Project 3

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Numerical Methods ME 355

Section A

Code Written on October 31, 2022

Report Submitted on October 31, 2022

I have neither given or received, nor have I tolerated others' use of unauthorized aid.

-Ethan Storer

Introduction:

Worldwide, Glaucoma is the second most common cause of vision loss. This loss of sight is synonymous with high intraocular pressure. The medical community currently thinks the abnormally high pressure causes damage to retinal ganglion cells, a type of neuron in the retina. A Glaucoma patient's data has been collected for examination. A spreadsheet with the patient's intraocular pressure and age is given to examine the vision loss over the patient's lifetime. The equation for vision loss is given:

$$VL = Aexp\left(k\int_{25}^{t}(P-13)\,dt\right)\%$$

Where A=4, k=0.013, P is the intraocular pressure, and t is the patient's age. An integration method is needed to find the vision loss at a given recorded age. The python script in this report finds the values for vision loss and and plots them in Figure 2 using multiple integration methods.

Results:

Table 1. As the patient ages, the intraocular pressure also increases.

Age (Years)	Intraocular Pressure (mm-Hg)
25	13
40	15
50	22
60	23
65	24

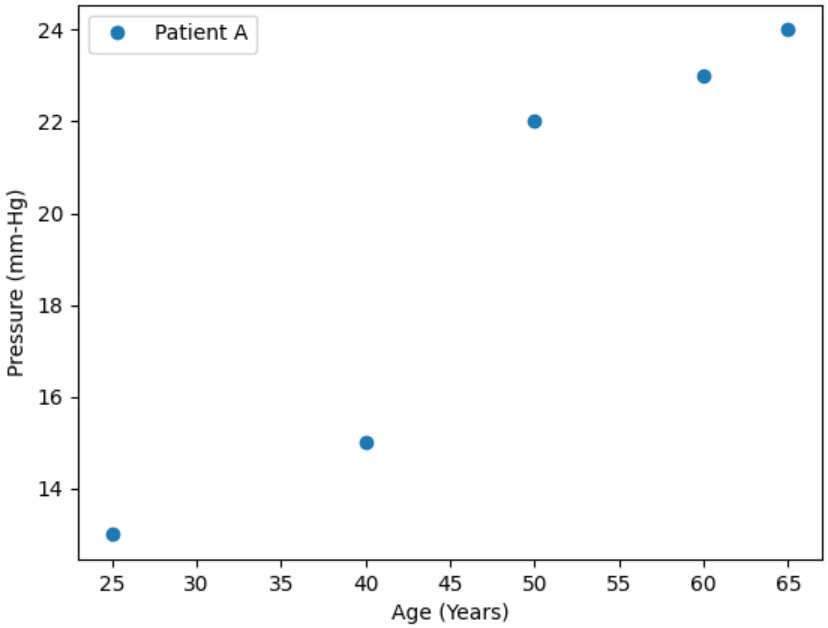

Figure 1. Patient A's Intraocular pressure increases over time.

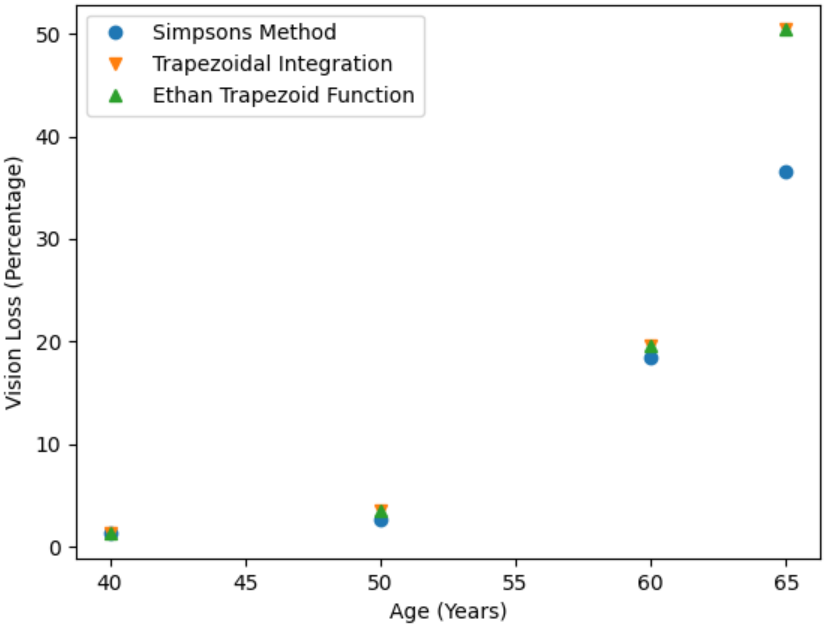

Figure 2. Different integration methods show vision loss over time.

Discussion:

a. The first integration method, Trapezoidal Integration, is the simplest and most inaccurate method. The formula uses two points from the Age column to find a step size. The sum of the corresponding Intraocular Pressures is halved and multiplied by the step size. The formula below illustrates this:

$$Vision\ Loss_{Trapezoidal} = (Age_1 - Age_0)(\frac{Pressure_0 + Pressure_1}{2})$$

In Figure 2, the called Trapezoidal Integration function was used to double check the Ethan Trapezoid function. The outputs overlap on the graph, so they are functionally identical.

The second integration method, Simpson's Method, uses a combination of Simpson's 1/3 method, Simpson's 3/8 method, and Trapezoidal Integration. This combination of methods lowers the overall error:

$$Vision\ Loss_{Simpson's\frac{1}{3}} = (Age_2 - Age_0)(\frac{Pressure_0 + 4 * Pressure_1 + Pressure_2}{6})$$

$$Vision\ Loss_{Simpson's\frac{3}{8}} = (Age_3 - Age_0)(\frac{Pressure_0 + 3 * Pressure_1 + 3 * Pressure_2 + Pressure_3}{6})$$

The Simpson's Method function decides which method to utilize based on the step size. Three consecutive, identical step sizes will have the Simpson's 1/3 method. Inconsistent step sizes are best integrated by the Trapezoidal Method. Simpson's 3/8 method is generally used to integrate 4 consecutive, identical step sizes. A set of step sizes integrated by the Simpson's 1/3 method and an adjacent set of step sizes integrated by the trapezoid method can also be combined under the Simpson's 3/8 method.

Given these rules, it can be seen that the Simpson's method function uses the trapezoid method in intervals [25, 40] and [60, 65] since they do not match the 10-year intervals of the other 3 data points. These 3 data points can be integrated with the Simpson's 1/3 rule.

However, since these 3 data points are adjacent to a set of points integrated by the Trapezoidal Method, Simpson's 3/8 method can join them together and reduce the error.

b. If Vision Loss does not require a tight tolerance to be useful, the three methods plot very closely together until age 65. The Trapezoidal Integration is an overestimate of the true value of Vision Loss since the Simpson's Method function has a lower error than Trapezoidal Integration. The Simpson's Method function is more exact at age 65 since it utilizes Simpson's 3/8 method for the last four points.