Proof. Let $a, b \in \mathbb{R}^2$ where $a = (x_1, y_1), b = (x_2, y_2),$ and $x_1 \neq x_2$. So,

$$(y - y_1) = m(x - x_1), \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1 \qquad \text{(combining equations)}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}x - \frac{x_1y_2 - x_1y_1}{x_2 - x_1} + y_1$$

$$= \frac{y_2 - y_1}{x_2 - x_1}x - \frac{x_1y_2 - x_1y_1}{x_2 - x_1} + \frac{x_2 - x_1}{x_2 - x_1}y_1$$

$$= \frac{y_2 - y_1}{x_2 - x_1}x - \frac{x_1y_2 - x_1y_1}{x_2 - x_1} + \frac{x_2y_1 - x_1y_1}{x_2 - x_1}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}x + \frac{-x_1y_2 + x_1y_1 + x_2y_1 - x_1y_1}{x_2 - x_1}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}x + \frac{x_2y_1 - x_1y_2}{x_2 - x_1}$$

