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MaxOverBTDescendants

Algorithm
Perform an in-order traversal of the binary tree in which both children of a parent are processed prior to the processing of the parent. For each person i, starting from the bottom of the tree, if $value_i$ is greater than that of his parent, change the value of the parent to the value of the child.

Proof
The value of any given person is determined by the maximum of his own value and the value of his highest valued descendant. By starting from the bottom of the tree and the changing the value the parent to the maximum of its value and the value of its child/pair of children, we can ensure that before any parent is processed, its children's values will be the maximum of their (the children's) values and their descendant's values. Therefore, by taking the maximum of the values of a parent and his children, the parent's value will be the maximum of his value and the values of his descendants.
The algorithm follows an in-order traversal of the tree which has runtime O(n) and does O(1) work at each node, namely, taking the maximum of the values of the node and its children. Therefore, the runtime of the algorithm is O(n).