Proof. Let $a, b \in \mathbb{R}^2$ where $a = (x_1, y_1)$, $b = (x_2, y_2)$, and $x_1 \neq x_2$. So,

$$
\begin{aligned}
(y - y_1) &= m(x - x_1), && m = \frac{y_2 - y_1}{x_2 - x_1} \\
y &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1 && \text{(combining equations)} \\
&= \frac{y_2 - y_1}{x_2 - x_1}x - \frac{x_1y_2 - x_1y_1}{x_2 - x_1} + y_1 \\
&= \frac{y_2 - y_1}{x_2 - x_1}x - \frac{x_1y_2 - x_1y_1}{x_2 - x_1} + \frac{x_2 - x_1}{x_2 - x_1}y_1 \\
&= \frac{y_2 - y_1}{x_2 - x_1}x - \frac{x_1y_2 - x_1y_1}{x_2 - x_1} + \frac{x_2y_1 - x_1y_1}{x_2 - x_1} \\
&= \frac{y_2 - y_1}{x_2 - x_1}x + \frac{-x_1y_2 + x_1y_1 + x_2y_1 - x_1y_1}{x_2 - x_1} \\
&= \frac{y_2 - y_1}{x_2 - x_1}x + \frac{x_2y_1 - x_1y_2}{x_2 - x_1}
\end{aligned}
$$

□