

1.

The degree of a particular vertex cannot be more than the amount of vertices in the graph to which it belongs. Thus, if there are v vertices, the maximum degree of a particular vertex is $v-1$ and the minimum degree is 0.

Thus, there are v vertices and only $v-1$ possibilities of the degree for a particular vertex. By the pigeonhole principle, 2 vertices will share the same degree. In other words, if we assign all v vertices a degree, one degree value must repeat because there are fewer degree values than there are vertices.

2.

Explanation of algorithm:

Analyze flips in sets of 2 flips. If a set of 2 flip results contains both a 0 and a 1 (as opposed to 0 and 0, or 1 and 1), add the first flip result of that set to the final result.

Pseudocode:

```
flip_results = array of flip results with elements containing either 1
or 0
random_sequence = array to hold the produced random sequence
while i < length of flip_results{
    if flip_results[i] != flip_results[i+1]{
        append flip_results[i] to random_sequence
    }
    i = i + 2
}
```

Proof:

- If the probability of heads is p , the probability of tails is $1 - p$
 - In other words: $\Pr(\text{heads}) = p$, $\Pr(\text{tails}) = 1 - p$
- The probability of a result of sequence {heads, tails} is equal to $\Pr(\text{heads}) \cdot \Pr(\text{tails})$
- Similarly, the probability of a result of sequence {tails, heads} is equal to $\Pr(\text{tails}) \cdot \Pr(\text{heads})$
 - $\Pr(\{\text{heads, tails}\}) = p \cdot (1-p)$
 - $\Pr(\{\text{tails, heads}\}) = (1-p) \cdot p$
- Therefore, $\Pr(\{\text{heads, tails}\}) = \Pr(\{\text{tails, heads}\})$
- Therefore, if we consider each set of two results containing a heads and tails, by repeatedly picking the first of each set of two, we can produce a random sequence.
- QED