how do we know how far things are

Ethan Snyder

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how do we know how far things are?

this will be our leading question

enjoy :)

parallax, mostly

parallax

this may be what you're picturing :)

Figure: Parallax as seen looking out a moving car's window[0].

[0] Image: https://stock.adobe.com/images/view-out-the-car-window-as-the-scenery-blurs-by/193746850

parallax

two types of parallax

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moving parallax

stationary parallax

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moving parallax

- involves movement

stationary parallax

- does not

parallax

two types of parallax

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- involves movement
- things close to observer appear to move more, things farther appear to move less

stationary parallax

- does not
- the change in an objects appearance from two different locations (at once or at different times)

parallax

two types of parallax

moving parallax

- involves movement
- things close to observer appear to move more, things farther appear to move less

stationary parallax

- does not
- the change in an objects appearance from two different locations (at once or at different times)

(these are actually the same, kinda. motion is just being in two places at different times :))

moving parallax

moving parallax car picture again here look

Figure: Parallax as seen looking out a moving car's window again[0].

[0] Image: https://stock.adobe.com/images/view-out-the-car-window-as-the-scenery-blurs-by/193746850

stationary parallax

like human eyes, for example
this is how depth perception works

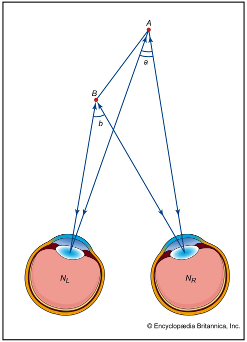


Figure: Eyes doing parallax[0].

[0] Image: https://cdn.britannica.com/85/4085-050-5575ACA3/parallax-points-NL-eyes-NR-left.jpg

summary

we know how far things are away from us
because we have EYES dipshit

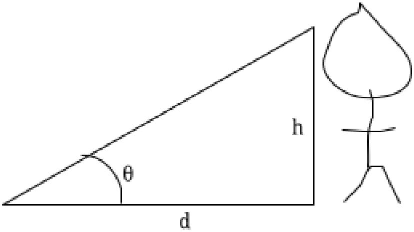
ok but what about numbers

like what if we want to MEASURE a distance

Apparent Size

What if you *know the size of a distant object?*

Easiest solution for measuring a distance.



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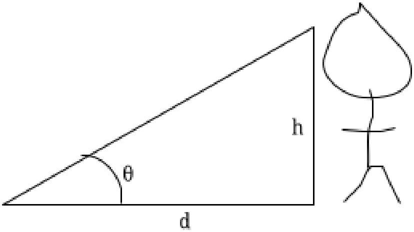

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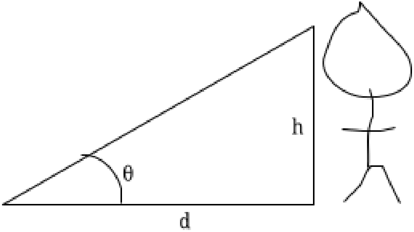

$$\sin\theta = \frac{h}{d} \implies d = \frac{h}{\sin\theta}$$

(you just need to know the size of the distant object and be able to measure the *angular extent* θ)

dawg wt f

what if I don't have a protractor or konw the size of the thing?

Figure: Man scratching his head, confused about how to measure a distance without prior knowledge of a distant object's size or the ability to measure angular extent[0].

[0] Image: https://stock.adobe.com/images/portrait-of-a-mixed-race-man-scratching-his-head-in-confusion/68695988


ITC
25'/7.5M
MOD. NO. ITM-425R
PROD. NO. 022020

Units :)

How many *things* span this distance?

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- Historically we've used things like a foot but truly make up anything
- If you want other people to use this unit just make sure their unit is the same as yours

Measuring Devices

Figure: A tape measure and a rolly measurey thing[1].

[1] Image: https://cdn.shopify.com/s/files/1/0404/0048/6557/products/7_CDM1001_1024x10242x.png?v=1615341175

how far is this fucker?

Figure: Moon[2].

[2] Image: Me :)

The Distance Ladder

Earth $\implies$ Moon $\implies$ Sun $\implies$ Planets $\implies$ Stars $\implies$ Galaxies

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Earth $\implies$ Moon $\implies$ Sun $\implies$ Planets $\implies$ Stars $\implies$ Galaxies

We need to know each distance/size in order to find out the next object's size/distance.
The goal of measuring the Earth, Moon, Sun, and stars is as old as humanity.

(so let's go back)

What have we always known?

(This is essentially the same question as what can be observed with the eyes.)

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- The Moon is closer than the Sun

Solar eclipses, the moon passing between the Earth and Sun

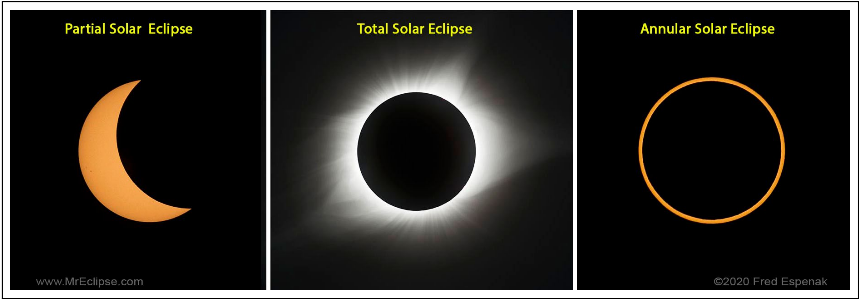

Figure: Partial, total, and annular eclipse images[3].

[3] Image: https://galamcdougal.blogspot.com/2022/10/solar-eclipse.html

What have we always known?

(This is essentially the same question as what can be observed with the eyes.)

- The Moon is closer than the Sun (solar eclipses) and *same angular size*
- The Earth is round

During a lunar eclipse, the Earth passes between a full Moon and the Sun

Figure: Two stages of a lunar eclipse[4].

[4] Images: me :)

What have we always known?

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- The Moon is closer than the Sun (solar eclipses) and *same angular size*
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- The Sun, Moon, planets, and stars are cyclical and move in ‘perfect circles’ around the Earth on a flat plane
  - we were wrong about this but it was a very compelling, mostly unproblematic explanation for a LONG time
  - we also knew slower moving things were farther (parallax), so we knew Jupiter and Saturn are far and Venus and Mars are close

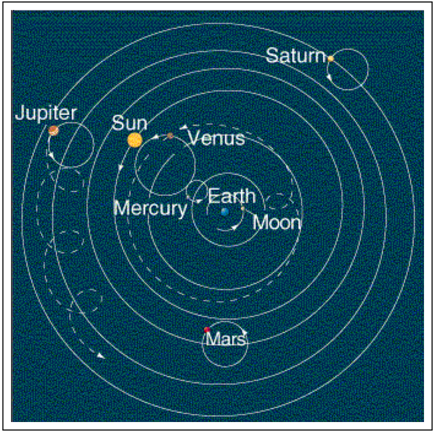
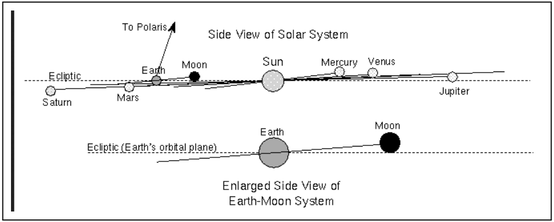


Figure: Geocentric model with subcircles that explain retrograde motion[5].

5 Image: https://starrythoughts.weebly.com/uploads/1/6/3/0/16304784/2180964_orig.gif



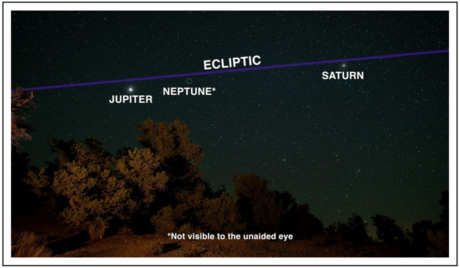


Figure: Ecliptic visuals[6].

[6] Images: https://www.astronomynotes.com/nakedeye/phases/solarsys.gif, NASA

In summary,

(this information is all we needed to figure out the distance to the stars. the rest is all measurements and math)

- The Earth is spherical
- The Moon is closer than the Sun and *same angular size*
- All solar system bodies orbit in a flat plane in perfect circles around the Earth

In summary,

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- The Earth is spherical
- The Moon is closer than the Sun and *same angular size*
- All solar system bodies orbit in a flat plane in perfect circles around the Earth
  - (no physics behind this, but it was a pretty viable, elegant explanation)

The First Recorded Attempt of Measuring Things

Aristarchus of Samos, Greek, 270BC, Heliocentrist

Realized the distance of the sun would change when we observe a half-moon. Instead of starting humble with the size of the Earth bro went wacko and tried to get THIS distance first.

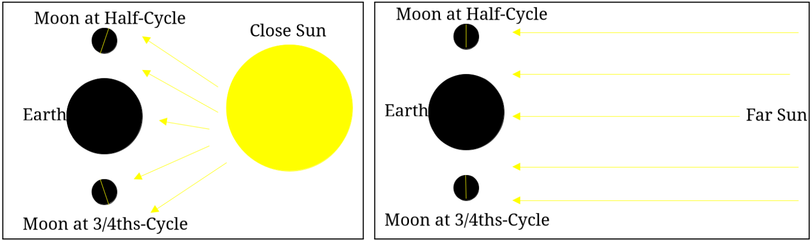
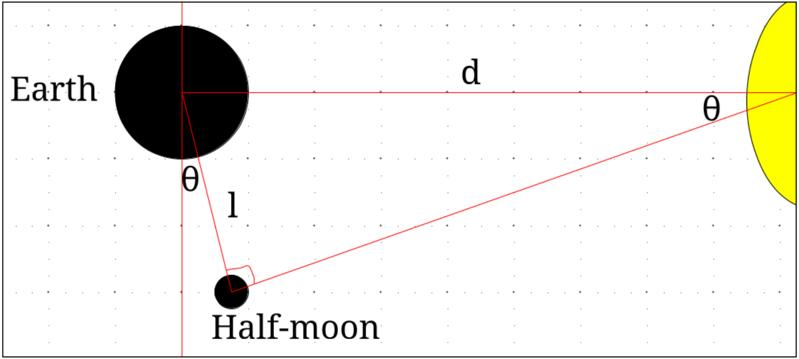


Figure: Near-Sun rays make a half-moon appear sooner in the lunar cycle, while far-Sun parallel rays mean half-moons happen at $\frac{1}{2}$ and $\frac{3}{4}$ in the lunar cycle.

Aristarchus' Method

Measure the angle θ at exactly half-moon.



$$\sin\theta = \frac{l}{d} \implies \frac{l}{d} = \sin\theta$$

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The lunar cycle is 29.5 days (which was known)
Nowadays, we know that half-moons occur 30 mins after it reaches those 90° marks
This meant measuring $\frac{1}{1400}$th of a lunar cycle.

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Aristarchus measured 87°, meaning $\frac{l}{d} = \sin\left(90^\circ - 87^\circ\right) = 19.1$ the Sun is 19x farther away than the Moon.
In reality, this half-moon angle is 89.88°, which gives a much larger number: the Sun is 409x farther away than the Moon.

The First Recorded Attempt of Measuring Things

Aristarchus of Samos, Greek, 270BC, Heliocentrist

good effort dipshit

Eratosthenes and the Size of the Earth

Greece, 230BC

The first good measurement of the size of the Earth

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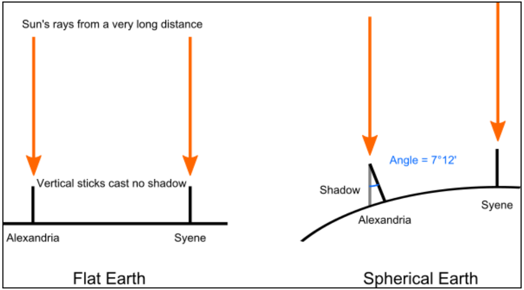


Figure: Eratosthenes' experimental setup[7].

[7] Image: https://www.mezzacotta.net/100proofs/images/002-SyeneAlexandria.png

Eratosthenes and the Size of the Earth

Greece, 230BC

Eratosthenes knew the city of Syene was on the Tropic of Cancer (no shadows on summer solstice, sun right overhead)

He also knew the distance between Alexandria and Syene

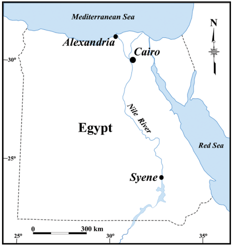
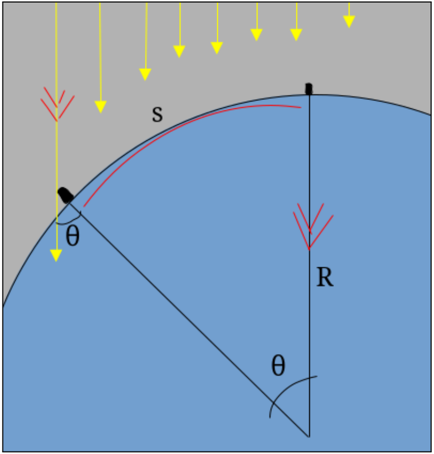


Figure: Alexandria and Syene along the Nile River in Egypt[8].

[8] Image: https://www.researchgate.net/profile/Alok-Kumar-111/publication/253596660/figure/fig2/

Eratosthenes' Method



Arc length:

$$s = \frac{\pi R \theta}{180^\circ}$$

Rearrange for R, Earth's radius:

$$R = \frac{180 s}{\pi \theta}$$

s was already known from geographical data, and θ was measureable from shadows cast in Alexandria.

Eratosthenes and the Size of the Earth

Greece, 230BC

To avoid doing difficult division with π, he opted to calculate the circumference, not the radius:

$$C = 2\pi R = \frac{360s}{\theta}$$

To which he found a value of 252,000 stadia.
In ancient Greece, they used a unit called a 'stade'. We don't really know how big a stade was, but we estimate 1 stade is about 525ft, making his calculation *less than 1% off.*

Eratosthenes: 25,050mi.
Modern value: 24,901mi.

Hipparchus and the Distance to the Moon

Greece, 189BC

Looking at a solar eclipse from two different places allowed for the calculation of the distance to the moon.

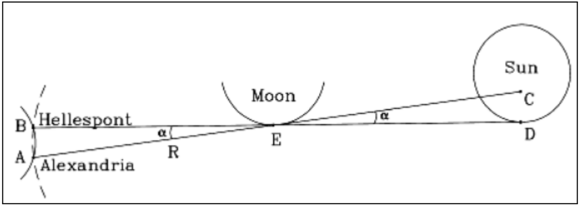


Figure: Parallax configuration that allowed our boy to measure the size of the moon[9].

[9] Image: https://pwg.gsfc.nasa.gov/stargaze/Shipparc.htm

Hipparchus

Enjoy this drawing, I'm getting lazier and I'm just going to use MSPaint

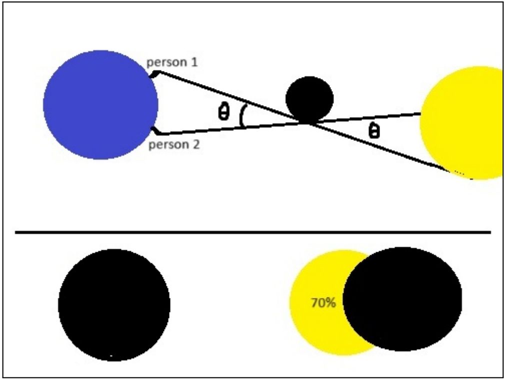


Figure: Moon measuring setup geometry and on-Earth POV

[9] Image: me ;)

Hipparchus

Distance to the moon, the math

Givens:

The distance between Hellespont and Alexandria was 9° of lattitude exactly

The moon and sun are in set locations, but only appear to be in different places because of parallax

They knew the sun spanned 0.5° in the sky

Hipparchus

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The distance between Hellespont and Alexandria was 9° of lattitude exactly

The moon and sun are in set locations, but only appear to be in different places because of parallax

They knew the sun spanned 0.5° in the sky

Unknowns/Measurements:

At the same point in time, the moon covered both 100% and 80% of the sun

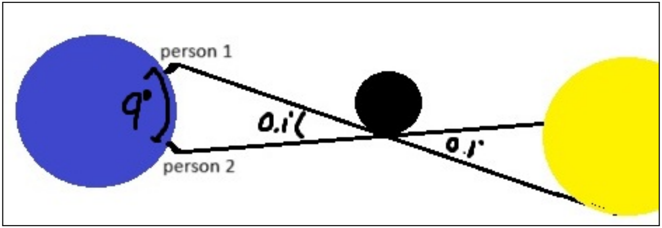
$\theta = (1 - 0.80) * 0.5^\circ = 0.1^\circ$ parallax of the moon

this means 9° of arc length on the Earth results in a 0.1° parallax angle in the moon :))))

now we have numbers

let us not lose sight of the purpose of such knowledge

"9° of arc length on the Earth results in a 0.1° parallax angle in the moon"



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$$\text{Arc length: } s = r\theta$$

$$s_{Earth} = R_{Earth}(9^\circ) \;\&\; s_{Moon} = r_{Moon}(0.1^\circ)$$

set these equal

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Arc length: $s = r\theta$

$$s_{Earth} = R_{Earth}(9^\circ) \;\&\; s_{Moon} = r_{Moon}(0.1^\circ)$$

set these equal

$$R_{Earth}(9^\circ) = r_{Moon}(0.1^\circ)$$

$$\frac{R_{Earth}}{r_{Moon}} = \frac{9}{0.1} \implies r_{moon} = \frac{R_{Earth}}{90}$$

look!!! the unknown variable in terms of known values!!!!!

Putting it all together

Forming the distance ladder

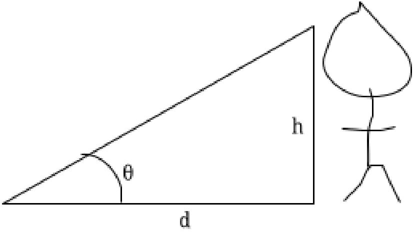
This measurement gave Hipparchus the result that the moon is $90x$ smaller than the Earth, so...

Remember apparent size:

Apparent Size

What if you *know the size of a distant object?*

Easiest solution for measuring a distance.



$$\sin\theta = \frac{h}{d} \implies d = \frac{h}{\sin\theta}$$

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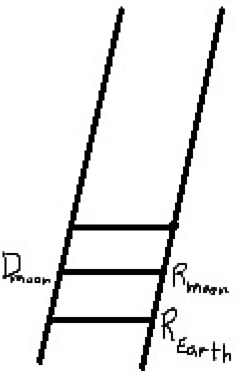
Remember apparent size: $d = \frac{h}{\sin\theta}$

where $h = r_{Moon} = \frac{R_{Earth}}{90}$ and $\theta = 0.5^\circ$ (measureable)

$$d_{Moon} = \frac{h}{\sin\theta} = \frac{r_{Moon}}{\sin\theta} = \frac{\frac{R_{Earth}}{90}}{\sin 0.5^\circ}$$

Distance Ladder

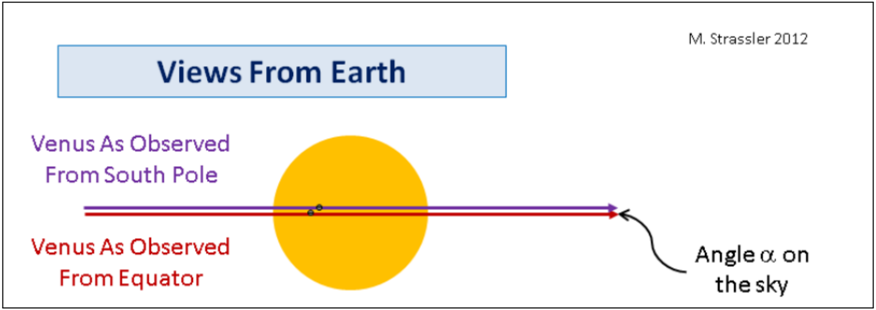


- $R_{Earth} = \frac{360 * 1070\mathrm{km}}{7.2^\circ}$
- $d_{Moon} = \frac{\frac{R_{Earth}}{90}}{\sin 0.5^\circ}$

How far is the sun?

For the sake of time this is going to be hand-wavey

This is the goal — use a transit of Venus and parallax
Find d_{EV} and d_{VS}, add them to get d_{ES}

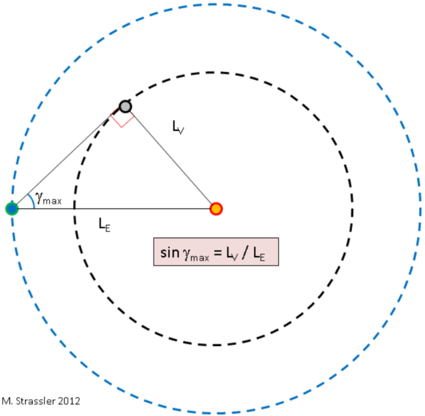


[9] Image: https://profmattstrassler.com/articles-and-posts/relativity-space-astronomy-and-cosmology/transit-of-venus-and-the-distance-to-the-sun/

How far is the sun?

Calculate distance to Venus, distance from Venus to the Sun, then add these :))))

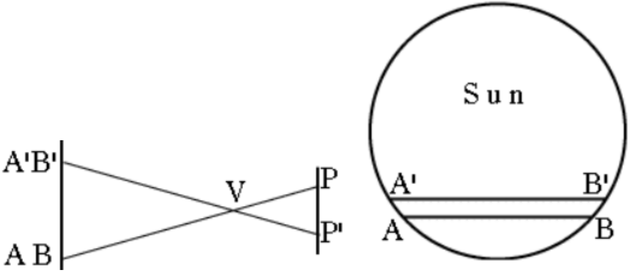
Self explanatory. We know *how many times smaller* Venus' orbit is than ours, but how big Earth's orbit is *is the question.*



How far is the sun?

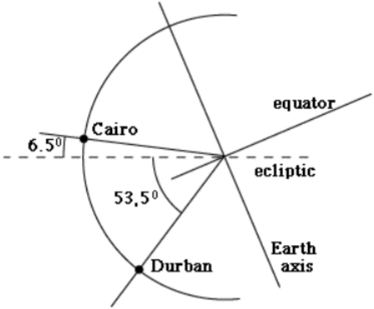
Calculate distance to Venus, distance from Venus to the Sun, then add these :))))

Kind of the same as measuring the distance to the moon with parallax :)
You can also just time how long Venus takes to transit from two spots, much less precise than measuring angles



[9] Image: http://www.phy6.org/stargaze/Svenus1.htm

it's actually super super hard



Now **in Figure** 7 we have a long narrow triangle whose side is the Earth-Venus distance, estimated as R_V=0.292AU . Since the angle D' is very small, PP' may be regarded as small part of a circle, giving very nearly

$$\mathbf{PP'/\ 2\pi R_V = D'/360°}$$

In Figure 5 the same argument can be applied, except the angle is now F and the side of the triangle is the Earth-Sun distance, estimated as R_S=1.015AU. Hence (very nearly)

$$\mathbf{PP'/\ 2\pi R_S = F/360°}$$

Dividing right by right, left by left

$$\mathbf{R_S / R_V \;=\; D' / F \;=\; 1.015/0.292 \;=\; 3.476 \qquad (12a)}$$

Meaning that at the time of the transit, the Sun is about three and a half times more distant than Venus. From (10)

$$\mathbf{x \;=\; (PP' / F)\ [21600/(6.2832.\ 1.015)] \;=\; 3386.9\ (PP' / F) \qquad (12b)}$$

Combining equations (11) and (12a),

$$\mathbf{D \;=\; D' - F \;=\; F\ [(D'/F) - 1] \;=\; 2.476\ F}$$

The value of D was **0.310306 minutes of arc.** From (12b)

$$x \;=\; 3386.9\ PP'\ [2.476/D] \;=\; 3386.9\ PP'\ [2.476/0.310306]$$

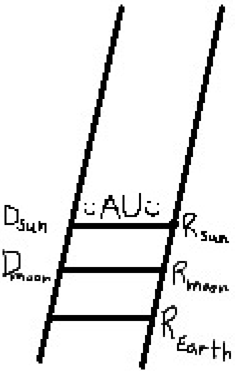
$$\mathbf{x \;=\; 27024.8\ PP' \qquad (13)}$$

(Thanks to Prof. Udo Backhaus of the Univ. of Essen for pointing out some shortcuts here.)

You also don't need the distance to the moon, just the Earth's radius but oh well

Distance Ladder

Edmund Halley (the comet guy) came up with this Venus transit method and died before it was done in 1761 and 1769.

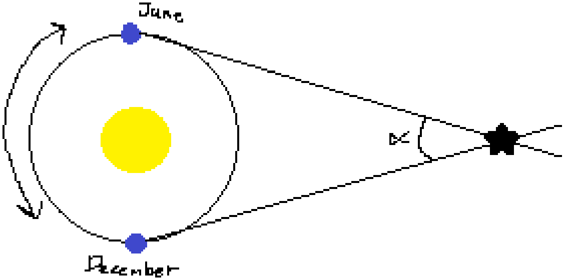


- $R_{Earth} = \frac{360 * 1070\mathrm{km}}{7.2^\circ}$
- $d_{Moon} = \frac{\frac{R_{Earth}}{90}}{\sin 0.5^\circ}$
- $1\mathrm{AU} = d_{Sun} \propto R_{Earth}$

How far are the stars?

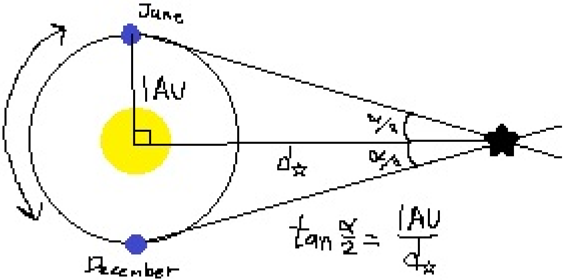
We can only calculate this if we know the astronomical unit (AU)

IT'S PARALLAX AGAIN IT'S PARALLAX



How far are the stars?

Genuinely the easiest math yet



$$\tan\frac{\alpha}{2} = \frac{1\mathrm{AU}}{d_{star}} \implies \tan\alpha \approx \frac{1\mathrm{AU}}{d_{star}}$$

How far are the stars?

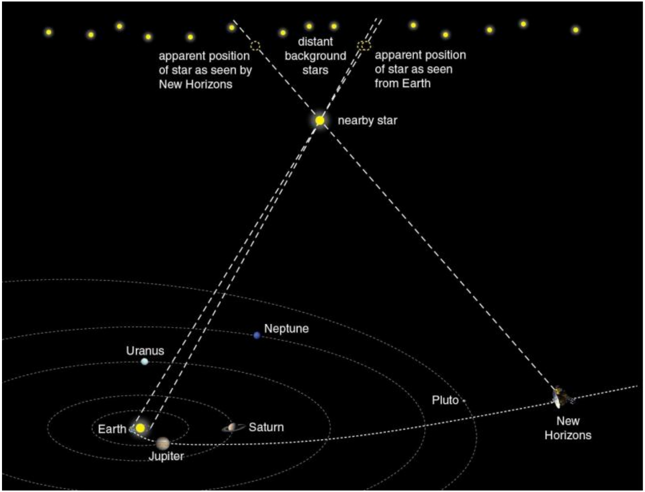
Genuinely the easiest math yet, that's all there is to it

$$d_{star} = \frac{1\mathrm{AU}}{\tan \alpha}$$

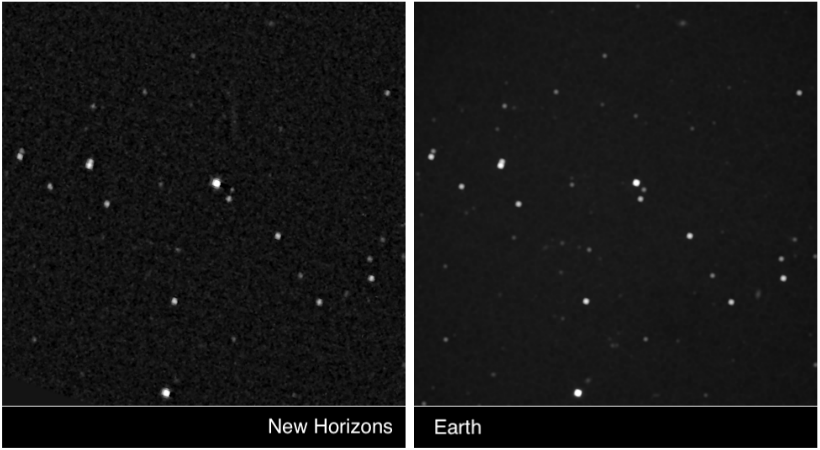
Literally just measure the angle to a star six months apart easy

How far are the stars?

The largest scale we've ever done this on is with the New Horizons probe

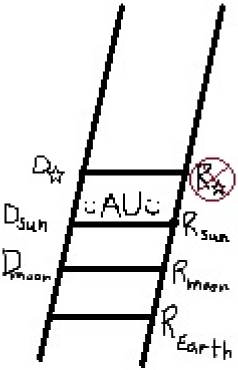


How far are the stars?



9 Image: NASA

Distance Ladder



- $R_{Earth} = \frac{360*1070\mathrm{km}}{7.2°}$
- $d_{Moon} = \frac{\frac{R_{Earth}}{90}}{\sin 0.5°}$
- $1\mathrm{AU} = d_{Sun} \propto R_{Earth}$
- $d_{star} = \frac{d_{sun}}{\tan\alpha} \propto \frac{R_{Earth}}{\tan\alpha}$

How far are galaxies?

:(

How far are galaxies?

NOT IN THE SPIRIT OF THE PRESENTATION, NO PARALLAX :(

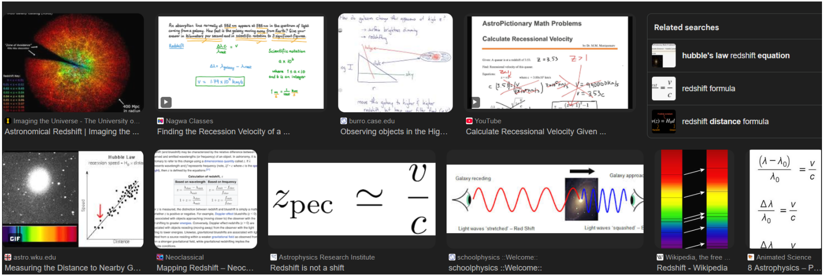
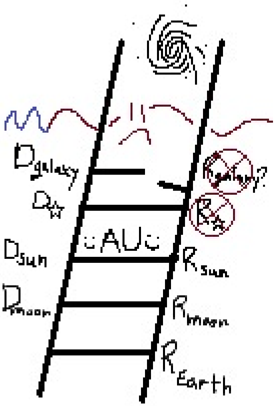


Figure: misery

Distance Ladder

Conclusion



- $R_{Earth} = \frac{360 * 1070\mathrm{km}}{7.2^\circ}$
- $d_{Moon} = \frac{\frac{R_{Earth}}{90}}{\sin 0.5^\circ}$
- $1\mathrm{AU} = d_{Sun} \propto R_{Earth}$
- $d_{star} = \frac{d_{sun}}{\tan\alpha} \propto \frac{R_{Earth}}{\tan\alpha}$
- $d_{galaxy} =$ idiot math

d_{galaxy} stops being cool geometry and starts being math so, not including that BS

questions?