Fibonacci Research Paper

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Abstract- **The Fibonacci Sequence is a sequence of numbers where the nth digit in the sequence is generated by adding the two previous digits in the sequence which can be denoted as $F(n) = (n-1) + (n-2)$ where $F(0) = 0$ and $F(1) = 1$. The generation of this sequence is often used in computer science to teach run-time analysis and showcase different optimization methods. These optimization methods often include using different data structures, avoiding recursion, or adding support for the use of multiple threads. In this project, we will be analyzing 3 different optimization methods (top-down dynamic programming, bottom-up dynamic programming, and the use of matrices) and analyze how the inclusion of multi-threading can improve or inhibit the run time of these algorithms on the generation of a Fibonacci Sequence.**

I. Introduction

The Fibonacci Sequence is a sequence of Fibonacci numbers (F_n) in which each number is the sum of the two preceding numbers. The first two entries of the sequence are commonly considered to be 0 and 1 although some people including Fibonacci himself start from 1 and 1.

For the purposes of this paper we will define the Fibonacci numbers by the following recurrence relation for $n \in \mathbb{N}$:

$$F_n = \begin{cases} 0, & \text{if n = 0} \\ 1, & \text{if n = 1} \\ F_{n-1} + F_{n-2} & \text{if n > 1} \end{cases}$$

As time has gone on more and more optimized approaches to calculating Fibonacci numbers were developed where often time, the new methodology was built off the previous, similar in nature to the Fibonacci Sequence itself.

A. Problem Statement

Integrating multithreading with optimized Fibonacci algorithms will provided a significant run-time boost to finding the nth term of the Fibonacci Sequence. We will be using the naive recursive algorithm as our benchmark algorithm to which compare our multithreaded approaches. We seek to provide further insight into the effectiveness of various non-concurrent algorithms for calculating the Fibonacci Sequence when compared to their concurrent counterparts, and additionally. provide insight for individuals wondering if the trade-off of implementing the required overhead for various concurrent algorithms for calculating the Fibonacci Sequence is worth the change in run-time.

II. Motivation

The Fibonacci numbers appear often enough in nature that they are impossible to ignore. This means that mathematicians, engineers, and other professions frequently find themselves using Fibonacci numbers. The only issue with this is the Fibonacci sequence , while not strictly exponential, exhibits exponential-like growth between numbers. As such the larger the n the exponentially more computation power it takes to calculate F_n. Our hope is that by integrating multithreading with some of the more common optimized Fibonacci algorithms we can further improve the overall run-time for larger values in the sequence, and in doing so, provide a way for algorithms relying on the Fibonacci Sequence to be improved as well.

III. Implementation

The following is the breakdown of tasks for this project:

- Write the Fibonacci algorithm
  - Using naive approach
  - Using top-down dynamic programming (memoization recurssive)
  - Using bottom-up dynamic programming (iterative)
  - Using matrices
- Rewrite each Fibonacci algorithm with multi-threading
- Create a testing environment to evaluate performance based on a variety of benchmarks

A. Plan to Implement

For each Fibonacci algorithm we plan to write it up and then either write up a mutlithread version following similar logic or write an explanation as to why a multithread approach is impossible for said algorithm.

For the multithread approaches we plan to create a thread that will calculate the Fibonacci sequence and place the sequence in a vector that is shared amongst the threads. As the order is sequential the parent thread will output the child threads value but will have to wait for the child thread to first finish. This will facilitate an easy data structure to either output the whole sequence of fibonacci number up to an index n or just to output the fibonacci number at the index n.

As we would like to test the algorithms performance on calculating relatively large Fibonacci numbers, a larger data type than the commonly available data types in C++ is needed. For initial testing we will be using long long int which only allows calculating up to the 92nd Fibonacci number. To address this we plan to create a class that will dynamically create and add bit sets with some type of if statement to identify overflows , in which case it will add a bit set of a higher dimension after the former bit set.

B. Challenges

We anticipated the following problems to arise: overflow from large Fibonacci numbers, exponential growth becomes more apparent with higher numbers.

Overflow is a simple result of the data type that we are using to store the Fibonacci numbers not being large enough so the problem can only be mitigated by implementing a new data type.

As already mentioned Fibonacci is not strictly exponential; however, it is exponential like as such we can assume that larger Fibonacci number calculations will result in longer and longer runtimes.

An unexpected problem that was thankfully highlighted by our professor was the issue with the dynamic programming Fibonacci algorithm. As the algorithm is linear in nature the max number of threads that can be used does not allow for multithreading as such this and the memoization algorithms will simply be baselines without a multithread equivalent algorithm.

IV. Testing

The following is the breakdown of how testing will work Record the time of each algorithm to generate n Fibonacci numbers

- Each test will output data in the following form

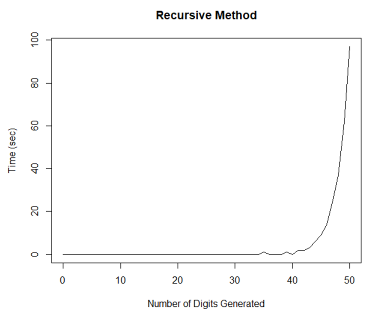
Number of Digits Generated	Time
n	x seconds

- Each test will be put through an R script to:
  - Generate a graph to visualize each individual test
  - Generate graphs that compare each method to the naive method (control)
  - Generate graphs that compare each method with their multi-threaded counterpart
  - Run a statistical analysis on whether implementing multi-threading had a notable impact on efficiency

V. Evaluation

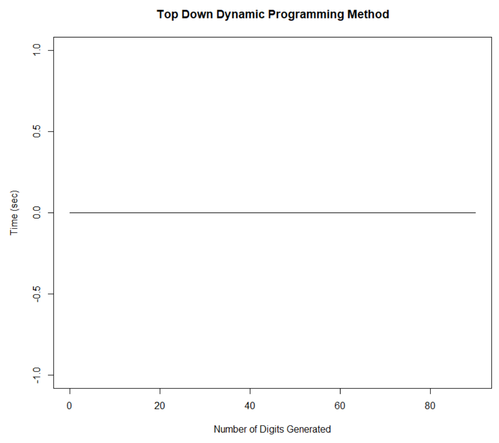
Algorithm 1: Naive Fibonacci

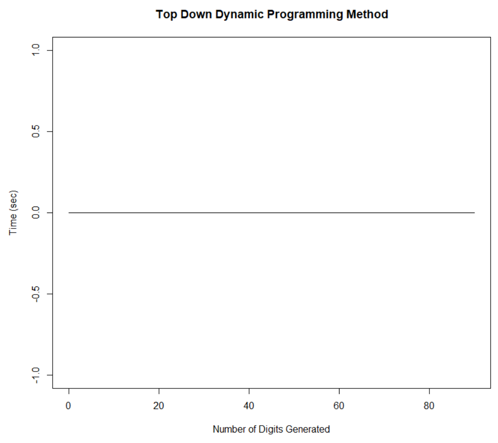
```
Data: $n \geq 0$
fib(n):
if $n \leq 2$ then
  | Result: n
end
Result: fib(n-1) + fib(n-2)
```



Algorithm 2: Top-Down Dynamic Programming Fibonacci

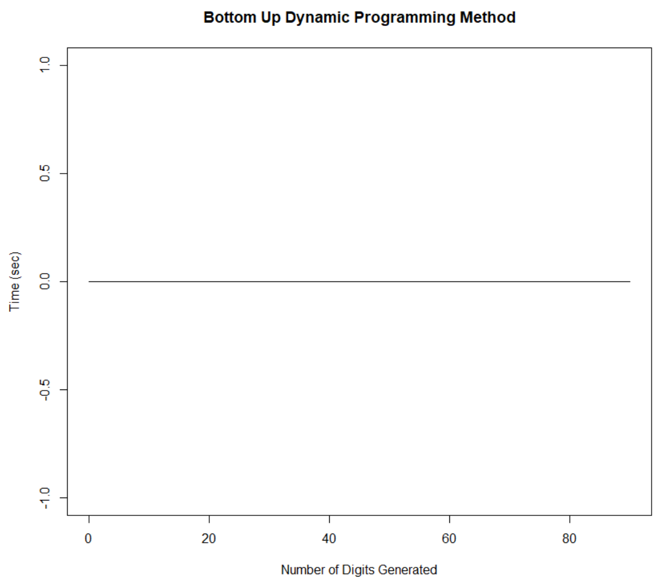
```
Data: $n \geq 0$
Result: $F$
```





Algorithm 3: Bottom-Up Dynamic Programming Fibonacci

```
Data: $n \geq 0$
Result: $F$
```



Algorithm 4: Fibonacci with Matrices

```
Data: $n \geq 0$, $F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
if $n == 0$ then
    Result: 0
end
$M \leftarrow F$
$i \leftarrow 2$
while $i \leq (n-1)$ do
    $F_{00} \leftarrow F_{00} \cdot M_{00}$
    $F_{01} \leftarrow F_{01} \cdot M_{01}$
    $F_{10} \leftarrow F_{10} \cdot M_{10}$
    $F_{11} \leftarrow F_{11} \cdot M_{11}$
    $i \leftarrow i + 1$
end
Result: $F_{00}$
```