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CS 3339.001
Homework 3

1.a)

Iteration	Step	Divisor	Remainder
1	Initialization	010101	000001 000101
2	$\text{Remainder}(\text{left}) = \text{Remainder}(\text{left}) - \text{Divisor}$	010101	101100 000101
	$\text{Remainder} < 0 \rightarrow \text{Remainder} + \text{Divisor}$	010101	000001 000101
	Shift Remainder left; $\text{Remainder}(0) = 0$	010101	000010 001010
3	$\text{Remainder}(\text{left}) = \text{Remainder}(\text{left}) - \text{Divisor}$	010101	101101 001010
	$\text{Remainder} < 0 \rightarrow \text{Remainder} + \text{Divisor}$	010101	000010 001010
	Shift Remainder left; $\text{Remainder}(0) = 0$	010101	000100 010100
4	$\text{Remainder}(\text{left}) = \text{Remainder}(\text{left}) - \text{Divisor}$	010101	101111 010100
	$\text{Remainder} < 0 \rightarrow \text{Remainder} + \text{Divisor}$	010101	000100 010100
	Shift Remainder left; $\text{Remainder}(0) = 0$	010101	001000 101000
5	$\text{Remainder}(\text{left}) = \text{Remainder}(\text{left}) - \text{Divisor}$	010101	110011 101000
	$\text{Remainder} < 0 \rightarrow \text{Remainder} + \text{Divisor}$	010101	001000 101000
	Shift Remainder left; $\text{Remainder}(0) = 0$	010101	010001 010000
6	$\text{Remainder}(\text{left}) = \text{Remainder}(\text{left}) - \text{Divisor}$	010101	111100 010000
	$\text{Remainder} < 0 \rightarrow \text{Remainder} + \text{Divisor}$	010101	010001 010000
	Shift Remainder left; $\text{Remainder}(0) = 0$	010101	100010 100000
7	$\text{Remainder}(\text{left}) = \text{Remainder}(\text{left}) - \text{Divisor}$	010101	001101 100000
	$\text{Remainder} > 0 \rightarrow \text{keep Remainder}$	010101	001101 100000
	Shift Remainder left; $\text{Remainder}(0) = 1$	010101	011011 000001
8	$\text{Remainder}(\text{left}) = \text{Remainder}(\text{left}) - \text{Divisor}$	010101	000110 000001
	$\text{Remainder} > 0 \rightarrow \text{keep Remainder}$	010101	000110 000001
	Shift Remainder left; $\text{Remainder}(0) = 1$	010101	001100 000011
	At the end: shift Remainder (left) right	010101	000110 000011

1.b) $\frac{69}{21} = 3$ Remainder 6 \rightarrow quotient = 3 = 000011 remainder = 6 = 000110

The process took 7 steps, excluding initialization, which is $n+1$ iterations as our divisor was 6 bits

2.a) Convert 0.02875:

$$0.02875 \cdot 2 = 0.0575$$

$$0.0575 \cdot 2 = 0.115$$

$$0.115 \cdot 2 = 0.23$$

$$0.23 \cdot 2 = 0.46$$

$$0.46 \cdot 2 = 0.92$$

$$0.92 \cdot 2 = 1.84$$

$$0.84 \cdot 2 = 1.68$$

$$0.68 \cdot 2 = 1.36$$

$$0.36 \cdot 2 = 0.72$$

$$0.72 \cdot 2 = 1.44$$

$$0.44 \cdot 2 = 0.88$$

$$0.88 \cdot 2 = 1.76$$

$$0.76 \cdot 2 = 1.52$$

$$0.52 \cdot 2 = 1.04$$

$$0.04 \cdot 2 = 0.08$$

$$0.08 \cdot 2 = 0.16$$

$$\underbrace{010110}_s \underbrace{1101011100}_f$$

The precision of this format is less accurate because only 10 bits are dedicated to the fraction instead of 23. This reduction can lead to less accurate fractions in situations where the fraction cannot be expressed in 10 bits.

2.b) Convert 37.125

$$0.125 \cdot 2 = 0.25$$

$$0.25 \cdot 2 = 0.5$$

$$0.5 \cdot 2 = 1$$

$$1.0010100100 \cdot 2^5 + 1.0000110011 \cdot 2^{-1}$$

$$1.0010100100 \cdot 2^5 + 0.0000100000 \cdot 2^5$$

$$= 1.00101100111 \cdot 2^5$$

$$= 1.0010110110 \cdot 2^5 = 37.6875$$

Convert 0.5250390625

$$0.5250390625 \cdot 2 = 1.050078125$$

$$0.050078125 \cdot 2 = 0.10015625$$

$$0.10015625 \cdot 2 = 0.2003125$$

$$0.2003125 \cdot 2 = 0.400625$$

$$0.400625 \cdot 2 = 0.80125$$

$$0.80125 \cdot 2 = 1.6025$$

$$0.6025 \cdot 2 = 1.205$$

$$0.205 \cdot 2 = 0.41$$

$$0.41 \cdot 2 = 0.82$$

$$0.82 \cdot 2 = 1.64$$

$$0.64 \cdot 2 = 1.28$$

2.) Convert -7.3125

$$0.3125 \cdot 2 = 0.625$$

$$0.625 \cdot 2 = 1.25$$

$$0.25 \cdot 2 = 0.5$$

$$0.5 \cdot 2 = 1$$

Convert .2203125

$$0.2203125 \cdot 2 = 0.440625$$

$$0.440625 \cdot 2 = 0.88125$$

$$0.88125 \cdot 2 = 1.7625$$

$$0.7625 \cdot 2 = 1.525$$

$$0.525 \cdot 2 = 1.05$$

$$0.05 \cdot 2 = 0.1$$

$$0.1 \cdot 2 = 0.2$$

$$0.2 \cdot 2 = 0.4$$

$$0.4 \cdot 2 = 0.8$$

$$0.8 \cdot 2 = 1.6$$

$$0.6 \cdot 2 = 1.2$$

$$1.1101010000 \cdot 10^2 \cdot 1.1100001100111 \cdot 10^{-3}$$

$$1.1101010000000$$

$$\times 1.1100001100111$$

$$11101010000000$$

$$11101010000000$$

$$11101010000000$$

$$11101010000000$$

$$11101010000000$$

$$11101010000000$$

$$11101010000000$$

$$11101010000000$$

$$11.00111000111000100110000000 \cdot 10^{-1}$$

16-bit floating format: 1100001001100100 Decimal: -1.61328125

There is no overflow as $16 \geq -1 \geq -15$, and my answer is relatively close to the correct answer of -1.61103515625