CHEN 2450 - HOMEWORK 7
NUMERICAL DIFFERENTIATION & INTEGRATION

Submit all your homework using Jupyter notebooks. You are expected to include proper text and discussion for each and every problem including appropriate headings and formatting. You will not get credit if your notebooks are not readable or just include code with a few print out statements. You must upload a PDF version of your notebook along with the ipynb notebook itself. Failure to do so will result in no credit at all. If you are having trouble converting your jupyter notebook to PDF, please upload your ipynb, indicate this in your upload comment, and email Prof. Saad and the TAs immediately.

Problem 1 (50 pts)

A UFO has been spotted in the Salt Lake airspace and is being tracked by radar in polar coordinates r and θ. The data recorded is given in the following table

t (s)	200	202	204	206	208	210
θ (rad)	0.75	0.72	0.7	0.68	0.67	0.66
r (m)	5120	5370	5560	5800	6030	6240

Table 1: Radial and angular location of UFO as a function of time

Naturally, the radar automatically calculates the speed and acceleration of the UFO but that part of the radar system has been dysfunctional for a while. You are one of the engineers running the radar station and the governor has tasked you to find the speed and acceleration of the UFO.

1. Use **second order** finite differencing to find the vector expressions for the velocity $\mathbf{u}$ and acceleration $\mathbf{a}$ of the UFO at $t = 210$ s. Recall that the velocity and acceleration in radial coordinates are given by:

$$\mathbf{u} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta; \quad \mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta, \tag{1}$$

where $\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t}$, $\dot{\theta} = \frac{\mathrm{d}\theta}{\mathrm{d}t}$, etc... and $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are unit vectors in the radial and tangential directions, respectively. Do this "by hand" - (but you can use Python to verify your results). Note that you can find second order finite difference formulas for first and second derivatives on the slides.

Problem 2 (50 pts)

The amount of energy required to change a the temperature of compound from T_1 to T_2, at constant pressure, can be written as

$$q = \int_{T_1}^{T_2} c_p \mathrm{d}T, \tag{2}$$

where q is the required energy and c_p is the compound's *heat capacity* and is a function of temperature.

1. Using the data for $c_p(T)$ from Table 2, determine how much energy is required to heat one kg of CO_2, steel, and graphite from $T_1 = 400$ K to $T_2 = 1000$ K. Do this using the Trapezoid rule (implement your own - you can adopt the codes developed in class). You may use Python's `trapz` function to check your results. Summarize your results in a table listing the heat required for each compound.

Table 2: Heat capacity (c_p) in $\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$ as a function of temperature.

Temperature (K)	400	450	500	550	600	650	700	750	800	850	900	950	1000
c_p CO_2	0.942	0.981	1.02	1.05	1.08	1.10	1.13	1.15	1.17	1.187	1.204	1.220	1.234
c_p Steel	487				559				685				1169
c_p Graphite	992				1406				1650				1793