A4-Q3: Convolutional Autoencoders

Preliminaries

In [70]:
```
import numpy as np
import torch
import torch.nn as nn
import torchvision
import matplotlib.pyplot as plt
from tqdm import tqdm
```

In [ ]:
```

```

In [71]:
```
# In case you are fortunate enough to have access to a GPU...
device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
```

Dataset: MNIST

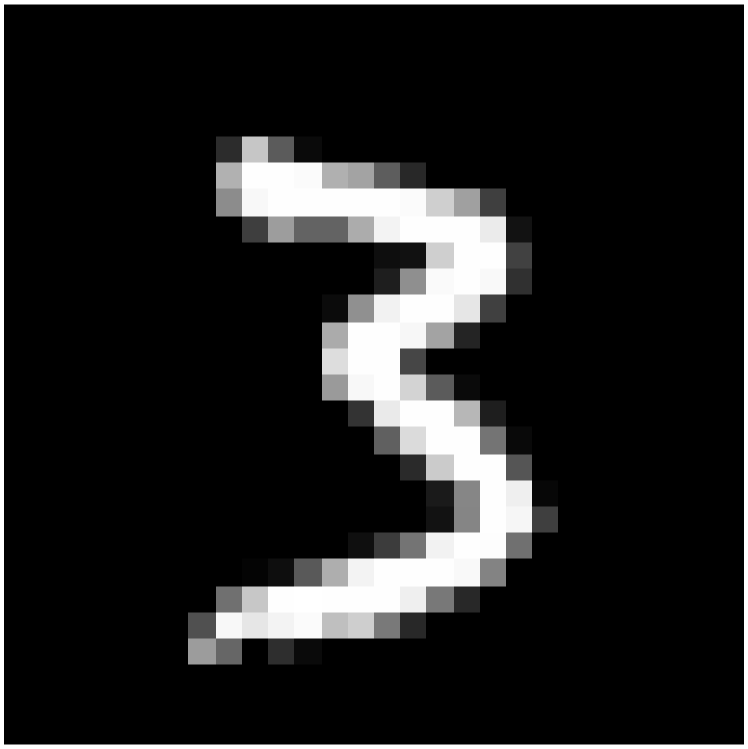
In [72]:
```
# You can change img_size to 14 if you want to use smaller (14x14) images.
img_size = 28
ds_full = torchvision.datasets.MNIST('./files/', train=True, download=True,
                            transform=torchvision.transforms.Compose([
                            torchvision.transforms.Resize((img_size,img_size)),
                            torchvision.transforms.ToTensor(),
                            ]))
```

In [73]:
```
ds = torch.utils.data.Subset(ds_full, range(1024))
```

In [74]:
```
def Draw(x):
    with torch.no_grad():
        plt.imshow(x.squeeze().detach().numpy(), cmap='gray');
        plt.axis('off');
```

In [77]:
```
with torch.no_grad():
    x,t = ds.__getitem__(130)
    Draw(x)
    plt.title(f'Size: {list(x.size())}')
plt.show()
```

Size: [1, 28, 28]



Create some `DataLoader`s

In [78]:
```
# Batched, for training
batch_size_train = 8
train_dl = torch.utils.data.DataLoader(ds, batch_size=batch_size_train, shuffle=Tru
```

In [79]:
```
# A single batch, for plotting
train_all = torch.utils.data.DataLoader(ds, batch_size=1024, shuffle=False)
```

A. Complete the `ConvAE` class

In [80]:
```
class ConvAE(nn.Module):
    '''
     net = ConvAE(img_size=28, embedding_dim=3)

     Create a convolutional autoencoder for imput images of size (img_size x img_si
     with an embedding (latent) layer of (embedding_dim) neurons.

     Inputs:
       img_size       size of input images, [1, img_size, img_size]
       embedding_dim  number of nodes in embedding (latent) layer

     Usage:
       net = ConvAE()
       y = net(x)
       h = net.encode(x)  # returns latent vectors
    '''
    def __init__(self, img_size=28, embedding_dim=3):
        self.img_size = img_size
        self.embedding_dim = embedding_dim
        self.losses = []
        super().__init__()

        #===== YOUR CODE HERE =====
        self.encoder = nn.Sequential(
            nn.Conv2d(1, 16, kernel_size=3, stride=2, padding=1),
            nn.ReLU(True),
            nn.Conv2d(16, 32, kernel_size=3, stride=2, padding=1),
            nn.ReLU(True),
            nn.Conv2d(32, 64, kernel_size=3, stride=2, padding=1),
            nn.ReLU(True),
            nn.Flatten()
        )
        self.fc_enc = nn.Sequential(
            nn.Linear(64 * 4 * 4, 128),
            nn.ReLU(True),
            nn.Linear(128, embedding_dim),
            nn.Tanh()
        )

        self.fc_dec = nn.Sequential(
            nn.Linear(embedding_dim, 128),
            nn.ReLU(True),
            nn.Linear(128, 64 * 4 * 4),
            nn.ReLU(True),
            nn.Unflatten(1, (64, 4, 4))
        )
        self.decoder = nn.Sequential(
            nn.ConvTranspose2d(64, 32, kernel_size=3, stride=2, padding=1, output_p
            nn.ReLU(True),
            nn.ConvTranspose2d(32, 16, kernel_size=3, stride=2, padding=1, output_p
            nn.ReLU(True),
            nn.ConvTranspose2d(16, 1, kernel_size=3, stride=2, padding=1, output_pa
            nn.Sigmoid()
        )

    def encode(self, x):
        #===== YOUR CODE HERE =====
        x = self.encoder(x)
        h = self.fc_enc(x)
        return h


    def decode(self, x):
        #===== YOUR CODE HERE =====
        h = self.fc_dec(x)
        x = self.decoder(h)
        x = x[:, :, :self.img_size, :self.img_size]
        return x


    def forward(self, x):
        #===== YOUR CODE HERE =====
        h = self.encode(x)
        y = self.decode(h)
        return y
```

B. Create and train the network

In [81]:
```
net = ConvAE(img_size=img_size, embedding_dim=3)
```

In [82]:
```
# Train it
optimizer = torch.optim.Adam(net.parameters(), lr=1e-3, weight_decay=1e-5)
criterion = nn.MSELoss()
epochs = 100

for epoch in range(epochs):
    net.train()
    running_loss = 0

    for x, _ in tqdm(train_dl, desc=f"Epoch {epoch+1}/{epochs}"):
        x = x.to(device)

        y_pred = net(x)

        loss = criterion(y_pred, x)

        optimizer.zero_grad()
        loss.backward()
        optimizer.step()

        running_loss += loss.item()

    avg_loss = running_loss / len(train_dl)
    net.losses.append(avg_loss)
    print(f"Epoch [{epoch+1}/{epochs}]  Average Loss: {avg_loss:.6f}")

print("\nTraining complete!")
```

```
Epoch 1/100: 100%|██████████| 128/128 [00:00<00:00, 298.93it/s]
Epoch [1/100]  Average Loss: 0.094311
Epoch 2/100: 100%|██████████| 128/128 [00:00<00:00, 333.11it/s]
Epoch [2/100]  Average Loss: 0.066325
Epoch 3/100: 100%|██████████| 128/128 [00:00<00:00, 297.87it/s]
Epoch [3/100]  Average Loss: 0.066098
Epoch 4/100: 100%|██████████| 128/128 [00:00<00:00, 305.37it/s]
Epoch [4/100]  Average Loss: 0.066066
Epoch 5/100: 100%|██████████| 128/128 [00:00<00:00, 316.04it/s]
Epoch [5/100]  Average Loss: 0.066016
Epoch 6/100: 100%|██████████| 128/128 [00:00<00:00, 282.23it/s]
Epoch [6/100]  Average Loss: 0.065992
Epoch 7/100: 100%|██████████| 128/128 [00:00<00:00, 321.09it/s]
Epoch [7/100]  Average Loss: 0.066038
Epoch 8/100: 100%|██████████| 128/128 [00:00<00:00, 326.99it/s]
Epoch [8/100]  Average Loss: 0.065589
Epoch 9/100: 100%|██████████| 128/128 [00:00<00:00, 336.16it/s]
Epoch [9/100]  Average Loss: 0.061889
Epoch 10/100: 100%|██████████| 128/128 [00:00<00:00, 301.06it/s]
Epoch [10/100]  Average Loss: 0.058413
Epoch 11/100: 100%|██████████| 128/128 [00:00<00:00, 319.25it/s]
Epoch [11/100]  Average Loss: 0.053229
Epoch 12/100: 100%|██████████| 128/128 [00:00<00:00, 347.75it/s]
Epoch [12/100]  Average Loss: 0.050628
Epoch 13/100: 100%|██████████| 128/128 [00:00<00:00, 326.38it/s]
Epoch [13/100]  Average Loss: 0.047173
Epoch 14/100: 100%|██████████| 128/128 [00:00<00:00, 314.01it/s]
Epoch [14/100]  Average Loss: 0.044147
Epoch 15/100: 100%|██████████| 128/128 [00:00<00:00, 335.90it/s]
Epoch [15/100]  Average Loss: 0.042780
Epoch 16/100: 100%|██████████| 128/128 [00:00<00:00, 331.97it/s]
Epoch [16/100]  Average Loss: 0.041491
Epoch 17/100: 100%|██████████| 128/128 [00:00<00:00, 334.59it/s]
Epoch [17/100]  Average Loss: 0.040398
Epoch 18/100: 100%|██████████| 128/128 [00:00<00:00, 336.09it/s]
Epoch [18/100]  Average Loss: 0.039726
Epoch 19/100: 100%|██████████| 128/128 [00:00<00:00, 339.70it/s]
Epoch [19/100]  Average Loss: 0.038845
Epoch 20/100: 100%|██████████| 128/128 [00:00<00:00, 312.60it/s]
Epoch [20/100]  Average Loss: 0.038110
Epoch 21/100: 100%|██████████| 128/128 [00:00<00:00, 328.88it/s]
Epoch [21/100]  Average Loss: 0.037718
Epoch 22/100: 100%|██████████| 128/128 [00:00<00:00, 339.22it/s]
Epoch [22/100]  Average Loss: 0.037302
Epoch 23/100: 100%|██████████| 128/128 [00:00<00:00, 336.29it/s]
Epoch [23/100]  Average Loss: 0.036799
Epoch 24/100: 100%|██████████| 128/128 [00:00<00:00, 336.74it/s]
Epoch [24/100]  Average Loss: 0.036237
Epoch 25/100: 100%|██████████| 128/128 [00:00<00:00, 337.17it/s]
Epoch [25/100]  Average Loss: 0.036009
Epoch 26/100: 100%|██████████| 128/128 [00:00<00:00, 334.59it/s]
Epoch [26/100]  Average Loss: 0.035396
Epoch 27/100: 100%|██████████| 128/128 [00:00<00:00, 330.67it/s]
Epoch [27/100]  Average Loss: 0.035097
Epoch 28/100: 100%|██████████| 128/128 [00:00<00:00, 346.76it/s]
Epoch [28/100]  Average Loss: 0.034641
Epoch 29/100: 100%|██████████| 128/128 [00:00<00:00, 342.58it/s]
Epoch [29/100]  Average Loss: 0.034551
Epoch 30/100: 100%|██████████| 128/128 [00:00<00:00, 336.88it/s]
Epoch [30/100]  Average Loss: 0.034383
Epoch 31/100: 100%|██████████| 128/128 [00:00<00:00, 352.53it/s]
Epoch [31/100]  Average Loss: 0.033887
Epoch 32/100: 100%|██████████| 128/128 [00:00<00:00, 353.96it/s]
Epoch [32/100]  Average Loss: 0.033738
Epoch 33/100: 100%|██████████| 128/128 [00:00<00:00, 356.37it/s]
Epoch [33/100]  Average Loss: 0.033376
Epoch 34/100: 100%|██████████| 128/128 [00:00<00:00, 314.48it/s]
Epoch [34/100]  Average Loss: 0.033217
Epoch 35/100: 100%|██████████| 128/128 [00:00<00:00, 325.81it/s]
Epoch [35/100]  Average Loss: 0.033032
Epoch 36/100: 100%|██████████| 128/128 [00:00<00:00, 319.99it/s]
Epoch [36/100]  Average Loss: 0.032820
Epoch 37/100: 100%|██████████| 128/128 [00:00<00:00, 334.34it/s]
Epoch [37/100]  Average Loss: 0.032732
Epoch 38/100: 100%|██████████| 128/128 [00:00<00:00, 344.01it/s]
Epoch [38/100]  Average Loss: 0.032297
Epoch 39/100: 100%|██████████| 128/128 [00:00<00:00, 355.82it/s]
Epoch [39/100]  Average Loss: 0.032170
Epoch 40/100: 100%|██████████| 128/128 [00:00<00:00, 329.93it/s]
Epoch [40/100]  Average Loss: 0.032210
Epoch 41/100: 100%|██████████| 128/128 [00:00<00:00, 344.81it/s]
Epoch [41/100]  Average Loss: 0.031973
Epoch 42/100: 100%|██████████| 128/128 [00:00<00:00, 301.96it/s]
Epoch [42/100]  Average Loss: 0.031800
Epoch 43/100: 100%|██████████| 128/128 [00:00<00:00, 310.39it/s]
Epoch [43/100]  Average Loss: 0.031709
Epoch 44/100: 100%|██████████| 128/128 [00:00<00:00, 283.62it/s]
Epoch [44/100]  Average Loss: 0.031687
Epoch 45/100: 100%|██████████| 128/128 [00:00<00:00, 310.97it/s]
Epoch [45/100]  Average Loss: 0.031582
Epoch 46/100: 100%|██████████| 128/128 [00:00<00:00, 293.78it/s]
Epoch [46/100]  Average Loss: 0.031273
Epoch 47/100: 100%|██████████| 128/128 [00:00<00:00, 335.98it/s]
Epoch [47/100]  Average Loss: 0.031180
Epoch 48/100: 100%|██████████| 128/128 [00:00<00:00, 300.94it/s]
Epoch [48/100]  Average Loss: 0.031075
Epoch 49/100: 100%|██████████| 128/128 [00:00<00:00, 323.50it/s]
Epoch [49/100]  Average Loss: 0.030884
Epoch 50/100: 100%|██████████| 128/128 [00:00<00:00, 336.76it/s]
Epoch [50/100]  Average Loss: 0.030669
Epoch 51/100: 100%|██████████| 128/128 [00:00<00:00, 332.24it/s]
Epoch [51/100]  Average Loss: 0.030905
Epoch 52/100: 100%|██████████| 128/128 [00:00<00:00, 326.23it/s]
Epoch [52/100]  Average Loss: 0.030613
Epoch 53/100: 100%|██████████| 128/128 [00:00<00:00, 329.22it/s]
Epoch [53/100]  Average Loss: 0.030437
Epoch 54/100: 100%|██████████| 128/128 [00:00<00:00, 327.41it/s]
Epoch [54/100]  Average Loss: 0.030602
Epoch 55/100: 100%|██████████| 128/128 [00:00<00:00, 334.88it/s]
Epoch [55/100]  Average Loss: 0.030307
Epoch 56/100: 100%|██████████| 128/128 [00:00<00:00, 330.00it/s]
Epoch [56/100]  Average Loss: 0.030160
Epoch 57/100: 100%|██████████| 128/128 [00:00<00:00, 332.22it/s]
Epoch [57/100]  Average Loss: 0.030249
Epoch 58/100: 100%|██████████| 128/128 [00:00<00:00, 322.36it/s]
Epoch [58/100]  Average Loss: 0.030046
Epoch 59/100: 100%|██████████| 128/128 [00:00<00:00, 336.16it/s]
Epoch [59/100]  Average Loss: 0.029871
Epoch 60/100: 100%|██████████| 128/128 [00:00<00:00, 329.00it/s]
Epoch [60/100]  Average Loss: 0.029734
Epoch 61/100: 100%|██████████| 128/128 [00:00<00:00, 334.79it/s]
Epoch [61/100]  Average Loss: 0.029833
Epoch 62/100: 100%|██████████| 128/128 [00:00<00:00, 276.68it/s]
Epoch [62/100]  Average Loss: 0.029598
Epoch 63/100: 100%|██████████| 128/128 [00:00<00:00, 321.75it/s]
Epoch [63/100]  Average Loss: 0.029699
Epoch 64/100: 100%|██████████| 128/128 [00:00<00:00, 318.65it/s]
Epoch [64/100]  Average Loss: 0.029592
Epoch 65/100: 100%|██████████| 128/128 [00:00<00:00, 291.10it/s]
Epoch [65/100]  Average Loss: 0.029303
Epoch 66/100: 100%|██████████| 128/128 [00:00<00:00, 336.37it/s]
Epoch [66/100]  Average Loss: 0.029678
Epoch 67/100: 100%|██████████| 128/128 [00:00<00:00, 220.99it/s]
Epoch [67/100]  Average Loss: 0.029353
Epoch 68/100: 100%|██████████| 128/128 [00:00<00:00, 222.24it/s]
Epoch [68/100]  Average Loss: 0.029115
Epoch 69/100: 100%|██████████| 128/128 [00:00<00:00, 275.35it/s]
Epoch [69/100]  Average Loss: 0.029124
Epoch 70/100: 100%|██████████| 128/128 [00:00<00:00, 293.04it/s]
Epoch [70/100]  Average Loss: 0.028942
Epoch 71/100: 100%|██████████| 128/128 [00:00<00:00, 302.34it/s]
Epoch [71/100]  Average Loss: 0.028969
Epoch 72/100: 100%|██████████| 128/128 [00:00<00:00, 273.53it/s]
Epoch [72/100]  Average Loss: 0.028793
Epoch 73/100: 100%|██████████| 128/128 [00:00<00:00, 297.84it/s]
Epoch [73/100]  Average Loss: 0.028687
Epoch 74/100: 100%|██████████| 128/128 [00:00<00:00, 299.20it/s]
Epoch [74/100]  Average Loss: 0.029036
Epoch 75/100: 100%|██████████| 128/128 [00:00<00:00, 284.33it/s]
Epoch [75/100]  Average Loss: 0.028865
Epoch 76/100: 100%|██████████| 128/128 [00:00<00:00, 314.80it/s]
Epoch [76/100]  Average Loss: 0.028479
Epoch 77/100: 100%|██████████| 128/128 [00:00<00:00, 311.05it/s]
Epoch [77/100]  Average Loss: 0.028582
Epoch 78/100: 100%|██████████| 128/128 [00:00<00:00, 304.87it/s]
Epoch [78/100]  Average Loss: 0.028654
Epoch 79/100: 100%|██████████| 128/128 [00:00<00:00, 314.92it/s]
Epoch [79/100]  Average Loss: 0.028408
Epoch 80/100: 100%|██████████| 128/128 [00:00<00:00, 271.30it/s]
Epoch [80/100]  Average Loss: 0.028271
Epoch 81/100: 100%|██████████| 128/128 [00:00<00:00, 307.28it/s]
Epoch [81/100]  Average Loss: 0.028317
Epoch 82/100: 100%|██████████| 128/128 [00:00<00:00, 292.36it/s]
Epoch [82/100]  Average Loss: 0.028206
Epoch 83/100: 100%|██████████| 128/128 [00:00<00:00, 311.44it/s]
Epoch [83/100]  Average Loss: 0.028313
Epoch 84/100: 100%|██████████| 128/128 [00:00<00:00, 306.14it/s]
Epoch [84/100]  Average Loss: 0.028031
Epoch 85/100: 100%|██████████| 128/128 [00:00<00:00, 306.73it/s]
Epoch [85/100]  Average Loss: 0.028088
Epoch 86/100: 100%|██████████| 128/128 [00:00<00:00, 302.02it/s]
Epoch [86/100]  Average Loss: 0.028096
Epoch 87/100: 100%|██████████| 128/128 [00:00<00:00, 298.97it/s]
Epoch [87/100]  Average Loss: 0.027919
Epoch 88/100: 100%|██████████| 128/128 [00:00<00:00, 287.16it/s]
Epoch [88/100]  Average Loss: 0.027987
Epoch 89/100: 100%|██████████| 128/128 [00:00<00:00, 308.92it/s]
Epoch [89/100]  Average Loss: 0.027994
Epoch 90/100: 100%|██████████| 128/128 [00:00<00:00, 292.34it/s]
Epoch [90/100]  Average Loss: 0.027649
Epoch 91/100: 100%|██████████| 128/128 [00:00<00:00, 305.27it/s]
Epoch [91/100]  Average Loss: 0.027555
Epoch 92/100: 100%|██████████| 128/128 [00:00<00:00, 312.63it/s]
Epoch [92/100]  Average Loss: 0.027720
Epoch 93/100: 100%|██████████| 128/128 [00:00<00:00, 308.72it/s]
Epoch [93/100]  Average Loss: 0.027724
Epoch 94/100: 100%|██████████| 128/128 [00:00<00:00, 303.76it/s]
Epoch [94/100]  Average Loss: 0.027465
Epoch 95/100: 100%|██████████| 128/128 [00:00<00:00, 309.01it/s]
Epoch [95/100]  Average Loss: 0.027553
Epoch 96/100: 100%|██████████| 128/128 [00:00<00:00, 310.87it/s]
Epoch [96/100]  Average Loss: 0.027673
Epoch 97/100: 100%|██████████| 128/128 [00:00<00:00, 304.92it/s]
Epoch [97/100]  Average Loss: 0.027372
Epoch 98/100: 100%|██████████| 128/128 [00:00<00:00, 310.46it/s]
Epoch [98/100]  Average Loss: 0.027316
Epoch 99/100: 100%|██████████| 128/128 [00:00<00:00, 267.35it/s]
Epoch [99/100]  Average Loss: 0.027343
Epoch 100/100: 100%|██████████| 128/128 [00:00<00:00, 272.13it/s]
Epoch [100/100]  Average Loss: 0.027298

Training complete!
```

In [83]:
```python
# Uncomment the following line if you want to save your network.
#torch.save(net.to('cpu'), 'my_ConvAE.pt')
# The corresponding code to reload the network is below.
#net = torch.load('my_ConvAE.pt')
# Remember to send it to the GPU, if you're using one.
net.to(device)
```

Out[83]:
```
ConvAE(
  (encoder): Sequential(
    (0): Conv2d(1, 16, kernel_size=(3, 3), stride=(2, 2), padding=(1, 1))
    (1): ReLU(inplace=True)
    (2): Conv2d(16, 32, kernel_size=(3, 3), stride=(2, 2), padding=(1, 1))
    (3): ReLU(inplace=True)
    (4): Conv2d(32, 64, kernel_size=(3, 3), stride=(2, 2), padding=(1, 1))
    (5): ReLU(inplace=True)
    (6): Flatten(start_dim=1, end_dim=-1)
  )
  (fc_enc): Sequential(
    (0): Linear(in_features=1024, out_features=128, bias=True)
    (1): ReLU(inplace=True)
    (2): Linear(in_features=128, out_features=3, bias=True)
    (3): Tanh()
  )
  (fc_dec): Sequential(
    (0): Linear(in_features=3, out_features=128, bias=True)
    (1): ReLU(inplace=True)
    (2): Linear(in_features=128, out_features=1024, bias=True)
    (3): ReLU(inplace=True)
    (4): Unflatten(dim=1, unflattened_size=(64, 4, 4))
  )
  (decoder): Sequential(
    (0): ConvTranspose2d(64, 32, kernel_size=(3, 3), stride=(2, 2), padding=(1, 
1), output_padding=(1, 1))
    (1): ReLU(inplace=True)
    (2): ConvTranspose2d(32, 16, kernel_size=(3, 3), stride=(2, 2), padding=(1, 
1), output_padding=(1, 1))
    (3): ReLU(inplace=True)
    (4): ConvTranspose2d(16, 1, kernel_size=(3, 3), stride=(2, 2), padding=(1, 1), 
output_padding=(1, 1))
    (5): Sigmoid()
  )
)
```

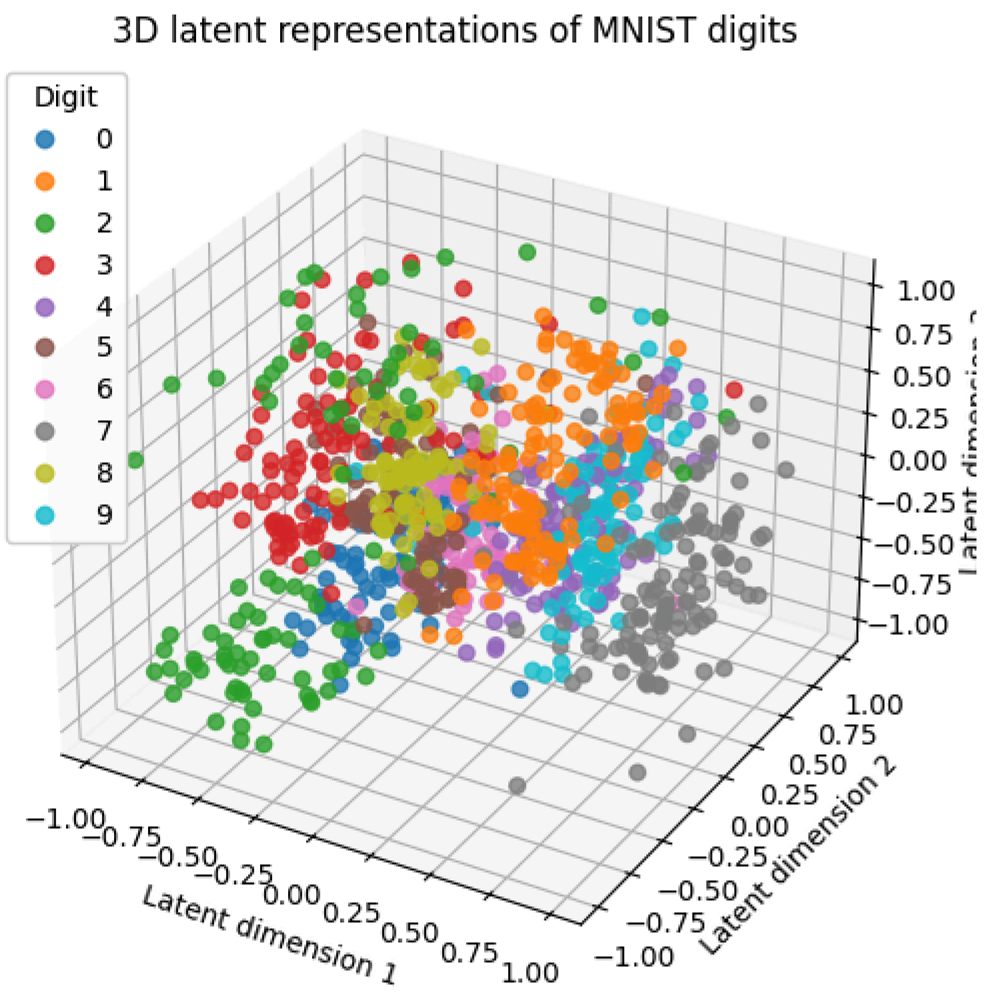
C. Plot the latent space

In [84]:
```
# Here are 10 colours you can use. But feel free to use others, too.
colour_options = ['k', 'tab:brown', 'r', 'orange', 'gold', 'lawngreen', 'forestgree
```

In [ ]:
```
# Compute the latent-space representation for all the samples.
```

In [ ]:
```
# You can create three 2D planar projections.
```

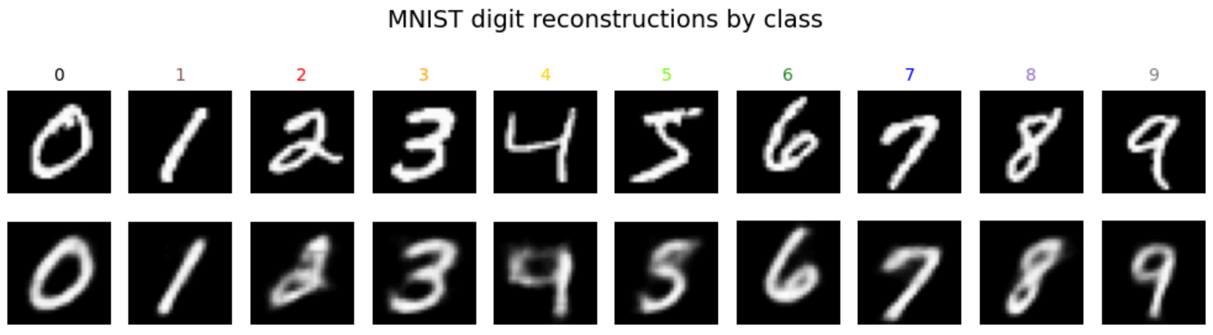
In [85]:
```
# Or you can plot a 3D scatter plot.
fig = plt.figure(figsize=(7,6))
ax = fig.add_subplot(111, projection='3d')

#===== YOUR CODE HERE =====
net.eval()
all_x, all_t = next(iter(train_all))
all_x = all_x.to(device)

with torch.no_grad():
    h = net.encode(all_x).cpu().numpy()  # shape [1024, 3]
labels = all_t.numpy()

sc = ax.scatter(h[:,0], h[:,1], h[:,2],
                c=labels, cmap='tab10', s=30, alpha=0.8)

ax.set_xlabel('Latent dimension 1')
ax.set_ylabel('Latent dimension 2')
ax.set_zlabel('Latent dimension 3')
ax.set_title('3D latent representations of MNIST digits')

legend = ax.legend(*sc.legend_elements(), title="Digit")
ax.add_artist(legend)

plt.show()
```



D. Plot reconstructed digit images

In [88]:

```
net.eval()

all_x, all_t = next(iter(train_all))
all_x = all_x.to(device)
all_t = all_t.numpy()

with torch.no_grad():
    y = net(all_x).cpu()

examples = []
for digit in range(10):
    idx = np.where(all_t == digit)[0][0]  # <-- fixed line
    examples.append((digit, all_x[idx], y[idx]))

plt.figure(figsize=(10, 3))

# Top row (inputs)
for i, (digit, orig, recon) in enumerate(examples):
    plt.subplot(2, 10, i + 1)
    plt.imshow(orig.squeeze().cpu(), cmap='gray')
    plt.title(str(digit), color=colour_options[digit], fontsize=10)
    plt.axis('off')
    if i == 0:
        plt.ylabel("Input", fontsize=12)

# Bottom row (reconstructions)
for i, (digit, orig, recon) in enumerate(examples):
    plt.subplot(2, 10, 10 + i + 1)
    plt.imshow(recon.squeeze().cpu(), cmap='gray')
    plt.axis('off')
    if i == 0:
        plt.ylabel("Output", fontsize=12)

plt.suptitle("MNIST digit reconstructions by class", fontsize=14)
plt.tight_layout()
plt.show()
```



Digit Reconstructions