

Table 1: Model Performance Metrics

Model	Regime	Performance Metrics				Count
		RMSE	MAPE (%)	R^2	Q-LIKE	
Common Dates Test Set Predictions ¹						
One-Regime HAR	High Volatility	0.003371	49.90	0.0163	0.3470	22
One-Regime RF	High Volatility	0.003721	55.04	0.1982	0.5218	22
Two-Regime HAR	High Volatility	0.003346	50.49	0.0313	0.3356	22
Two-Regime RF	High Volatility	0.003406	58.44	0.0041	0.3681	22
One-Regime HAR	Low Volatility	0.000870	68.89	0.1350	0.3251	385
One-Regime RF	Low Volatility	0.000887	127.70	0.1003	0.3298	385
Two-Regime HAR	Low Volatility	0.000869	66.81	0.1376	0.3297	385
Two-Regime RF	Low Volatility	0.000874	106.14	0.1264	0.3476	385
One-Regime HAR	Overall	0.001153	67.86	0.3463	0.3263	407
One-Regime RF	Overall	0.001222	123.77	0.2664	0.3402	407
Two-Regime HAR	Overall	0.001148	65.92	0.3520	0.3300	407
Two-Regime RF	Overall	0.001162	103.56	0.3366	0.3487	407
Individual Model Performance on Original Test Sets						
One-Regime Models ²						
One-Regime HAR	High Volatility	0.003371	49.90	0.0163	0.3470	22
One-Regime RF	High Volatility	0.003721	55.04	0.1982	0.5218	22
One-Regime HAR	Low Volatility	0.001074	61.47	0.0815	0.3013	535
One-Regime RF	Low Volatility	0.001093	108.49	0.0501	0.3091	535
One-Regime HAR	Overall	0.001248	61.02	0.2460	0.3031	557
One-Regime RF	Overall	0.001301	106.38	0.1802	0.3175	557
Two-Regime Models						
Two-Regime HAR	High Volatility	0.007944	44.12	0.0897	0.5782	172
Two-Regime RF	High Volatility	0.008050	53.04	0.0653	0.7126	172
Two-Regime HAR	Low Volatility	0.000869	66.81	0.1376	0.3297	385
Two-Regime RF	Low Volatility	0.000874	106.14	0.1264	0.3476	385
Two-Regime HAR	Overall	0.004473	59.80	0.2343	0.4064	557
Two-Regime RF	Overall	0.004532	89.74	0.2141	0.4603	557

¹Common dates determined by the intersection of one-regime and two-regime test sets.²Models were not differentiated by regime during training; regime classification performed post-hoc using a Hidden Markov Model.

Table 2: Diebold-Mariano Test Results Across Volatility Regimes

Models Compared	Volatility Regime	DM Statistic	P-Value	Conclusion
Overall Performance				
OneRegHAR vs OneRegRF	Overall	-2.5835	0.0101	OneRegHAR outperforms
OneRegHAR vs TwoRegHAR	Overall	1.4386	0.1510	No significant difference
OneRegHAR vs TwoRegRF	Overall	-0.3600	0.7190	No significant difference
OneRegRF vs TwoRegHAR	Overall	2.6217	0.0091	TwoRegHAR outperforms
OneRegRF vs TwoRegRF	Overall	2.489	0.0132	TwoRegRF outperforms
TwoRegHAR vs TwoRegRF	Overall	-0.5820	0.5609	No significant difference
High Volatility Regime				
OneRegHAR vs OneRegRF	High Vol	-2.5612	0.0182	OneRegHAR outperforms
OneRegHAR vs TwoRegHAR	High Vol	2.7839	0.0111	TwoRegHAR outperforms
OneRegHAR vs TwoRegRF	High Vol	-0.2450	0.1806	No significant difference
OneRegRF vs TwoRegHAR	High Vol	2.6562	0.0148	TwoRegHAR outperforms
OneRegRF vs TwoRegRF	High Vol	2.6493	0.0150	TwoRegRF outperforms
TwoRegHAR vs TwoRegRF	High Vol	-0.4309	0.6709	No significant difference
Low Volatility Regime				
OneRegHAR vs OneRegRF	Low Vol	-1.1798	0.2388	No significant difference
OneRegHAR vs TwoRegHAR	Low Vol	0.3104	0.7564	No significant difference
OneRegHAR vs TwoRegRF	Low Vol	-0.3366	0.7366	No significant difference
OneRegRF vs TwoRegHAR	Low Vol	1.1161	0.2651	No significant difference
OneRegRF vs TwoRegRF	Low Vol	0.8352	0.4041	No significant difference
TwoRegHAR vs TwoRegRF	Low Vol	-0.4816	0.6304	No significant difference

Table 3: HAR Model Coefficients Comparison Across Regimes

Predictor	One-Regime		Two-Regime High		Two-Regime Low	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
const	-0.6024	0.0002	-7.0526	0.0000	-7.1480	0.0000
ln_RV_d,t	0.4808	0.0000	0.4799	0.0000	0.4471	0.0000
ln_RV_w,t	0.3076	0.0000	0.2517	0.0006	0.2542	0.0000
ln_RV_m,t	0.1406	0.0007	0.0953	0.0393	0.1068	0.0103

Table 4: Random Forest Feature Importances Comparison Across Regimes

Predictor	One-Regime	Two-Regime High	Two-Regime Low
ln_RV_d,t	0.5883	0.4461	0.4366
ln_RV_w,t	0.1507	0.2560	0.1791
ln_RV_m,t	0.0670	0.0697	0.0923
volume	0.0462	0.0671	0.0629
high	0.0445	0.0457	0.0852
Fear_Greed_Value	0.0424	0.0741	0.0567
UEMP15T26	0.0178	0.01178	0.0218
TB3SMFFM	0.0163	0.0098	0.0282
PERMITW	0.0134	0.0179	0.0201
UEMP27OV	0.0132	0.0087	0.0171

Table 5: Model Performance Metrics Across Forecasting Approaches

Model-Regime	RMSE	MAPE (%)	R^2	QLIKE	Count
One-Regime Common High - HAR	0.0034	0.4990	0.0163	0.3470	22
One-Regime Common High - RF	0.0037	0.5504	-0.1982	0.5218	22
One-Regime Common Low - HAR	0.0009	0.6889	0.1350	0.3251	385
One-Regime Common Low - RF	0.0009	1.2770	0.1003	0.3298	385
One-Regime Common Overall - HAR	0.0012	0.6787	0.3463	0.3263	407
One-Regime Common Overall - RF	0.0012	1.2377	0.2664	0.3402	407
One-Regime High - HAR	0.0034	0.4990	0.0163	0.3470	22
One-Regime High - RF	0.0037	0.5504	-0.1982	0.5218	22
One-Regime Low - HAR	0.0011	0.6147	0.0815	0.3013	535
One-Regime Low - RF	0.0011	1.0849	0.0501	0.3091	535
One-Regime Overall - HAR	0.0012	0.6102	0.2460	0.3031	557
One-Regime Overall - RF	0.0013	1.0638	0.1802	0.3175	557
Two-Regime Common High - HAR	0.0033	0.5049	0.0313	0.3356	22
Two-Regime Common High - RF	0.0036	0.5279	-0.1458	0.5360	22
Two-Regime Common Low - HAR	0.0009	0.6681	0.1376	0.3297	385
Two-Regime Common Low - RF	0.0009	0.8856	0.1283	0.3841	385
Two-Regime Common Overall - HAR	0.0011	0.6592	0.3520	0.3300	407
Two-Regime Common Overall - RF	0.0012	0.8662	0.2939	0.3923	407
Two-Regime High - HAR	0.0079	0.4412	0.0897	0.5782	172
Two-Regime High - RF	0.0081	0.5159	0.0583	0.7688	172
Two-Regime Low - HAR	0.0009	0.6681	0.1376	0.3297	385
Two-Regime Low - RF	0.0009	0.8856	0.1283	0.3841	385
Two-Regime Overall - HAR	0.0045	0.5980	0.2343	0.4064	557
Two-Regime Overall - RF	0.0045	0.7714	0.2084	0.5029	557