NSCI 247 HW5

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Written Portion

1) a) $P(X,Y,Z,V,U) = P(X) \cdot P(Z) \cdot P(V \mid X) \cdot P(Y \mid X, Y) \cdot P(U \mid Y)$

b) X and Z have common child Y, so they are dependent. X and Z become independent when conditioned on Y as once Y is known it blocks additional influence between X and Z (conditionally independent). Conditioning on V doesn't make X and Z independent as V is decendant of X and Y is still a common child.

c) For X

- no parents
- Children: V and Y
- other parents of children: Z

So, markov blanket is $\{V, Y, Z\}$

2)
$$P_{combined}(x) \propto P_1(x) P_2(x)$$
$$\propto \exp(-|x-\mu_1|) \exp(-|x-\mu_2|)$$
$$\propto \exp(-|x-\mu_1|-|x-\mu_2|)$$

With $\mu_1 = -1$, $\mu_2 = 1$

$$P_{combined}(x) \propto \exp(-|x+1|-|x-1|)$$

$x < -1$:

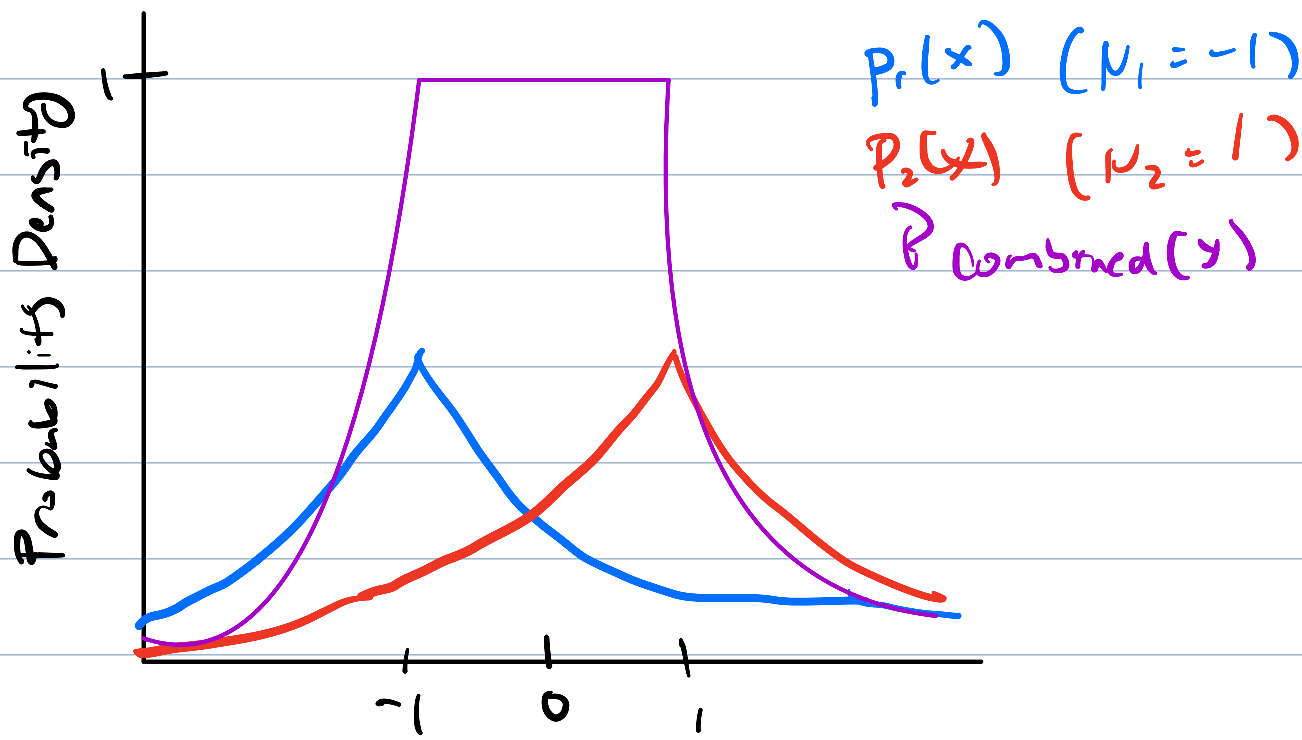
$$P_{combined}(x) \propto \exp(-(x-1)-(-x+1)) = \exp(-2x)$$

$-1 < x < 1$

$$P_{combined}(x) \propto \exp(-(x+1)-(-x+1)) = \exp(-2)$$

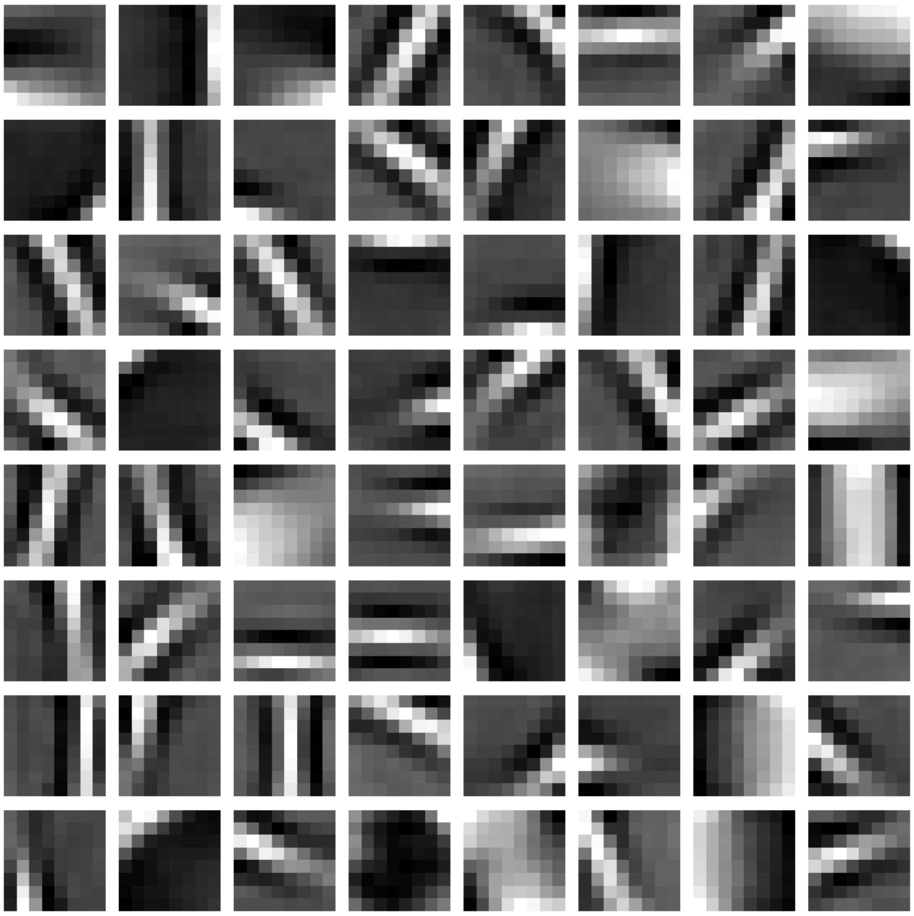
$x > 1$

$$P_{combined}(x) \propto \exp(-(x+1)-(x-1)) = \exp(-2x)$$



Computer Portion

1)



2)

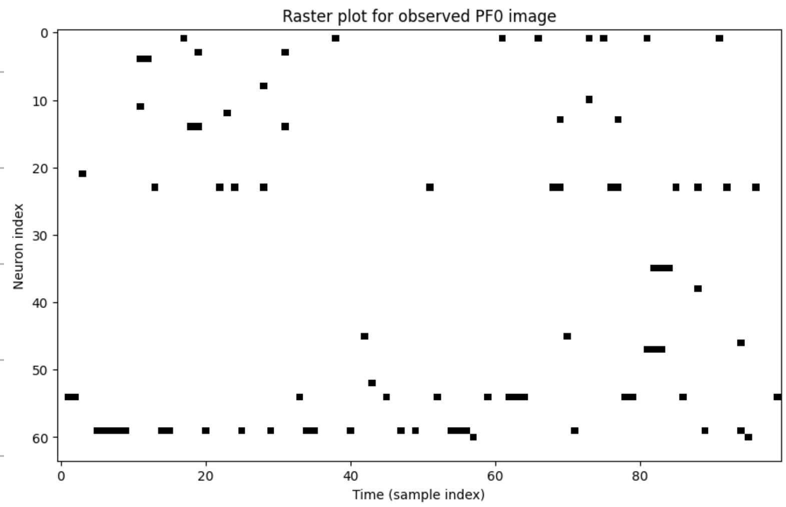
Gibbs Sampling Equation for the k-th latent: [p(r_k = 1 | I, r_{-k}) = \frac{p(I | r_k=1, r_{-k}) p(r_k=1)}{p(I | r_k=1, r_{-k}) p(r_k=1) + p(I | r_k=0, r_{-k}) p(r_k=0)}]

$$p(r_k=1 \mid I, r_{-k}) = \frac{P(I \mid r_k=1, r_{-k})\, p(r_k=1)}{p(I \mid r_k=1, r_{-k})\, p(r_k=1) + p(I \mid r_k=0, r_{-k})\, P(r_k=0)}$$

3) See code

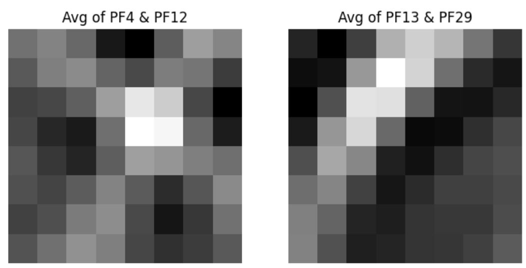
4) See code

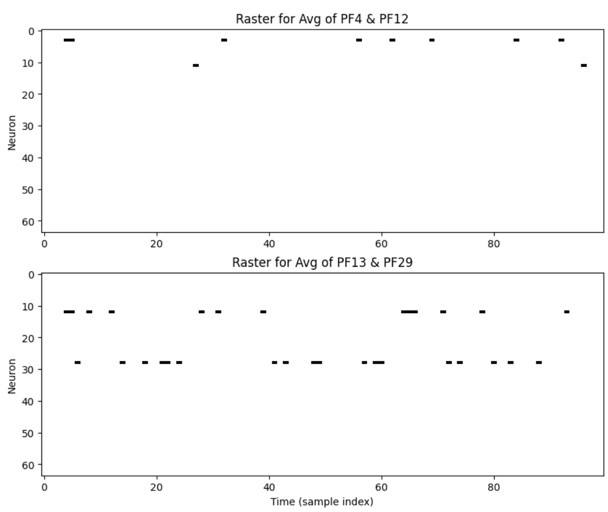
5)



The activity is distributed across neurons overtime. This suggest the Gibbs sampler is transitioning states effectively. Generally I think it is behaving as it should as it reflects the expected variability in neuron activation.

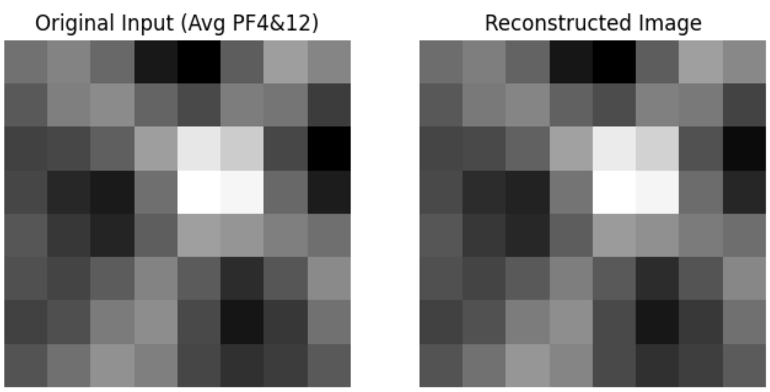
6) a)



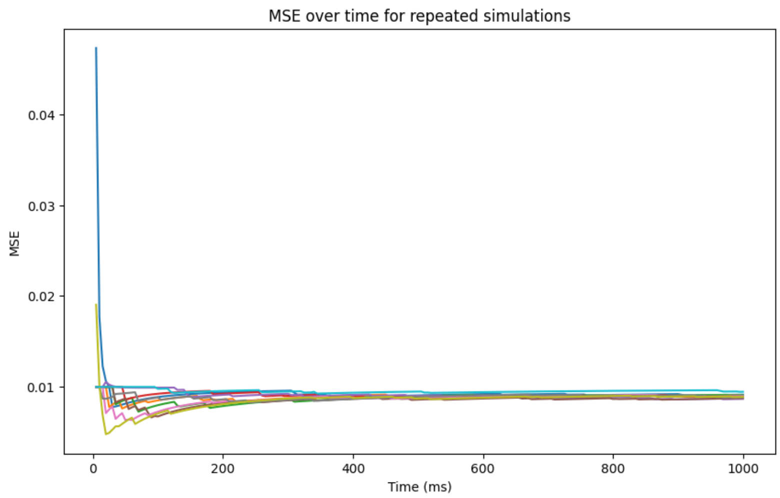


This shows distinct firing patterns for the two different sets of averaged projective fields. PF4 & PF12 raster plot shows sparse and localized neural response which is consistent with the averaged PF. For PF13 and PF29, the raster plot shows a more distributed and varied neural response, which also reflects the broader distribution of activity in input representation. As such, this aligns with expectation that averaged PF should produce a stimulus that combines features of individual fields.

b)

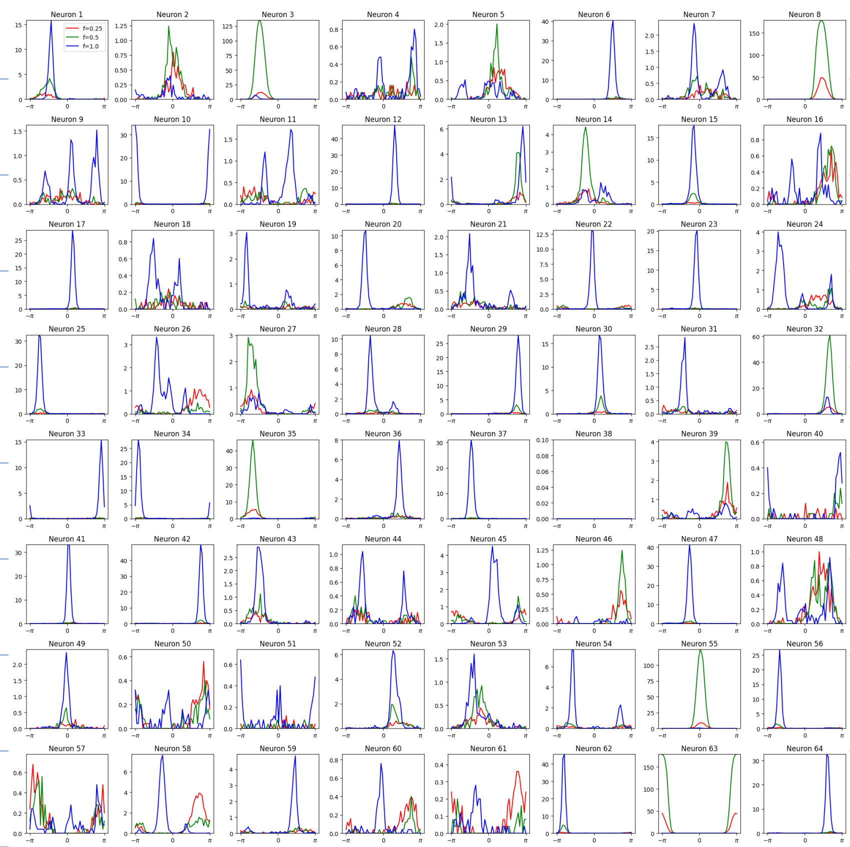


Reconstructed Image = $PFs^T \cdot$ Posterior Probability c)



As more time is given, I would expect the error to decrease as more samples are able to be drawn. This aligns with the results observed in the graph.

7) a) view code
b)



The amplitude is determined by strength of contrast and spatial alignment. The prefered orientation is determined by the dominant orientation of the edges or gradients in PF. There are latents with weak orientation tuning even with having oriented projective fields. This can happen due to inhibitory interactions, sub~~optimal~~ spatial frequency, and intrinsic variability.