Display Advertisement Pricing Model

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Display advertisement is bought and sold in online marketplaces – to further understand the drivers of display advertisment spend, a Two-Stage Least Squares Regression applied to a system of equations is used to estimate advertisement eqilibrium price. Equilibrium price is analagous to Cost per Advertisement, and may be placed in a budgeting model (Expected Quantity × Cost per Advertisement) to predict spend.

Simultaneous Equations Model

Structural Equations

$P_{Dt} = \beta_0 + \beta_1 Q_t + \beta_2 Cash_t + \beta_3 Print_t + u_{Dt}$

$P_{St} = \alpha_0 + \alpha_1 Q_t + \alpha_2 Serve_t + u_{St}$

P_{Dt} = Inverse Demand

P_{St} = Inverse Supply

Q_t = Exchange Quantity of Display Advertisements

$Cash_t$ = Net Cash Flow

$Print_t$ = Price of Print Advertisement

$Serve_t$ = Cost to Serve Advertisement

u_{Dt} = Error term Demand

u_{St} = Error term Supply

Reduced-Form Equation

Through algebraic manipulation, the reduced-form equation for Quantity is determined. It becomes apparent that Q_t is correlated with the error terms in both structural equations:

$$\mathbf{Q}_t = \frac{\alpha_0 + \alpha_3 Serve_t + \mathbf{u}_{St} - \beta_0 - \beta_2 Cash_t - \beta_3 Print_t - \mathbf{u}_{Dt}}{\beta_1 - \alpha_1}$$

This defines Q_t as an endogenous variable, meaning it is determined within the system of equations. This introduces an endogeneity bias into the simultaneous equations model, which occurs when an independent variable, such as Q_t, is endogenous. This violates the independence assumption of least squares. A different method is necessary to produce unbiased parameters – this analysis uses Two-Stage Least Squares.

Two-Stage Least Squares

In this instance, the Two-Stage Least Squares method is used to absolve the simultaneous equations model of endogeneity bias. It does this by estimating a fitted value for the endogenous independent variable through the use of Instrumental Variables. $Print_t$ and $Cash_t$ are instrumental varibles for Quantity Demanded, and $Serve_t$ is an instrumental variable for Quantity Supplied.

Three criteria must be met for a variable to be use as an instrument – they are as follows:

1. Correlated with the endogenous independent variable
2. Uncorrelated with the error term
3. Not already included in the structural equation

Statisical tests such as the Durbin-Wu-Hausman and Stock-Yogo can test for endogeneity and the strength of instruments, respectively.

Quantity Demanded

$$\hat{Q}_{Dt} = \gamma_0 + \gamma_1 Print_t + \gamma_2 Serve_t + \gamma_3 Cash_t + u_{Dt}$$

Quantity Supplied

$$\hat{Q}_{St} = \delta_0 + \delta_1 Cash_t + \delta_2 Print_t + \delta_3 Serve_t + u_{St}$$

Identified Structural Equations

$$P_{Dt} = \beta_0 + \beta_1 \hat{Q}_{Dt} + \beta_2 Cash_t + \beta_3 Print_t + u_{Dt}$$

$$P_{St} = \alpha_0 + \alpha_1 \hat{Q}_{St} + \alpha_2 Serve_t + u_{St}$$

The identified structural equations are rid of endogeneity and are able to produce unbiased parameters to predict equilibrium price.