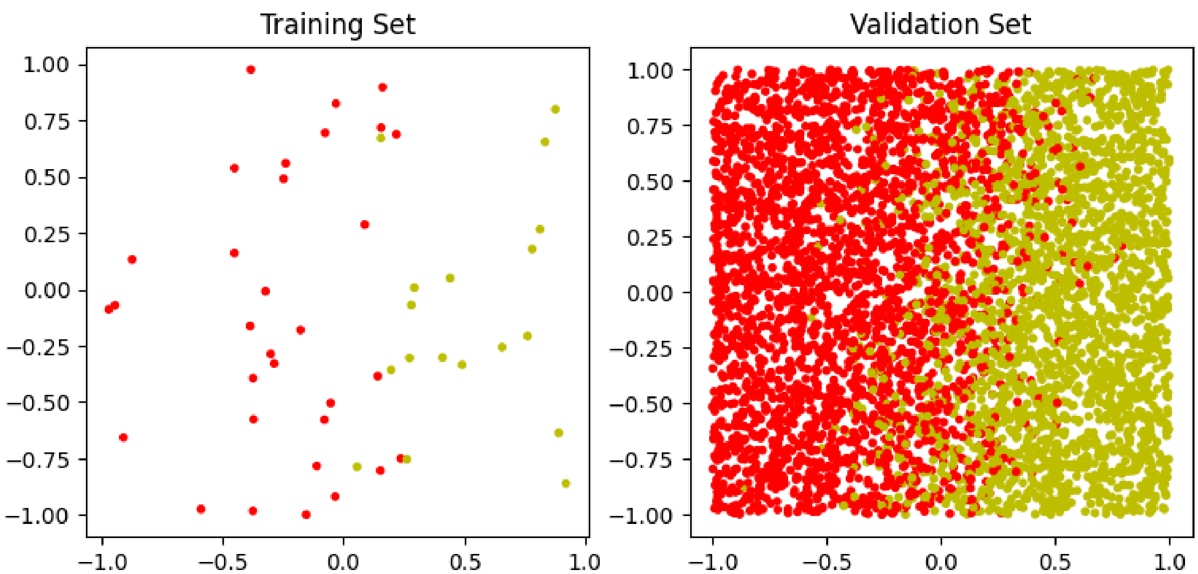
In [ ]:
```

```

Dataset

In [14]:
```
class DividedPlane(torch.utils.data.Dataset):
    def __init__(self, n=100, noise=0.1, seed=None):
        torch.manual_seed(seed)
        theta = torch.rand((1,))*2.*torch.pi
        a = torch.tensor([torch.cos(theta), torch.sin(theta), 0.1])
        def myfunc(x):
            y = a[0]*x[:,0] + a[1]*x[:,1] + a[2]
            return y
        self.x = torch.rand((n,2))*2. - 1.
        y = myfunc(self.x) + noise*torch.normal( torch.zeros((len(self.x))) )
        self.y = (y>0.).type(torch.float)

    def __len__(self):
        return len(self.x)

    def __getitem__(self, idx):
        return self.x[idx], self.y[idx]

    def inputs(self):
        return self.x

    def targets(self):
        return self.y.reshape( (len(self.y),1) )

    def plot(self, labels=None, *args, **kwargs):
        X = self.inputs()
        if labels is None:
            labels = self.targets()
        colour_options = ['y', 'r', 'g', 'b', 'k']
```

```
        if len(labels[0])>1:
            # one-hot labels
            cidx = torch.argmax(labels, axis=1)
        else:
            # binary labels
            cidx = (labels>0.5).type(torch.int)
        colours = [colour_options[k] for k in cidx]
        plt.scatter(X[:,0].detach(), X[:,1].detach(), color=colours, marker='.')
```

In [15]:
```
seed = np.random.randint(100000)
train = DividedPlane(n=50, noise=0.25, seed=seed)
validation = DividedPlane(n=5000, noise=0.25, seed=seed)
plt.figure(figsize=(9,4))
plt.subplot(1,2,1); train.plot(); plt.title(f'Training Set');
plt.subplot(1,2,2); validation.plot(); plt.title(f'Validation Set');
```



(a): `Dropout` layer

In [16]:
```
class Dropout(torch.nn.Module):
    '''
     lyr = Dropout()

     Creates a dropout layer in which each node is set to zero
     with probability lyr.dropprob.

     Usage:
       lyr = Dropout()
       lyr.set_dropprob(p) # set the dropout probability to p
       y = lyr(z)          # sets each node to 0 with probability p
    '''
    def __init__(self):
        super().__init__()
        self.dropprob = 0.

    def set_dropprob(self, p):
```

```
        self.dropprob = p

    def forward(self, z):
        # Drop nodes with prob dropprob
        #===== YOUR CODE HERE =====

        return z * torch.empty_like(z).bernoulli_(1 - self.dropprob) / (1 - self.dr
```

In [17]:

```
# Test for Dropout Layer
z = torch.ones((3,1000))
drop_layer = Dropout()
drop_layer.set_dropprob(0.75)
y = drop_layer(z)
drop_fraction = (torch.sum(y==0.)*100.)/torch.numel(y)
print(f'Dropped {drop_fraction:.1f}%')
print(f'Expected output is {torch.sum(y)}, which should be close to {torch.sum(z)}'
```

```
Dropped 75.3%
Expected output is 2960.0, which should be close to 3000.0
```

(b): `RobustNetwork`

In [18]:

```
class RobustNetwork(torch.nn.Module):
    def __init__(self, nodes=100):
        super().__init__()
        self.lyrs = torch.nn.ModuleList()
        self.lyrs.append(torch.nn.Linear(2, nodes//2))
        self.lyrs.append(torch.nn.Tanh())
        self.drop_lyr1 = Dropout()
        self.lyrs.append(self.drop_lyr1)
        self.lyrs.append(torch.nn.Linear(nodes//2, nodes))
        self.lyrs.append(torch.nn.Tanh())
        self.drop_lyr2 = Dropout()
        self.lyrs.append(self.drop_lyr2)
        self.lyrs.append(torch.nn.Linear(nodes, 1))
        self.lyrs.append(torch.nn.Sigmoid())
        self.loss_fcn = torch.nn.BCELoss(reduction='mean')

    def forward(self, x):
        y = x
        for lyr in self.lyrs:
            y = lyr(y)
        return y

    def learn(self, x, t, epochs=100, lr=0.1, l2_lambda=0.0):
        losses = []
        for epoch in range(epochs):
            y = self(x)
            data_loss = self.loss_fcn(y.squeeze(), t.squeeze())

            #Add L2 regulaization implementation here

            frob_norm = 0
            for p in self.parameters():
```

```python
                frob_norm += p.pow(2).sum()

            loss = data_loss + frob_norm * l2_lambda / 2

            losses.append(loss.item())
            self.zero_grad()
            loss.backward()
            with torch.no_grad():
                #replace me for gradient descent updates
                for p in self.parameters():
                    p -= l2_lambda * lr * p.grad
        plt.plot(np.array(losses))
        plt.yscale('log'); plt.xlabel('Epochs'); plt.ylabel('Log Loss');

        print(f'Final loss = {loss}')
        return losses
```

(c) Train and Validation

In [30]:

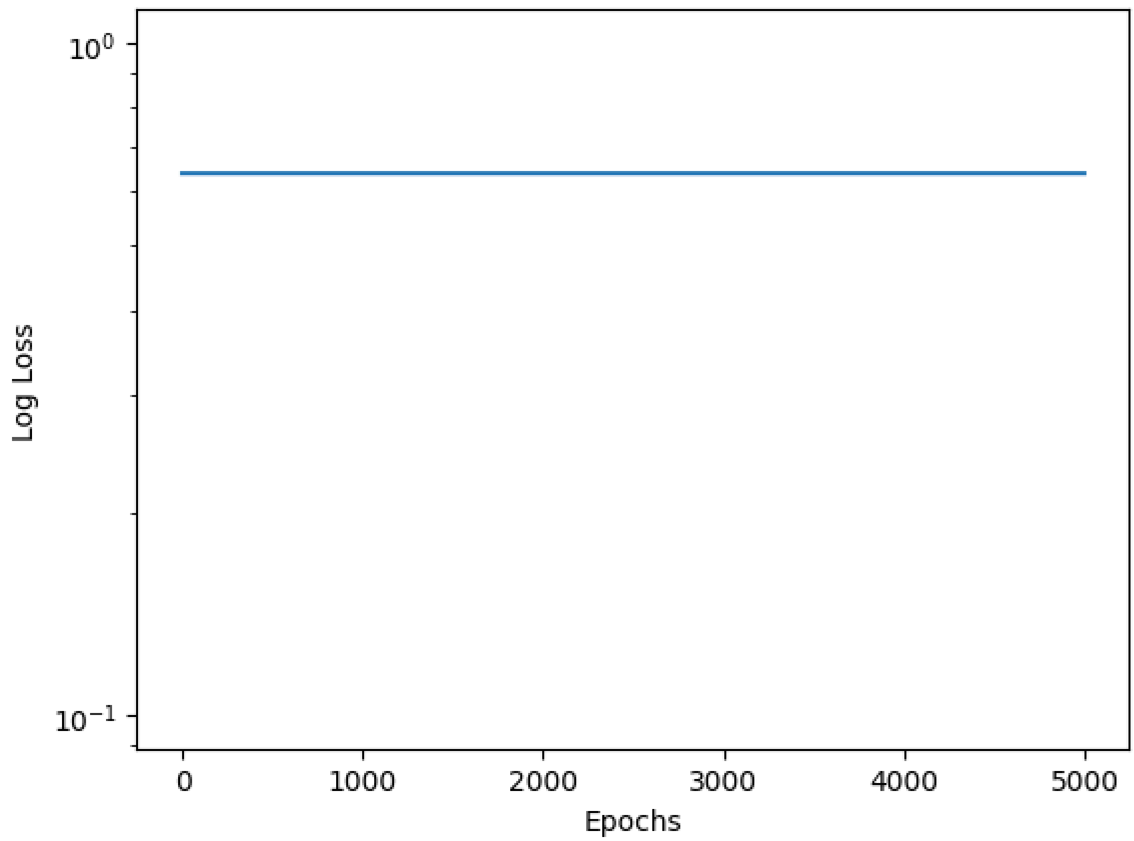
```python
net_orig = RobustNetwork(nodes=100)

# Duplicate the network for apples-to-apples comparison
net = copy.deepcopy(net_orig) #network without regularization
dnet = copy.deepcopy(net_orig) #network with dropout
l2net = copy.deepcopy(net_orig) #network with L2 regularization

# Set come common parameters
lr = 0.25
n_epochs = 5000
```

Train the models

In [31]:

```python
# No effort to guard against overfitting
net.drop_lyr1.dropprob = 0.
net.drop_lyr2.dropprob = 0.
losses = net.learn(train.inputs(), train.targets(), epochs=n_epochs, lr=lr)
```

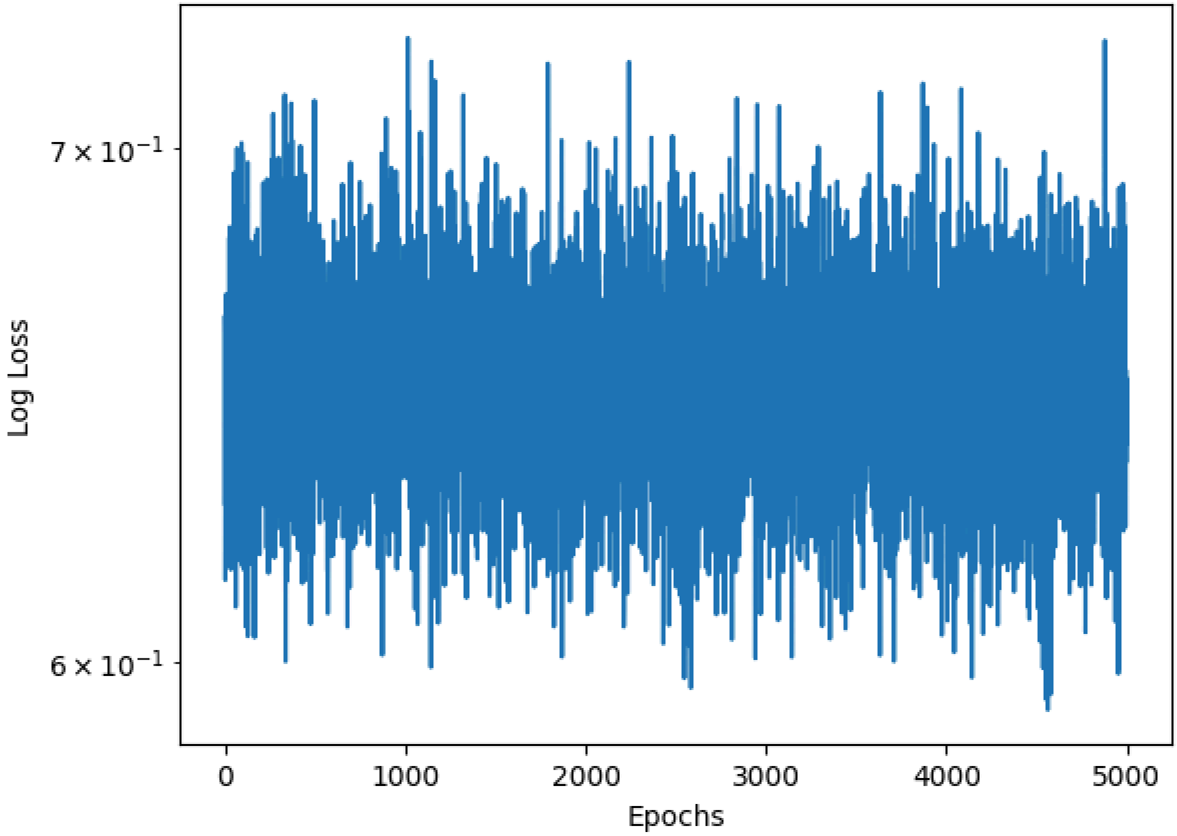
```
Final loss = 0.6395237445831299
```



In [32]:

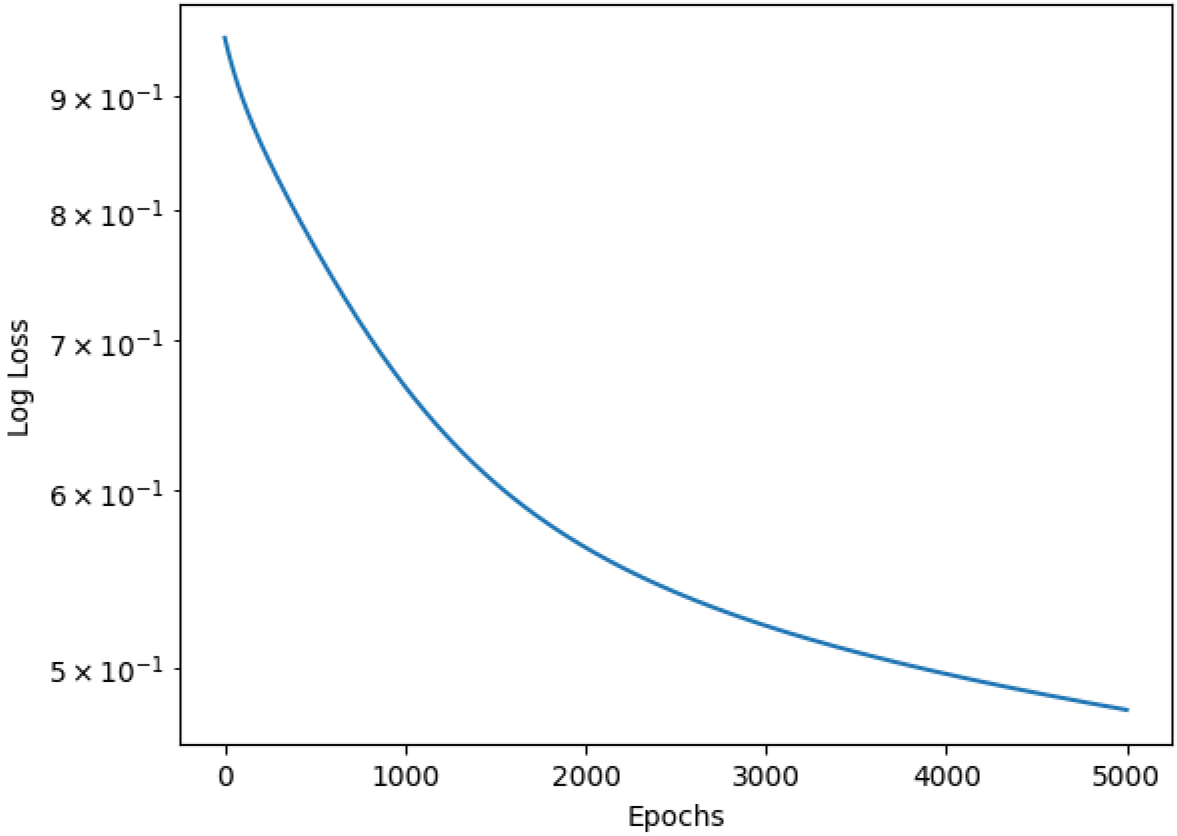
```
#network with dropout (dnet) training here
dnet.drop_lyr1.dropprob = 0.5
dnet.drop_lyr2.dropprob = 0.5
dlosses = dnet.learn(train.inputs(), train.targets(), epochs=n_epochs, lr=lr)
```

```
Final loss = 0.6372409462928772
```



In [33]:
```
#network with l2 regularization (l2net) training here
l2net.drop_lyr1.dropprob = 0.
l2net.drop_lyr2.dropprob = 0.
l2losses = l2net.learn(train.inputs(), train.targets(), epochs=n_epochs, lr=lr, l2_
```

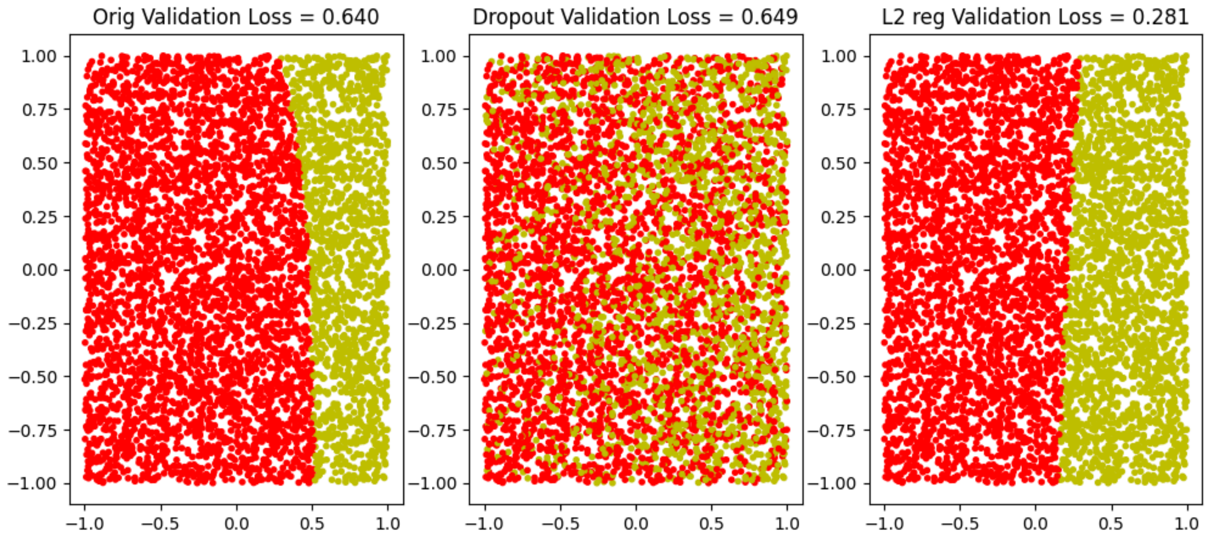
```
Final loss = 0.4785027503967285
```



Test the models

Let's see what the decision boundaries look like.

In [35]:
```
# Compute validation losses here: "validation_loss", "dvalidation_loss", "l2validat

def compute_validation_loss(model, x, t):
    model.eval()
    with torch.no_grad():
        y = model(x)
        val_data_loss = model.loss_fcn(y.squeeze(), t.squeeze()).item()
    return val_data_loss

validation_loss = compute_validation_loss(net, validation.inputs(), validation.targ
dvalidation_loss = compute_validation_loss(dnet, validation.inputs(), validation.ta
l2validation_loss = compute_validation_loss(l2net, validation.inputs(), validation.

# Displaying the results
plt.figure(figsize=(12,5))
plt.subplot(1,3,1)
#Original model
validation.plot(labels=net(validation.inputs())); plt.title(f'Orig Validation Loss 
plt.subplot(1,3,2)
#Model with dropout
validation.plot(labels=dnet(validation.inputs())); plt.title(f'Dropout Validation L
plt.subplot(1,3,3)
#Model with L2 regularization
validation.plot(labels=l2net(validation.inputs())); plt.title(f'L2 reg Validation L
```



Regularization has the lowest validation loss and is lower than dropout.