Section 6.3

1) a. $\vec{n} = (1, 2, -1)$

b. $\vec{n} = (4, -2, -3)$

c. $\vec{n} = (-1, 6, 1)$

d. $\vec{n} = (2, 3, -4)$

2)

a. $\vec{n}_1 = (1, -1, 1)$
$\vec{n}_2 = (2, -2, 2)$

$\vec{n}_1 \cdot \vec{n}_2 = 2 + 2 + 2 \neq 0$
Since $\vec{n}_2 = 2\vec{n}_1$, $\vec{n}_1 \parallel \vec{n}_2$

b. $\vec{n}_1 = (2, -1, 1)$
$\vec{n}_2 = (1, 1, -1)$

$\vec{n}_1 \cdot \vec{n}_2 = 2 - 1 - 1 = 0$
Since $\vec{n}_1 \cdot \vec{n}_2 = 0$, $\vec{n}_1 \perp \vec{n}_2$

c. $\vec{n}_1 = (2, -1, 3)$
$\vec{n}_2 = (1, 2, 1)$

$\vec{n}_1 \cdot \vec{n}_2 = 2 - 2 + 3 \neq 0$
The planes are not orthogonal or parallel

d. $\vec{n}_1 = (4, -2, 6)$
$\vec{n}_2 = (-2, 1, -3)$

$\vec{n}_1 \cdot \vec{n}_2 = -8 - 2 - 18 \neq 0$
Since $\vec{n}_2 = -\frac{1}{2}\vec{n}_1$, $\vec{n}_1 \parallel \vec{n}_2$

3) a. $\begin{cases} x - y + z = 2 \\ 2x - 3y + z = 1 \end{cases}$ $\qquad \begin{cases} x + 2z = 5 \\ y + z = 3 \end{cases}$

$$(A|\vec{b}) = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -3 & 1 & 1 \end{bmatrix} R_1(-2) + R_2$$
$$= \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & -1 & -1 & -3 \end{bmatrix} R_2(-1)$$
$$= \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 1 & 3 \end{bmatrix} R_2(1) + R_1$$
$$= \begin{bmatrix} 1 & 0 & 2 & 5 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

Let $z = t$
$x = 5 - 2t$
$y = 3 - t$
$z = t$

$\therefore x = 5 - 2t,\ y = 3 - t,\ z = t$

b. $\begin{cases} 3x - y + 3z = -1 \\ -4x + 2y - 4z = 2 \end{cases}$

$$(A|\vec{b}) = \begin{bmatrix} 3 & -1 & 3 & -1 \\ -4 & 2 & -4 & 2 \end{bmatrix} R_2(-\tfrac{1}{4})$$
$$= \begin{bmatrix} 3 & -1 & 3 & -1 \\ 1 & -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix} R_{2,1}$$
$$= \begin{bmatrix} 1 & -\frac{1}{2} & 1 & -\frac{1}{2} \\ 3 & -1 & 3 & -1 \end{bmatrix} R_1(-3) + R_2$$
$$= \begin{bmatrix} 1 & -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} R_2(1) + R_1$$
$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$\begin{cases} x + z = 0 \\ y = \frac{1}{2} \end{cases}$

Let $z = t$
$x = -t$
$y = \frac{1}{2}$
$z = t$

$\therefore x = -t,\ y = \frac{1}{2},\ z = t$

c. $\begin{cases} x - 3y - 2z = 4 \\ 3x + 2y - 4z = 6 \end{cases}$ $\qquad \begin{cases} x + \frac{-16}{11}z = \frac{62}{11} \\ y + \frac{2}{11}z = \frac{-6}{11} \end{cases}$

$$(A|\vec{b}) = \begin{bmatrix} 1 & -3 & -2 & 4 \\ 3 & 2 & -4 & 6 \end{bmatrix} R_1(-3) + R_2$$
$$= \begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 11 & 2 & -6 \end{bmatrix} R_2(\tfrac{1}{11})$$
$$= \begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 1 & \frac{2}{11} & -\frac{6}{11} \end{bmatrix} R_2(3) + R_1$$
$$= \begin{bmatrix} 1 & 0 & -\frac{16}{11} & \frac{62}{11} \\ 0 & 1 & \frac{2}{11} & -\frac{6}{11} \end{bmatrix}$$

Let $z = t$
$\therefore x = \frac{62}{11} + \frac{16}{11}t$
$y = -\frac{6}{11} - \frac{2}{11}t$
$z = t$

4) $\begin{cases} 2x + 2y + z = 2 \\ x - 3y = 5 \end{cases}$ $\quad P = (2, -3, 0)$

$\vec{n}_1 = (2, 2, 1)$
$\vec{n}_2 = (1, -3, 0)$

$$\vec{n}_1 \times \vec{n}_2 = \left(\begin{vmatrix} 2 & 1 \\ -3 & 0 \end{vmatrix}, -\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 2 & 2 \\ 1 & -3 \end{vmatrix} \right) = (3, 1, -8)$$

$x = 2 + 3t,\ y = -3 + t,\ z = -8t$

5) $\vec{v}_1 = (1, -2, -1)$ $\quad \vec{v}_2 = (-1, -1, 1)$

$$\vec{v}_1 \times \vec{v}_2 = \left(\begin{vmatrix} -2 & -1 \\ -1 & 1 \end{vmatrix} i, -\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} j, \begin{vmatrix} 1 & -2 \\ -1 & -1 \end{vmatrix} k \right) = (-3, 0, 1)$$

6) a. $x + 3y + 5z = 1 - 3 = -2$
b. $-x - z = -1 - 3 = -4$
c. $-x + y - z = -1 - 2 = -3$
d. $x + y + z = 1$

7) $4x - y + 3z = 8 - 7 - 3 = -2$

8) a. $\overrightarrow{P_1P_2} = (1, 1, 2)$ $\quad \overrightarrow{P_1P_3}(2, -3, 3)$

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \left(\begin{vmatrix} 1 & 2 \\ -3 & 3 \end{vmatrix}, -\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} \right)$$
$$\vec{n} = (9, 1, -5)$$

$\vec{n} \cdot \vec{P}_1 = 9 + 2 + 5 = 16$
$9x + y - 5 = 16$

b. $\overrightarrow{P_1P_2} = (-1, 1, -2)$ $\quad \overrightarrow{P_1P_3} = (0, 1, -3)$

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \left(\begin{vmatrix} 1 & -2 \\ 1 & -3 \end{vmatrix}, -\begin{vmatrix} -1 & -2 \\ 0 & -3 \end{vmatrix}, \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} \right)$$
$$\vec{n} = (-1, -3, -1)$$

$\vec{n} \cdot P_1 = -1 + -1 = -2$
$-x - 3y - z = -2$

9) $\vec{n}_1 = (2, 1, 1)$, $\vec{n}_2 = (1, -2, 3)$

$\vec{v}_1 = (\qquad)$ $\quad \vec{v}_2 = (6, 0, 1)$

$$\vec{n}_1 \times \vec{n}_2 = \left(\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix}, -\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} \right)$$
$$= (5, -5, -5)$$
$$\vec{v}_1 = 5(1, -1, -1)$$

$$\vec{v}_1 \times \vec{v}_2 = \left(\begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & -1 \\ 6 & 1 \end{vmatrix}, \begin{vmatrix} 1 & -1 \\ 6 & 0 \end{vmatrix} \right)$$
$$= (-1, -7, -6)$$

10) $\begin{cases} x - y + 3z = 5 \\ 2x - 2y + 7z = 0 \end{cases}$

$$(A|\vec{b}) = \begin{bmatrix} 1 & -1 & 3 & 5 \\ 2 & 2 & 7 & 0 \end{bmatrix} R_1(-2) + R_2$$
$$= \begin{bmatrix} 1 & -1 & 3 & 5 \\ 0 & 4 & 1 & -10 \end{bmatrix} R_2(\tfrac{1}{4})$$
$$= \begin{bmatrix} 1 & -1 & 3 & 5 \\ 0 & 1 & \frac{1}{4} & -\frac{10}{4} \end{bmatrix} R_2(1) + R_1$$
$$= \begin{bmatrix} 1 & 0 & \frac{13}{4} & \frac{5}{2} \\ 0 & 1 & \frac{1}{4} & -\frac{10}{4} \end{bmatrix}$$

$x = -\frac{13}{4}t + \frac{5}{2}$
$y = -\frac{5}{2} - \frac{1}{4}t$
$z = t$

Let $t = 2$, $(x, y, z) = (-4, -3, 2)$

$PQ = (-4, -3, 2) - (1, 4, 4) = (-5, -7, -2)$

$\vec{v} = (\frac{13}{4}, -\frac{1}{4}, 1) = (-4)(13, 1, -4)$

$$\vec{v} \times PQ = \left(\begin{vmatrix} 1 & -4 \\ -7 & -2 \end{vmatrix}, -\begin{vmatrix} 13 & -4 \\ -5 & -2 \end{vmatrix}, \begin{vmatrix} 13 & -1 \\ -5 & -7 \end{vmatrix} \right)$$
$$= -2(15, -23, 43)$$

$\vec{n} = (15, -23, 43) = 15x - 23y + 43z$

$(15, -23, 43) \cdot (1, 4, 4) = 95$

11) $\vec{n}_1 = (1, 1, 1)$, $\vec{n}_2 = (-1, 2, 3)$

$$\vec{n}_1 \times \vec{n}_2 = \left(\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}, -\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} \right)$$
$$= (1, -4, 3)$$

$x - 4y + 3z = (1, -4, 3) \cdot (1, 2, -1) = -10$

12) $\vec{n} = (-3, 1, 2)$

Then $-3x + y + 2z = (-3, 1, 2) \cdot (1, 1, 3) = 4$

13) $\vec{v}_1 = (1, 0, 2)$ $\quad \vec{v}_2 = (1, 1, 1)$

$$\vec{v}_1 \times \vec{v}_2 = \left(\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \right)$$
$$= (-2, 1, 1)$$

$-2x + y + z = (-2, 1, 1) \cdot (3, 5, 5) = 4$

14)

a) $D = \dfrac{(1)(1) + (-2)(-1) + (-1)(2) + 1}{\sqrt{1^2 + (-1)^2 + (2)^2}} = \dfrac{\sqrt{6}}{3}$

b) $D = \dfrac{(2)(0) + (1)(1) + (3)(2) - 4}{\sqrt{2^2 + 1^2 + 3^2}} = \dfrac{3}{\sqrt{14}} = \dfrac{3\sqrt{14}}{14}$

c) $D = \dfrac{(2)(1) + (1)(1) - 3}{\sqrt{2^2 + 5^2 + 1^2}} = 0$

d) $D = \dfrac{(-1)(-1) + (-1)(2) + (3)(1) - 1}{\sqrt{1^2 + 2^2 + 3^2}} = \dfrac{14}{\sqrt{14}}$

15)

a) $$(A|\vec{b}) = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 3 & -3 & 6 & 1 \end{bmatrix} R_1(-3) + R_2$$
$$= \begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 0 & 0 & 10 \end{bmatrix} R_2(\tfrac{1}{10})$$
$$= \begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} R_2(3) + R_1$$
$$= \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$x = \frac{1}{3},\ y = z = 0$

$D = \dfrac{(1)(\frac{1}{3}) + 3}{\sqrt{1^2 + 1^2 + 2^2}} = \dfrac{5\sqrt{6}}{9}$

b) $$(A|\vec{b}) = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 2 & -2 & -5 \end{bmatrix} R_1(-2) + R_2$$
$$= \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & -7 \end{bmatrix} R_2(-\tfrac{1}{7})$$
$$= \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} R_2(-1) + R_1$$
$$= \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let $y = z = 0$, $x = -\frac{5}{2}$

$D = \dfrac{(1)(-\frac{5}{2}) - 1}{\sqrt{1^2 + 1^2 + 1^2}} = \dfrac{7\sqrt{3}}{6}$

Section 7.1

1. $A = \begin{bmatrix} 2 & 0 & 1 \\ 4 & 0 & 2 \\ 6 & 0 & 3 \end{bmatrix}$ $\begin{matrix} R_1(1/2) \\ R_2(1/2) \\ R_3(1/3) \end{matrix}$

$= \begin{bmatrix} 1 & 0 & 1/2 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix}$ $\begin{matrix} R_1(-2)+R_2 \\ R_1(-3)+R_3 \end{matrix}$

$= \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 0 & 0 \\ 0 & 0 & -1/2 \end{bmatrix}$ $R(A) = 2$

Since $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ contains a zero vector $\vec{a}_2$, $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ And $r(A) = 2 < 3 = n$

2. $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 0 & 2 & 2 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\begin{matrix} R_1(-2)+R_2 \\ R_1(-1)+R_3 \end{matrix}$

$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -2 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $R_2(-1/2)$

$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Since $r(A) = 3 < 4 = n$ and $\vec{a}_1 - \vec{a}_2 = \vec{a}_3$, it is linearly dependent

3. $A = \begin{bmatrix} 1 & 7 & -1 & 6 \\ 3 & 9 & -2 & -2 \\ -4 & 8 & 1 & 5 \end{bmatrix}$ $\begin{matrix} R_1(-3)+R_2 \\ R_1(4)+R_3 \end{matrix}$

$= \begin{bmatrix} 1 & 7 & -1 & 6 \\ 0 & -12 & 1 & -20 \\ 0 & 36 & -3 & 29 \end{bmatrix}$ $R_2(3)+R_3$

$= \begin{bmatrix} 1 & 7 & -1 & 6 \\ 0 & -12 & 1 & -20 \\ 0 & 0 & 0 & -31 \end{bmatrix}$ $r(A) = 3$

Since $n = 4 > 3 = m$, S is linearly independent

4. $A = \begin{bmatrix} -1 & 3 & 1 \\ 0 & -1 & 2 \\ -1 & -6 & 3 \end{bmatrix}$ $R_1(-1)$

$= \begin{bmatrix} 1 & -3 & -1 \\ 0 & -1 & 2 \\ -1 & -6 & 3 \end{bmatrix}$ $R_1(1)+R_3$

$= \begin{bmatrix} 1 & -3 & -1 \\ 0 & -1 & 2 \\ 0 & -9 & 2 \end{bmatrix}$ $R_2(-9)+R_3$

$= \begin{vmatrix} 1 & -3 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & -16 \end{vmatrix} = 16 \neq 0$

Since $\det A \neq 0$, it is linearly independent