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Drill: Order Of Growth

3 Pick your constants: Big O

Given $f(n) = 32n^2 + 17n + 1$

1. $f(n) = O(n^2)$ because $32n^2 + 17n + 1 < 49n^2$.

2. $f(n) \mathrel{!}= O(n)$ - Proof by contradiction: In order for $f(n)$ to be $O(n)$, we must have positive constants c, n_0 such that:
   $32n^2 + 17n + 1 <= c*n$ for all $n >= n_0$.
   If we divide both sides by n we get $32n + 17 + 1/n <= c$ which is not true for $n = c + 1$.

   $f(n) \mathrel{!}= O(n\log n)$ - Proof by contradiction: In order for $f(n)$ to be $O(n\log n)$, we must have positive constants c, n_0 such that:
   $32n^2 + 17n + 1 <= c*n\log n$ for all $n >= n_0$.
   If we divide both sides by nlogn, we get $(32n + 17 + 1/n)/\log n <= c$. Because the limit of $(32n + 17 + 1/n)/\log n$ as n approaches ∞ is ∞, it is clear that there exists an n such that $(32n + 17 + 1/n)/\log n > c$.

4 Pick your constants: Ω

Given $f(n) = 32n^2 + 17n + 1$

1. $f(n) = \Omega(n^2)$ because $32n^2 + 17n + 1 > 31n^2$
   $f(n) = \Omega(n)$ because $32n^2 + 17n + 1 > n$

2. $f(n) \mathrel{!}= \Omega(n^3)$ - Proof by contradiction: In order for $f(n)$ to be $\Omega(n^3)$, we must have positive constants c, n_0 such that:
   $32n^2 + 17n + 1 >= c*n^3$ for all $n >= n_0$.
   If we divide both sides by n^3, we get $(32n^2 + 17n + 1)/n^3$. Because the limit of $(32n^2 + 17n + 1)/n^3$ as n approaches ∞ is 0, there is clearly a c such that $c > (32n^2 + 17n + 1)/n^3$ for all $n >= n_0$.

5 Pick your constants: Θ

Given $f(n) = 32n^2 + 17n + 1$

1. $f(n) = \Theta(n^2)$ because $32n^2 + 17n + 1 > 31n^2$ and $32n^2 + 17n + 1 < 49n^2$.
2. $f(n) \mathrel{!}= \Theta(n)$ - See 3.2 above for proof that $f(n) \mathrel{!}= O(n)$. Because $f(n) \mathrel{!}= O(n)$, $f(n) \mathrel{!}= \Theta(n)$.
   $f(n) \mathrel{!}= \Theta(n^3)$ - See 4.2 above for proof that $f(n) \mathrel{!}= \Omega(n^3)$. Because $f(n) \mathrel{!}= \Omega(n^3)$, $f(n) \mathrel{!}= \Theta(n^3)$.