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Estimate Secret Algorithms

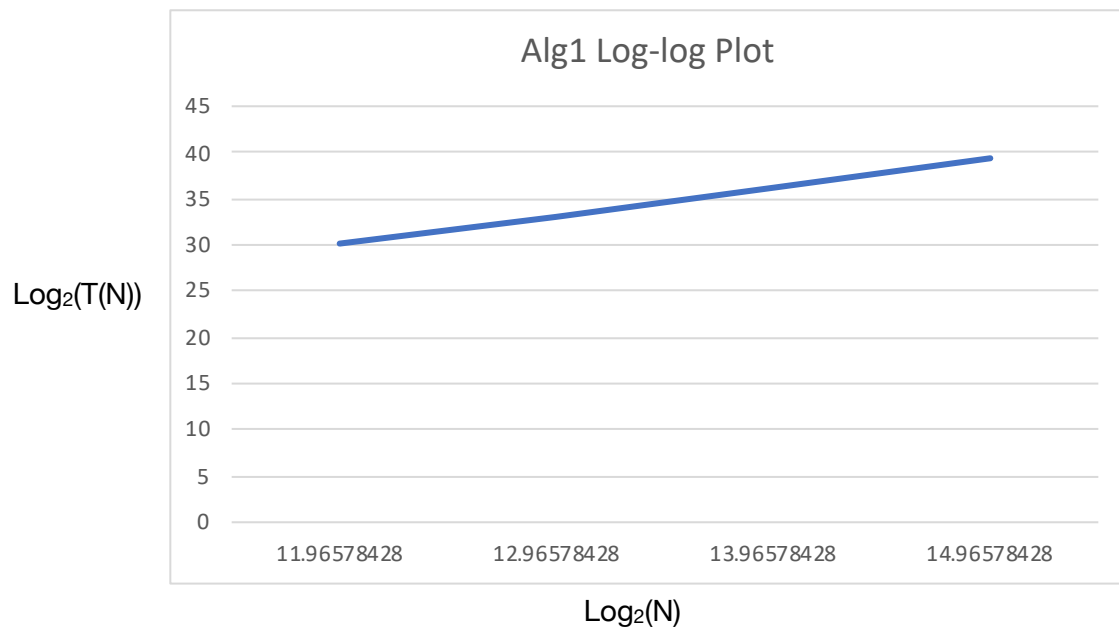
To attain my data, I began with an input of 500 and continuously doubled that with the largest input being 131072000 ($=500 \cdot 2^{18}$) (for algorithms 1 and 3, I did not get results for the larger inputs as they took too long to run). I checked the time before and after the execution using `System.nanoTime()` and subtracted the end time from the start time to get the duration of the program execution and printed out this calculation. All other calculations were performed in Excel.

Needless to say, the results were very inconsistent for smaller sized inputs and even for some larger sized inputs. To find consistent data, I ensured that no other processes were running on my computer and then ran my code several times in an attempt to identify a consistent order-of-growth. After each time that I ran the code, I applied the doubling method to find a series of consecutive inputs and execution duration that indicated a consistent order-of-growth. Thus, for example, in my analysis of algorithm 1, I focused on inputs from 4000-32000 as these inputs yielded the most consistent data (and larger inputs took too long to execute).

After identifying the series of consecutive inputs, I used those specific inputs and the measured duration to calculate and graph the log-log plot in excel using log base 2. The log of the input size is the x axis and the log of the duration of the execution is the y axis. Using the Excel slope function, I calculated the average slope of the log-log plot which provides the order of growth for the function. Additionally, I calculated the average of the ratios provided by the doubling method.

Algorithm 1

N	T(N) (nano-seconds)	Log ₂ (N)	Log ₂ (T(N))
4000	1159869881	11.9657843	30.11131582
8000	9044394321	12.9657843	33.07437675
16000	74014235573	13.9657843	36.10708373
32000	6.92174E+11	14.9657843	39.33234351



Average Slope of Log-Log Plot = 3.069579

Doubling Method:

T(8000)/T(4000)	7.79776634
T(16000)/T(8000)	8.18343749
T(32000)/T(16000)	9.35190181

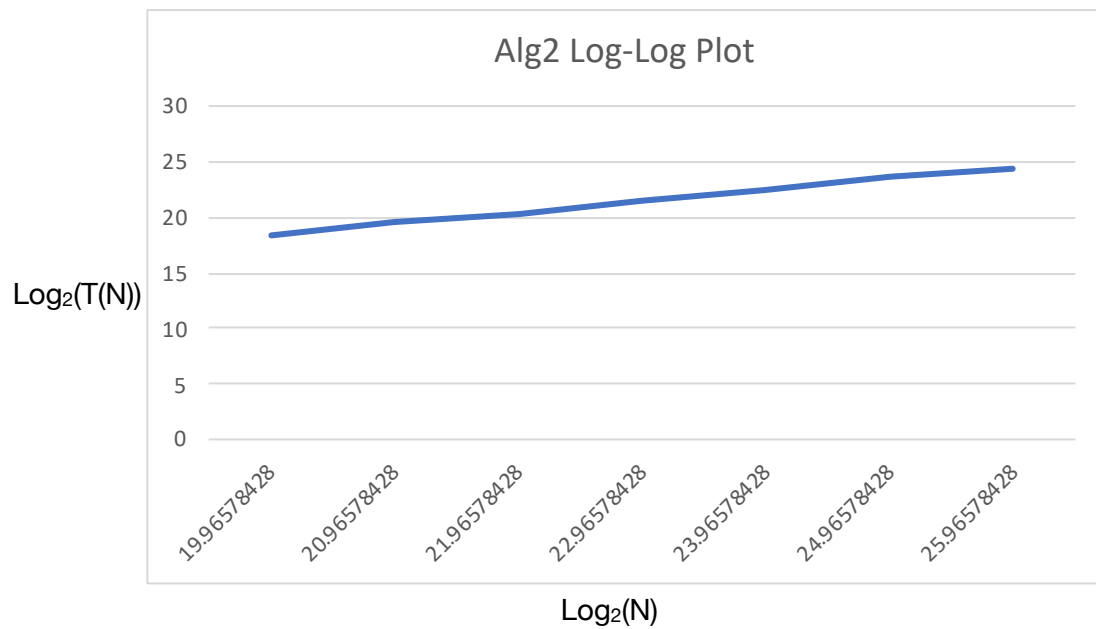
Average Ratio = 8.44436855

Conclusion: Algorithm 1 is likely $O(N^3)$. This is indicated by the fact that the slope of the log-log plot is close to 3, and the average ratio calculated by way of the doubling method is close to 8.

Algorithm 2

Log-log Method:

N	T(N) (nano-seconds)	Log ₂ (N)	Log ₂ (T(N))
1024000	372536	19.9657843	18.50702032
2048000	792820	20.9657843	19.59663383
4096000	1372205	21.9657843	20.3880646
8192000	2866234	22.9657843	21.45072497
16384000	5661062	23.9657843	22.43264129
32768000	12844102	24.9657843	23.61460269
65536000	23400882	25.9657843	24.48005957



Average Slope of Log-Log Plot = 0.99998686

Doubling Method:

T(1024000)/T(512000)	2.04262505
T(2048000)/T(1024000)	2.12817016
T(4096000)/T(2048000)	1.73079009
T(8192000)/T(4096000)	2.08877974

$T(16384000)/T(8192000)$	1.97508717
$T(32768000)/T(16384000)$	2.26885026
$T(65536000)/T(32768000)$	1.82191655

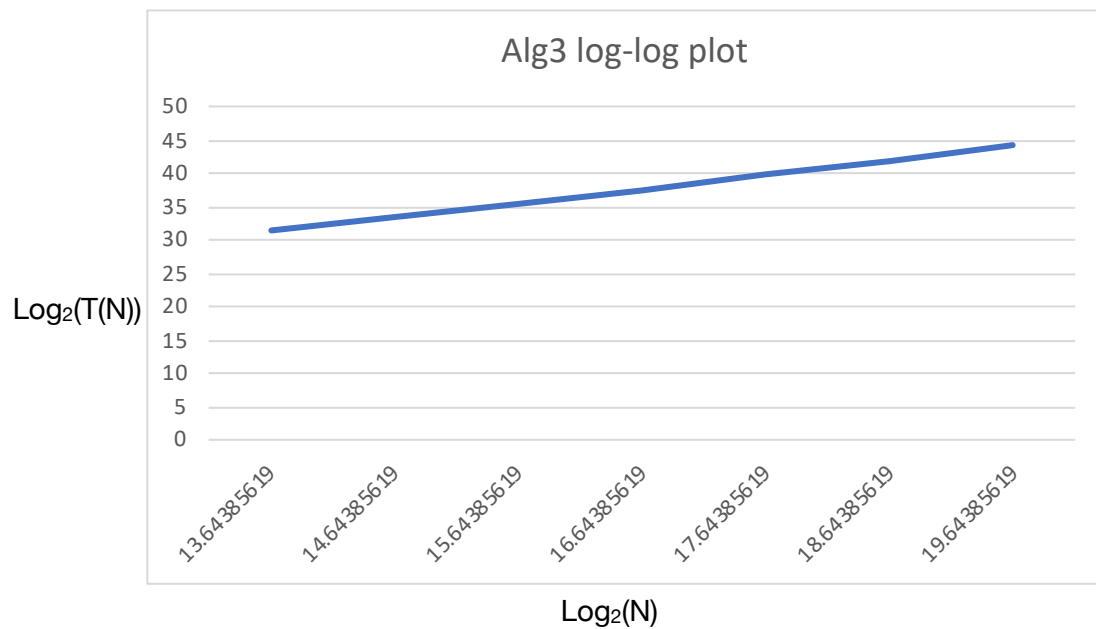
Average Ratio = 2.00803129

Conclusion: Algorithm 2 is likely $O(N)$. This is indicated by the fact that the slope of the log-log plot is close to 1, and the average ratio calculated by way of the doubling method is close to 2.

Algorithm 3

Log-log Method:

N	T(N) (nano-seconds)	Log ₂ (N)	Log ₂ (T(N))
12800	2913345237	13.6438562	31.44002953
25600	11778632809	14.6438562	33.45545304
51200	44715799314	15.6438562	35.38006561
102400	2.04461E+11	16.6438562	37.57303363
204800	1.01888E+12	17.6438562	39.89011964
409600	3.80836E+12	18.6438562	41.79230858
819200	1.85842E+13	19.6438562	44.07914212



Average Slope of Log-Log plot = 2.11075367

Doubling Method:

T(25600)/T(12800)	4.04299245
T(51200)/T(25600)	3.79634887
T(102400)/T(51200)	4.57245196
T(204800)/T(102400)	4.98324676

$T(409600)/T(204800)$	3.73779886
$T(819200)/T(409600)$	4.87983898

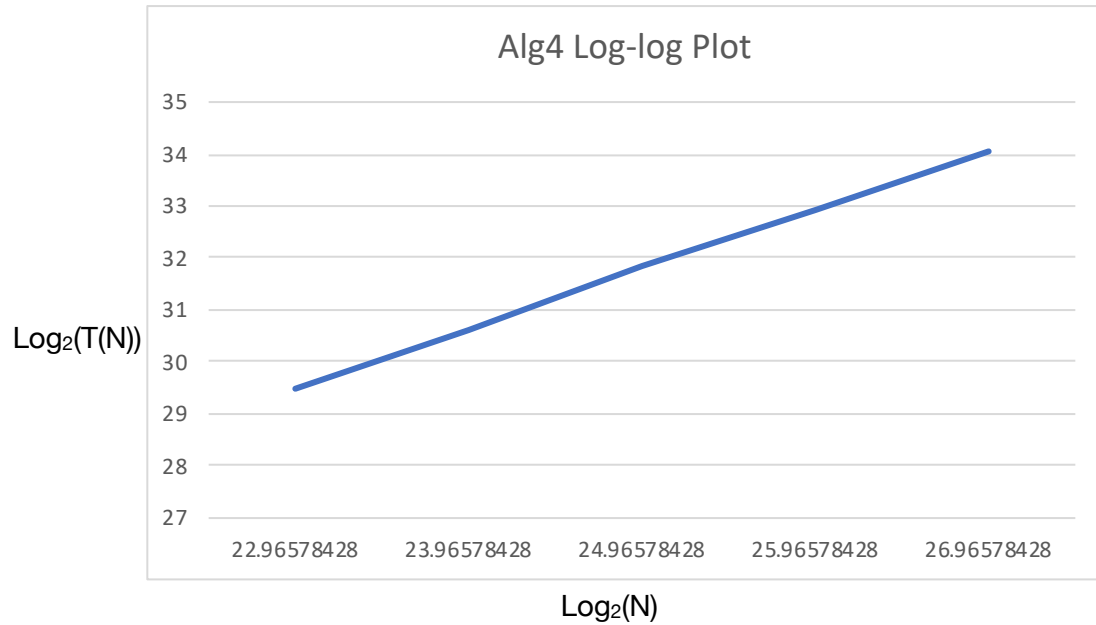
Average Ratio = 4.33544631

Conclusion: Algorithm 3 is likely $O(N^2)$ as indicated by the fact that the average slope of the log-log plot is close to 2 and the average ratio calculated by way of the doubling method is just over 4.

Algorithm 4

Log-log Method:

N	T(N) (nano-seconds)	Log(N)	Log(T(N))
8192000	740209653	22.9657843	29.46335871
16384000	1665389452	23.9657843	30.63321245
32768000	3865521034	24.9657843	31.84801574
65536000	8120374246	25.9657843	32.91889907
131072000	17869507734	26.9657843	34.05678084



Average Slope of Log-log plot = 1.14725309

Doubling Method

$T(500 \cdot 2^{14}) / T(500 \cdot 2^{13})$	2.24988886
$T(500 \cdot 2^{15}) / T(500 \cdot 2^{14})$	2.32109134
$T(500 \cdot 2^{16}) / T(500 \cdot 2^{15})$	2.1007192
$T(500 \cdot 2^{17}) / T(500 \cdot 2^{16})$	2.20057687

Average Ratio = 2.21806907

Conclusion: It seems that Algorithm 4 is $O(N)$. This is indicated by the fact that the slope of the log-log plot is slightly above 1 and the average ratio calculated by way of the doubling method is just above 2.