Display Advertisement Pricing Model

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Display advertisement is bought and sold in online marketplaces – firms purchase space on publishers' webpages to display their advertisements. For budgeting purposes, it is in a firm's interest to understand the determinants of advertisement price. This analysis applies a Two-Stage Least Squares Regression to a system of equations to estimate advertisement eqilibrium price. Price may be placed in a budgeting model (Expected Quantity of Advertisements × Equilibrium price) to predict the firm's advertising spend.

Simultaneous Equations Model

Structural Equations

$P_{Dt} = \beta_0 + \beta_1 Q_t + \beta_2 Cash_t + \beta_3 Print_t + u_{Dt}$

$P_{St} = \alpha_0 + \alpha_1 Q_t + \alpha_2 Serve_t + u_{St}$

P_{Dt} = Inverse Demand - Price bid

P_{St} = Inverse Supply - Price ask

Q_t = Exchange Quantity of Display Advertisements

$Cash_t$ = Index of Net Cash Flow for Buyers

$Print_t$ = Price of Print (Offline) Advertisement

$Serve_t$ = Cost to Serve Advertisement

u_{Dt} = Error term Demand

u_{St} = Error term Supply

Reduced-Form Equation

Through algebraic manipulation, the reduced-form equation for Quantity is determined. It becomes apparent that Q_t is correlated with the error terms in both structural equations:

$$\mathbf{Q}_t = \frac{\alpha_0 + \alpha_2 Serve_t + \mathbf{u}_{St} - \beta_0 - \beta_2 Cash_t - \beta_3 Print_t - \mathbf{u}_{Dt}}{\beta_1 - \alpha_1}$$

This defines Q_t as an endogenous variable, meaning it is determined within the system of equations. This introduces an endogeneity bias into the simultaneous equations model, which occurs when an independent variable, such as Q_t, is endogenous. A different method is necessary to produce unbiased parameters – this analysis uses Two-Stage Least Squares.

Two-Stage Least Squares

In this instance, the Two-Stage Least Squares method is used to absolve the simultaneous equations model of endogeneity bias. It does this by estimating a fitted value for the endogenous independent variable through the use of Instrumental Variables. $Print_t$ and $Cash_t$ are instruments for Quantity in the supply equation, and $Serve_t$ is an instrument for Quantity in the demand equation.

Three criteria must be met for a variable to be use as an instrument – they are as follows:

1. Correlated with the endogenous independent variable
2. Uncorrelated with the error term
3. Not already included in the structural equation

Statisical tests such as the Durbin-Wu-Hausman and Stock-Yogo can test for endogeneity and the strength of instruments, respectively.

Fitted Quantity

$$\hat{Q}_t = \gamma_0 + \gamma_1 Print_t + \gamma_2 Serve_t + \gamma_3 Cash_t + u_t$$

Identified Structural Equations

$$P_{Dt} = \beta_0 + \beta_1 \hat{Q}_t + \beta_2 Cash_t + \beta_3 Print_t + u_{Dt}$$

$$P_{St} = \alpha_0 + \alpha_1 \hat{Q}_t + \alpha_2 Serve_t + u_{St}$$

Barring serial correlation, the identified structural equations are rid of endogeneity and are able to produce unbiased parameters to predict equilibrium price.