

AEM 668 Project 4

MIMO LTI Feedback Control of Tail-Controlled Rocket

Learning Objective

This project is intended to introduce dynamics and control analysis in the presence of uncertainties for a tail-controlled rocket. The equations of motion are taken from R.T. Reichert, “Autopilot Design Using μ -Synthesis,” American Control Conference, 1990.

Dynamical System

The nonlinear state dynamics for a tail-controlled rocket can be modeled as

$$\begin{aligned} \begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} &= f(\alpha, q, \mathcal{M}) = \begin{bmatrix} f_1(\alpha, q, \mathcal{M}) \\ f_2(\alpha, q, \mathcal{M}) \end{bmatrix} = \begin{bmatrix} \frac{\cos \alpha}{mc_s \mathcal{M}} (Q_\infty S_f C_n(\alpha, \mathcal{M}, \delta)) + q \\ \frac{1}{I_{yy}} (Q_\infty S_f d C_m(\alpha, \mathcal{M}, \delta)) \end{bmatrix} \\ \begin{bmatrix} \alpha \\ q \\ a_z \end{bmatrix} &= \begin{bmatrix} \alpha \\ q \\ \frac{Q_\infty S_f C_n(\alpha, \mathcal{M}, \delta)}{mg_0} \end{bmatrix} \end{aligned} \quad (1)$$

where α is the angle of attack in degrees, q is the pitch rate in degrees/second, a_z is the vertical acceleration in g 's, g_0 is the standard acceleration due to gravity, \mathcal{M} is the Mach number, c_s is the speed of sound, Q_∞ is the free-stream dynamic pressure and can be modeled as

$$Q_\infty = \frac{1}{2} \gamma_{s-h} P_{s,\infty} \mathcal{M}^2 \quad (2)$$

γ_{s-h} is the specific heat ratio, $P_{s,\infty}$ is the static pressure of the free-stream, S_f is the fin reference area, d is the diameter of the rocket fuselage, m is the mass, I_{yy} is the pitch moment of inertia, g is the acceleration due to gravity, C_n is the nondimensional aerodynamic coefficient for the vertical force modeled as

$$C_n(\alpha, \mathcal{M}, \delta) = \text{sign}(\alpha) \left(n_3 |\alpha|^3 + n_2 \alpha^2 + (n_{1,\mathcal{M}} \mathcal{M} + n_{1,0}) |\alpha| \right) + n_0 \delta \quad (3)$$

C_m is the nondimensional aerodynamic coefficient for the pitching moment modeled as

$$C_m(\alpha, \mathcal{M}, \delta) = \text{sign}(\alpha) \left(m_3 |\alpha|^3 + m_2 \alpha^2 + (m_{1,\mathcal{M}} \mathcal{M} + m_{1,0}) |\alpha| \right) + m_0 \delta \quad (4)$$

and δ is the tail fin deflection in degrees which is actuated. The actuator can be modeled by a second-order transfer function

$$A_{fin}(s) = \frac{\delta(s)}{\delta_c(s)} = \frac{\omega_a^2}{s^2 + 2\zeta_a \omega_a s + \omega_a^2} \quad (5)$$

where $\omega_a = 150$ rad/s is the actuator undamped natural frequency and $\zeta_a = 0.7$ is the actuator damping ratio. In addition, this actuator is mechanically limited to $\pm 20^\circ$ deflection and a deflection rate of $\pm 600^\circ$ per second.

These equations can be used to form a linearized, time-invariant state-space model for the plant, $G(s)$, about trim values $\bar{\alpha}$, \bar{q} , $\bar{\delta}$, \bar{a}_z , for some Mach number, \mathcal{M} , as

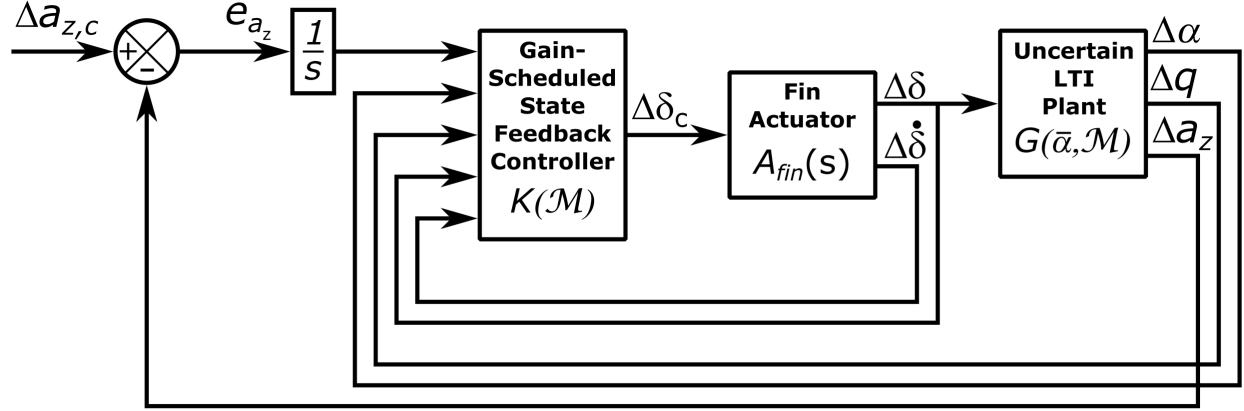
$$\begin{aligned} \begin{bmatrix} \Delta \dot{\alpha} \\ \Delta \dot{q} \end{bmatrix} &= \begin{bmatrix} \frac{\partial f_1(\alpha, q, \mathcal{M})}{\partial \alpha} & \frac{\partial f_1(\alpha, q, \mathcal{M})}{\partial q} \\ \frac{\partial f_2(\alpha, q, \mathcal{M})}{\partial \alpha} & \frac{\partial f_2(\alpha, q, \mathcal{M})}{\partial q} \end{bmatrix}_{\bar{\alpha}, \bar{q}, \bar{\delta}, \bar{a}_z} \begin{bmatrix} \Delta \alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1(\alpha, q, \mathcal{M})}{\partial \delta} \\ \frac{\partial f_2(\alpha, q, \mathcal{M})}{\partial \delta} \end{bmatrix}_{\bar{\alpha}, \bar{q}, \bar{\delta}, \bar{a}_z} \Delta \delta \\ \begin{bmatrix} \Delta \alpha \\ \Delta q \\ \Delta a_z \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\pi}{180} \left(\frac{Q_\infty S_f}{mg_0} \right) \frac{\partial C_n(\alpha, \mathcal{M}, \delta)}{\partial \alpha} & \left(\frac{Q_\infty S_f}{mg_0} \right) \frac{\partial C_n(\alpha, \mathcal{M}, \delta)}{\partial q} \end{bmatrix}_{\bar{\alpha}, \bar{q}, \bar{\delta}, \bar{a}_z} \begin{bmatrix} \Delta \alpha \\ \Delta q \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ \frac{\pi}{180} \left(\frac{Q_\infty S_f}{mg_0} \right) \frac{\partial C_n(\alpha, \mathcal{M}, \delta)}{\partial \delta} \end{bmatrix}_{\bar{\alpha}, \bar{q}, \bar{\delta}, \bar{a}_z} \Delta \delta \end{aligned} \quad (6)$$

where $\Delta \bullet = \bullet - \bar{\bullet}$ for \bullet as α , q , δ , and n_z .

For this project, the values of the model parameters for this rocket-powered flight vehicle are:

Model Parameter	Symbol	Value
Fin Reference Area	S_{ref}	0.44 ft ²
Diameter	d	0.75 ft
Mass	m	13.98 slugs
y-moment of inertia	I_{yy}	182.5 slug-ft ²
Standard acceleration due to gravity at 20,000 ft	g	32.2 ft/s ²
Specific heat ratio of air	γ_{s-h}	1.40
Static pressure at 20,000 ft	$P_{s,\infty}$	973.3 lbs/ft ²
First-order C_n term w.r.t δ	n_0	-0.034 /deg
First-order C_n term w.r.t α	$n_{1,0}$	-0.3392 /deg
First-order C_n w.r.t \mathcal{M} & α	$n_{1,\mathcal{M}}$	0.0565333 /deg
Second-order C_n term w.r.t α	n_2	-0.0094457 /deg ²
Third-order C_n term w.r.t α	n_3	0.000103 /deg ³
First-order C_m term w.r.t δ	m_0	-0.206 /deg
First-order C_m term w.r.t α	$m_{1,0}$	-0.357 /deg
First-order C_m w.r.t \mathcal{M} & α	$m_{1,\mathcal{M}}$	0.136 /deg
Second-order C_m term w.r.t α	m_2	-0.019546 /deg ²
Third-order C_m term w.r.t α	m_3	0.000215 /deg ³

A state feedback robust servomechanism control system is to be used to track acceleration commands to the rocket as



where

$$\Delta\delta_c = K \begin{bmatrix} \int a_{z,c} - a_z \\ \Delta\alpha \\ \Delta q \\ \Delta\delta \\ \Delta\dot{\delta} \end{bmatrix} \quad (7)$$

For this project, the control design objectives for this rocket-powered flight vehicle are:

1. Track commanded step inputs for acceleration maneuvers $a_{z,c}$ up to ± 0.1 g with:
 - (a) steady-state error of $< 1\%$;
 - (b) 5%-settling time < 20 seconds;
 - (c) overshoot $< 50\%$; and
 - (d) no position or rate saturation of the fin deflection, δ .
2. Robust stability to gain and phase variations in the actuator system and unmodeled high-frequency vibration modes across the range of flight conditions

Project Assignment and Deliverables

For this project, determine in MATLAB/Simulink:

- a) Construct a function (at the bottom of the script or as a separate file) to return the linearized, time-invariant state-space model for the plant at a given trim angle of attack, $\bar{\alpha}$, and Mach number, \bar{M} .

- As a check, the linearization at $\bar{\alpha} = 0^\circ$ and $\bar{M} = 3$ should be

$$A = \begin{bmatrix} -0.6 & 1.0 \\ 32.4 & 0 \end{bmatrix} \quad (8)$$

$$B = \begin{bmatrix} -0.12 \\ -130.9 \end{bmatrix} \quad (9)$$

$$C = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \\ -1.02 & 0 \end{bmatrix} \quad (10)$$

$$D = \begin{bmatrix} 0 \\ 0 \\ -0.2038 \end{bmatrix} \quad (11)$$

- and at $\bar{\alpha} = 20^\circ$ and $\bar{M} = 3$ should be

$$A = \begin{bmatrix} -1.30 & 1.0 \\ -300.2 & 0 \end{bmatrix} \quad (12)$$

$$B = \begin{bmatrix} -0.11 \\ -130.9 \end{bmatrix} \quad (13)$$

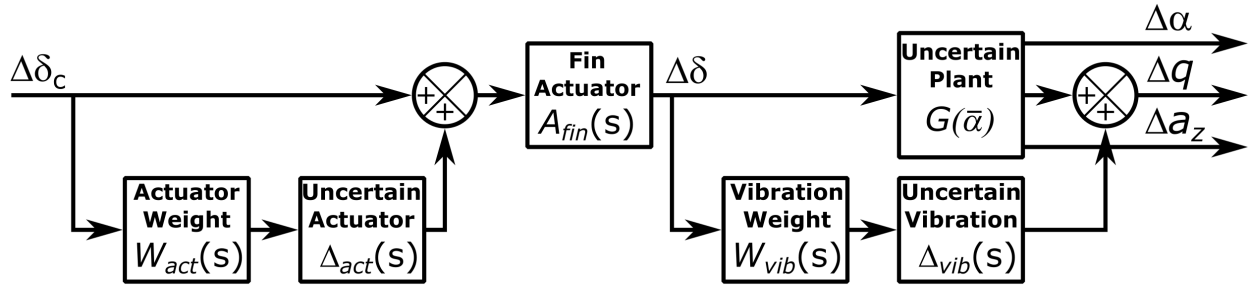
$$C = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \\ -2.54 & 0 \end{bmatrix} \quad (14)$$

$$D = \begin{bmatrix} 0 \\ 0 \\ -0.204 \end{bmatrix} \quad (15)$$

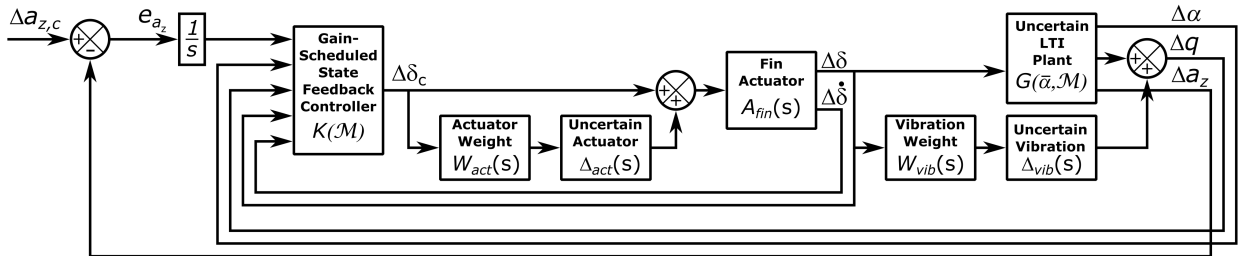
- b) Design a robust servomechanism, LTI, state feedback controller for the linearized plant at $\bar{\alpha} = 0^\circ$ and $\bar{M} = 3$

- use the `ss` function for A_{fin} as a state-space system to output both $\Delta\delta$ and $\Delta\dot{\delta}$;
- use the `tf` function for the tracking error integrator block, $\frac{1}{s}$;
- use the `connect` function to form the generalized plant;
- determine the controllability of the generalized plant;
- determine the gain and phase margins, use the 1×1 open-loop transfer function matrix at the plant input, i.e., $L_{in}(s) = [K(s)][T_{A\&G}(s)]$; and
- (optional) use the `place` function to set the poles of the feedback control system.

- c) Simulate the MIMO LTI system with your robust servomechanism state feedback controller at $\bar{\alpha} = 0^\circ$ and $M = 3$, actuator, and linearized and nonlinear plants with different step inputs for $a_{z,c}$
- start with the provided Simulink model, “rocket.mdl;”
 - output the step response with a suitably small $a_{z,c}$ such that the linearized and nonlinear plants agree; and
 - output the step response with five increasingly larger $a_{z,c}$ until the linearized and nonlinear plants disagree significantly, in your own opinion.
- d) Construct an uncertain LTI system for the linearized dynamics model as



- $\bar{\alpha} \in [0^\circ, 10^\circ]$ as real parametric uncertainty;
 - Δ_{vib} models high-frequency vibration modes;
 - $W_{act} = 0.6$: roughly corresponds to gain variations of 0.6 to 2.5 and phase variations of 35° ;
 - $W_{vib} = \frac{150s^3+13,000s^2+70,000s+48,000}{s^3+2,000s^2+2,000,000s+62,000,000}$;
 - use the `ureal` function on $\bar{\alpha}$ with the `ss` function to form the uncertain plant $G(\bar{\alpha})$ for $M = 3$ with the *quadratic approximation* for $\cos \alpha$ and $\sin \alpha$ (in radians);
 - use the `simplify` function with the ‘full’ option to eliminate redundant copies of the α uncertainty for this uncertain plant;
 - use the `ultidyn` function to form the LTI uncertainties, Δ_{act} and Δ_{vib} ;
 - use the `connect` function to form the system interconnections; and
 - output the Bode plot of the open-loop transfer functions, $\Delta\alpha/\Delta\delta_c$, $\Delta q/\Delta\delta_c$ and $\Delta a_z/\Delta\delta_c$, using the `bode` function of the uncertain LTI model to generate several samples of the open-loop response.
- e) Output to the command window the closed-loop robust stability margin, robust performance, and worst-case gain, for your robust servomechanism controller in part b) shown below for the flight condition $M = 3$



- use the `connect` function to form the uncertain LTI feedback control system with generalized disturbance, $a_{z,c}$, and generalized error, e_{a_z} ;
- use the `robstab` and `wcgain` functions to determine the stability margin, worst-case gain, and worst-case uncertainty of your uncertain system; and
- interpret the meaning of the outputs of these functions.