PHYS 3120 | HW 01 | Ethan Ryan

3.1 a) $z = 3+4i, \ w = 2-i$

$$z + w = 3+4i+2-i$$
$$= \underline{5+3i}$$

b) $w - z = 3+4i-2+i$
$$= \underline{1+5i}$$

c) $w \cdot z = (3+4i)(2-i) = 6-3i+8i-4i^2$
$$6+4-3i+8i$$
$$\underline{10+5i}$$

d) $$\frac{z}{w} = \frac{3+4i}{2-i} \cdot \frac{2+i}{2+i} = \frac{(3+4i)(2+i)}{(2-i)(2+i)}$$

$$= \frac{6+3i+8i+4i^2}{4+\cancel{2i}-\cancel{2i}-i^2} = \frac{6-4+3i+8i}{4+1}$$

$$= \frac{2+11i}{5} = \underline{\frac{2}{5} + \frac{11}{5}i}$$

e) $(z^* \cdot w) + (w^* \cdot z) = (3-4i)(2-i) + (2+i)(3+4i)$

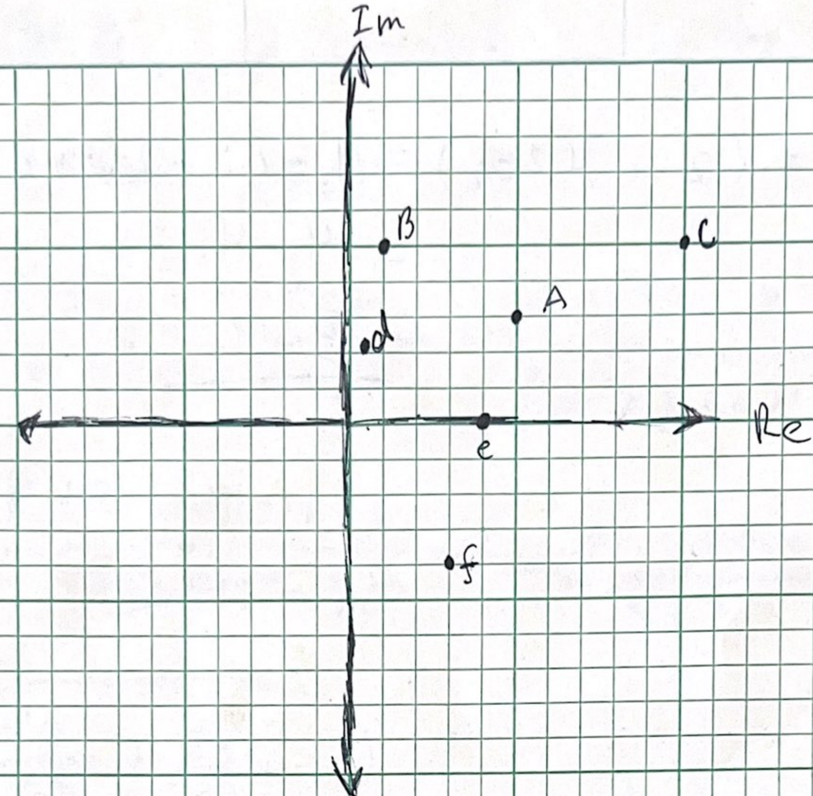
$$= 6 - \cancel{3i} - \cancel{8i} + 4i^2 + 6 + \cancel{8i} + \cancel{3i} + 4i^2$$

$$= 6+6+4i^2+4i^2 = 12-8 = \underline{4}$$

f) $\omega^2 = (2-i)(2-i) = 4 - 2i - 2i + i^2$

$= 4 - 4i - 1$

$= \underline{3 - 4i}$


Im
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1) $z_1 = 4+6i \quad , \quad z_2 = 2-3i$

$$\frac{4+6i}{2-3i} \cdot \frac{(2+3i)}{(2+3i)} = \frac{(4+6i)(2+3i)}{(2-3i)(2+3i)}$$

$$= \frac{8+12i+12i+18i^2}{4+\cancel{6i}-\cancel{6i}-9i^2} = \frac{8-18+24i}{4+9}$$

$$= \frac{-10}{13} + \frac{24i}{13}$$

$z_1 = 4+6i \quad ; \quad z_2 = 2-3i$

$r = \sqrt{4^2+6^2} \qquad r = \sqrt{2^2+3^2}$

$= \sqrt{16+36} \qquad = \sqrt{4+9}$

$= 2\sqrt{13} \qquad = \sqrt{13}$

$\tan(\theta) = \frac{6}{4} \qquad \tan(\theta) = \frac{-3}{2}$

$\theta = \arctan\left(\frac{6}{4}\right) \qquad \theta = \arctan\left(\frac{-3}{2}\right)$

$\theta = .983 \qquad \theta = -.983$

$z_1 = 2\sqrt{13}\, e^{i.983} \quad , \quad z_2 = \sqrt{13}\, e^{i.983}$

$$\frac{z_1}{z_2} = \frac{2\cancel{\sqrt{13}}\, e^{i.983}}{\cancel{\sqrt{13}}\, e^{i-.983}} = 2e^{i(.983+.983)} = 2e^{1.966i}$$

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$$\frac{-10}{13} + \frac{24i}{13} \rightarrow r = \sqrt{\left(\frac{10}{13}\right)^2 + \left(\frac{24}{13}\right)^2}$$

$$r = 2$$

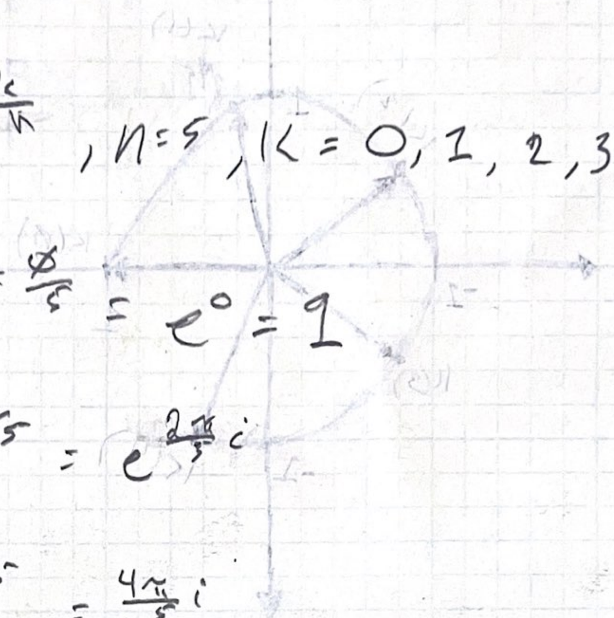
$$\theta = \tan^{-1}\left(\frac{\cancel{24/13}}{\cancel{-10/13}}\right) \Rightarrow \tan^{-1}\left(-\frac{24}{10}\right)$$

$$\theta = -1.176 + \pi = 1.966$$

$$\frac{-10}{13} + \frac{24i}{13} = \boxed{2e^{1.966i}}$$

2) $z^5 = 1$

$z = e^{i2\pi \frac{k}{n}}$, $n=5$, $k = 0, 1, 2, 3, 4$



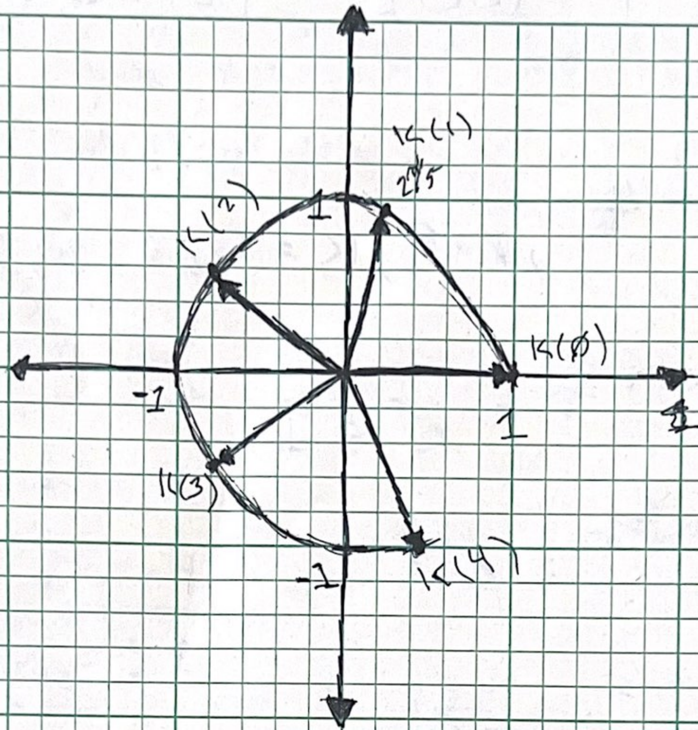
$k(\emptyset) = e^{i2\pi \frac{\emptyset}{5}} = e^0 = 1$

$k(1) = e^{i2\pi \, 1/5} = e^{\frac{2\pi}{5} i}$

$k(2) = e^{i2\pi \, 2/5} = e^{\frac{4\pi}{5} i}$

$k(3) = e^{i2\pi \, 3/5} = e^{\frac{6\pi}{5} i}$

$k(4) = e^{i2\pi \, 4/5} = e^{\frac{8\pi}{5} i}$



K(1)
2π/5
K(2)
1
K(ϕ)
-1
1
K(3)
-1
K(4)

3) $z^6 - z^5 + 4z^4 - 7z^3 + 3z^2 - 12z + 12 = \emptyset$

$(z^3 - 3)(z^2 + 4)(z - 1)$

- $z - 1 = \emptyset$

$\boxed{z_1 = 1}$

- $z^2 = 4$

$z^2 = -4$

$\boxed{z_{2,3} = \pm 2i}$

- $z^3 - 3 = \emptyset$

$z^3 = 3 \cdot 1 \qquad z^n = 1 = e^{i 2\pi k/n} \quad , k = 0, 1, 2$

$z_4(\emptyset) = 3 \cdot e^{\frac{\emptyset}{3} i} = 3^{1/3} \cdot 1 = \boxed{3^{1/3}}$

$z_5(1) = 3^{1/3} \cdot e^{2\pi(1)i/3} = \boxed{3^{1/3} \cdot e^{\frac{2\pi}{3} i}}$

$z_6(2) = 3^{1/3} \cdot e^{\frac{2\pi(2)i}{3}} = \boxed{3^{1/3} e^{\frac{4\pi}{3} i}}$

So, 2 real roots, 4 imaginary.

z^6, 6 roots total. ✓

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4) solve: $2\cosh(x) + 10\sinh(x) - 5 = \phi$

$2\cosh(x) = e^x + e^{-x}$

$10\sinh(x) = 5(e^x - e^{-x})$

$(e^x + e^{-x}) + 5(e^x - e^{-x}) - 5 = \phi$

$\star$ Multiply by $-e^x$

$-e^x(e^x + e^{-x}) - 5e^x(e^x - e^{-x}) + 5e^x = \phi$

$-e^{2x} - e^0 - 5e^{2x} + 5e^0 + 5e^x = \phi$

$\underline{-e^{2x}}\ \underline{-1}\ \underline{-5e^{2x}}\ \underline{+5} + 5e^x = \phi$

$-6e^{2x} + 5e^x + 4 = \phi$

$(-3e^x + 4)(2e^x + 1)$

$-3e^x + 4 = \phi$

$-3e^x = -4$

$e^x = \frac{4}{3}$

$x = \ln(\frac{4}{3})$

$\boxed{x = \ln(4) - \ln(3)}$

$2e^x + 1 = \phi$

$2e^x = -1$

$e^x = -\frac{1}{2}$

$x = \ln(-\frac{1}{2})$

$\boxed{x = \ln(2)}$