Project 2

by

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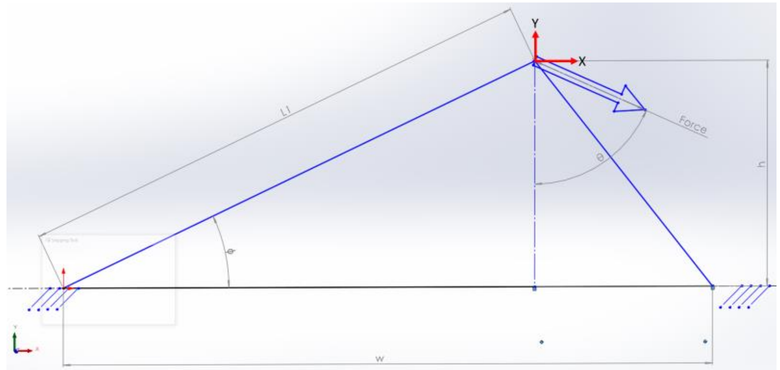
Due 10/22/2022

Model:



(a) (b)

Figure 1. (a) Figure of the system shows the context of the code: an amusement park ride. (b) Figure of the system modeled using spring elements. Input dimensions are labeled on the diagram.

Model Assumptions:

1. 2 Dimensional loading.
2. Key Parameters:

Parameter	Symbol	Value	Units
Young's Modulus	E	$2*10^{10}$	Pa
Cross-sectional area	A	0.01	m^2
Width	w	10	m
Length	L1	12	m
Height	h	10	m
Force	F	50000	N

3. Ride runs for 60 seconds starting at t=0 seconds.
4. The angle θ with respect to time t can be modeled as a cos/sin wave with 4 oscillations in 30 seconds.
5. Amplitude increases linearly.
6. Motions starts at amplitude = 0 radians.
7. Max amplitude = 3 radians.
8. At 30 seconds, angle θ decreases at same frequency.
9. Amplitude decreases linearly.
10. Beams can be modeled as truss elements.
11. Truss elements can be modeled as springs with stiffness *EA/l*.

Derivations:

This derivation shows how the potential energy is derived from the truss geometry and given assumptions listed above.

Using existing parameters, find key dimensions ϕ, w1, w2, and L2.

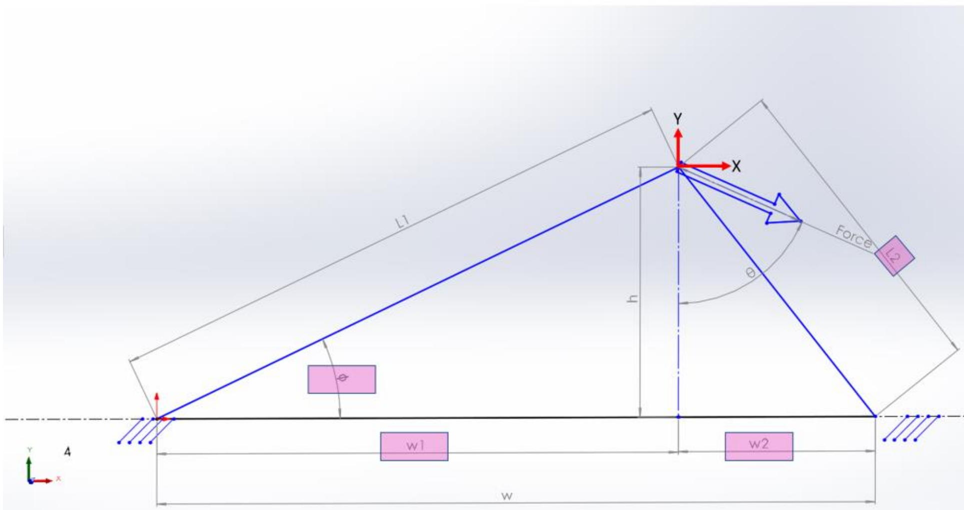


Figure 2. The first step is finding L2, the length of the other truss. The input parameters are used to find L2. The steps are listed below.

Find ϕ.

Step 1: Find ϕ using the sides L1 and h.

1. $\phi = \arcsin\left(\frac{h}{L1}\right)$

Step 2: Find w1

2. w1$=L1 * \cos(\phi)$

Step 3: Find w2

3. w2=w-w1

Step 4: Find L2 using the Pythagorean theorem

4. L2$=\sqrt{h^2 + w2^2}$

Using ϕ, w1, w2, L2, and input parameters x and y, the loaded lengths of w1, w2, and h can be found.: w1_loaded, w2_loaded, and h_loaded.

Step 1: Find w1_loaded by combining w1 and the displacement x.

1. w1_loaded =w1+x

Step 2: Find w2_loaded by finding the difference between w and w1_loaded

2. w2_loaded=w-w1_loaded

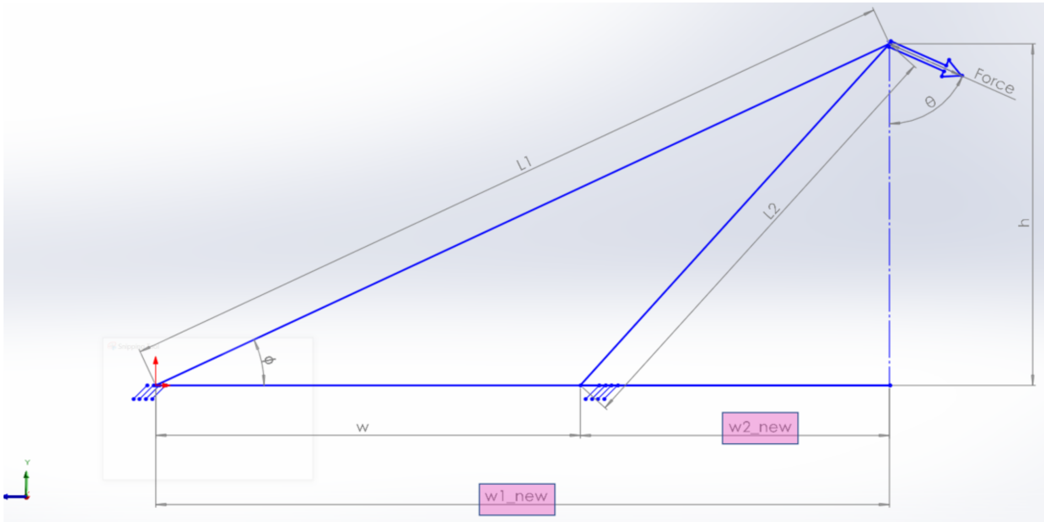


Figure 3. The model has now factored in the x and y displacement into its geometry. This model is a graphical justification for Step 2.

Step 3: Find h_loaded

3. h_loaded = h+y

Using w1_loaded, w2_loaded, and y_loaded, the loaded lengths L1_loaded and L2_loaded can be found using the Pythagorean Theorem:

$$L1\_loaded = \sqrt{w1\_loaded^2 + y\_loaded^2}$$

$$L2\_loaded = \sqrt{w2\_loaded^2 + y\_loaded^2}$$

After finding the final lengths of the truss members, the spring potential energy equation and work equations show how much potential energy the truss members have.

Generic potential energy equation: $PE = (\frac{1}{2}Kx^2) + (f * x)$

L1_loaded Spring PE:

$$PE\_Spring1 = \frac{1}{2}K * \left(L1_{loaded}{}^{2}\right) + (L1\_loaded - L1)^2$$

Step 2: Find L2_loaded Spring PE:

$$PE\_Spring2 = \frac{1}{2}K * \left(L2_{loaded}{}^{2}\right) + (L2\_loaded - L2)^2$$

Step 3: Work for x displacement and y displacement:

$$Work = Fsin(\theta) \text{ - } Fcos(\theta)$$

Step 4: Work is subtracted from the Spring Potential Energy to find the Net Potential Energy.

$$PE = PE\_Spring1 + PE\_Spring2 - Work$$

Step 5: Model equation:

$$PE = \frac{1}{2}K\left(\left(\sqrt{(L1 * \cos(\arcsin(\frac{h}{L1})) + x)^2 + (y + h)^2}\right)^2\right) +$$
$$\left(\sqrt{(L1 * \cos(\arcsin(\frac{h}{L1})) + \mathrm{x})^2 + (y + h)^2)} - L1\right)^2 +$$
$$\frac{1}{2}K\left(\sqrt{(\mathrm{w} - (L1 * \cos(\arcsin(\frac{h}{L1}) + \mathrm{x}))^2 + (y + h)^2}^{\,2}\right) +$$

$$\left((\mathrm{w} - (L1 * \cos(\arcsin(\frac{h}{L1}) + \mathrm{x}))^2 - \sqrt{h^2 + (\mathrm{w} - (L1 * \cos(\arcsin(\frac{h}{L1})))^2}\right)^2$$

The equation for theta is derived using assumptions 3-9. The fraction $\frac{3t}{30}$ ensures that when t=30, the amplitude will be 3 radians. The variable t also causes θ to increase at a linear rate. The frequency $\frac{\pi}{3.75}$ is derived from the requirement of 4 oscillations before reaching 30 seconds.

$$H_1(x) = \frac{3t}{30}\sin\left(\frac{\pi t}{3.75}\right)\ \{30 \leq x \leq 60\}$$

$$H_2(x) = -(\frac{3(t-60)}{30}\sin\left(\frac{\pi t}{3.75}\right)\ \{30 \leq x \leq 60\}$$

Analysis:

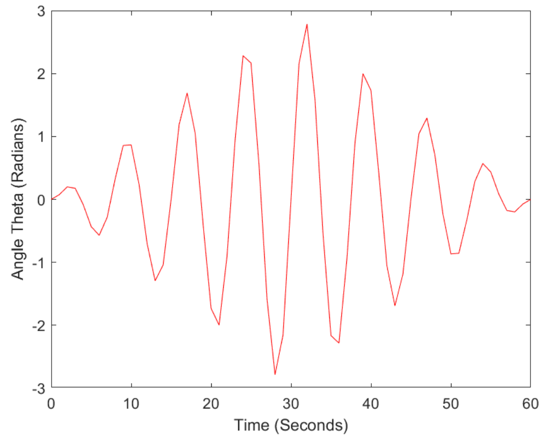


Figure 1. The periodic swings of the Evolution are modeled as a function of time.

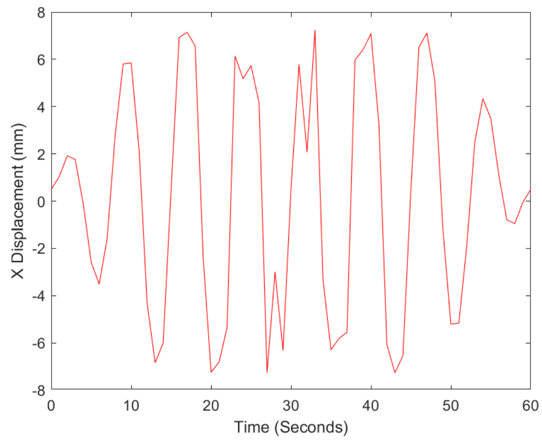


Figure 2. Eight peaks are counted which corresponds to the eight peaks of θ.

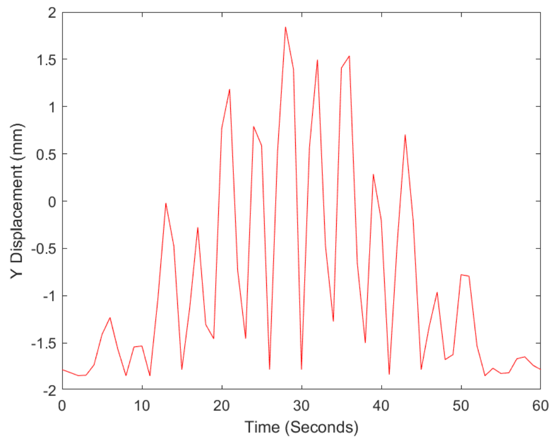


Figure 3. The Y Displacement peaks very frequently.

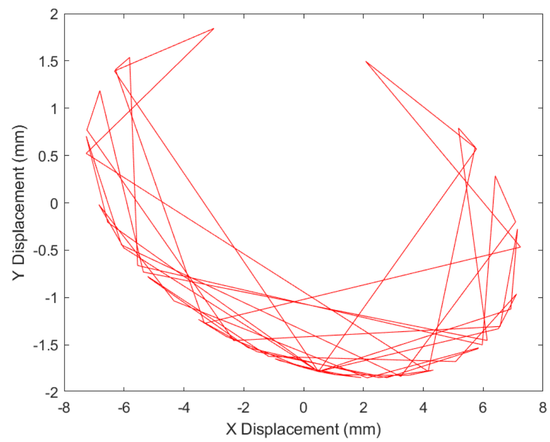


Figure 4. The cyclic displacement of the swing only goes up three radians. This is reflected in the oval shape because overall, the X Displacement is larger. If the Evolution's θ made it a full 3π, the overall shape should appear more circular.

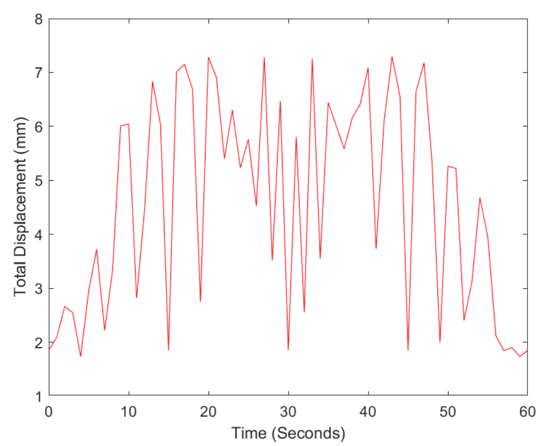


Figure 5. The Total Displacement peaks very frequently as a consequence of the Y displacement varying frequently.