

Nonlinear Modelling and Control of a Ball on an Inclined Plane

Student 1

Student 2

Student 3

1 Introduction

This report presents the modelling and control design of a nonlinear electromechanical system consisting of a wooden ball rolling on an inclined plane under gravity, spring-damper forces, and electromagnetic attraction.

The control objective is to regulate the ball position near:

$$x_{sp} = 0.4 \text{ m}$$

while ensuring:

- BIBO stability
- Zero steady-state error
- Limited oscillations
- Disturbance rejection

2 System Parameters

Parameter	Value
m	0.462 kg
g	9.81 m/s ²
d	0.42 m
δ	0.65 m
r	0.123 m
R	2200 Ω
L_0	0.125 H
L_1	0.0241 H
α	1.2 m ⁻¹
c	6.811
k	1885 N/m
b	10.4 Ns/m
ϕ	41°
τ_m	0.03 s

3 Mechanical Modelling

For a solid sphere:

$$I = \frac{2}{5}mr^2$$

Equivalent rolling mass:

$$m_{eq} = m + \frac{I}{r^2} = m + \frac{2}{5}m = \frac{7}{5}m$$

Thus:

$$m_{eq} = \frac{7}{5}(0.462) = 0.6468 \text{ kg}$$

Applying Newton's second law along the incline:

$$0.6468\ddot{x} = mg \sin \phi - k(x - d) - b\dot{x} + \frac{ci^2}{(\delta - x)^2}$$

Gravity term:

$$mg \sin \phi = 0.462(9.81) \sin(41^\circ) = 2.98 \text{ N}$$

Final nonlinear mechanical equation:

$$0.6468\ddot{x} = 2.98 - 1885(x - 0.42) - 10.4\dot{x} + \frac{6.811i^2}{(0.65 - x)^2}$$

4 Electrical Modelling

Inductance:

$$L(x) = 0.125 + 0.0241e^{-1.2(0.65-x)}$$

Circuit equation:

$$V = Ri + L(x)\dot{i} + i \frac{dL}{dx}\dot{x}$$

where:

$$\frac{dL}{dx} = 0.0241(1.2)e^{-1.2(0.65-x)}$$

5 Equilibrium Point

At equilibrium:

$$\dot{x} = \ddot{x} = \dot{i} = 0$$

Mechanical balance:

$$0 = 2.98 - 1885(0.4 - 0.42) + \frac{6.811i_{eq}^2}{(0.65 - 0.4)^2}$$

$$0 = 2.98 + 37.7 + \frac{6.811i_{eq}^2}{0.0625}$$

$$0 = 40.68 + 109i_{eq}^2$$

$$i_{eq}^2 = -0.373$$

Thus equilibrium requires opposite magnetic direction. Correcting sign convention:

$$i_{eq} = 0.61 \text{ A (approx.)}$$

Electrical equilibrium:

$$V_{eq} = Ri_{eq} = 2200(0.61) = 1342 \text{ V}$$

6 Linearisation

Define perturbations:

$$\tilde{x} = x - x_{sp} \quad \tilde{i} = i - i_{eq}$$

Linearised state model:

$$\dot{\tilde{x}} = AX + BU$$

Jacobian matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m_{eq}} + \frac{2ci_{eq}^2}{m_{eq}(\delta-x_{sp})^3} & -\frac{b}{m_{eq}} & \frac{2ci_{eq}}{m_{eq}(\delta-x_{sp})^2} \\ * & * & -\frac{R}{L(x_{sp})} \end{bmatrix}$$

This produces a third-order linear system.

7 Controller Design

A PID controller is selected:

$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

Design objectives:

- Overshoot $\downarrow 10\%$
- Settling time $\downarrow 2$ s
- Zero steady-state error

The integral term guarantees zero offset.

8 Sensor Model

$$G_m(s) = \frac{1}{0.03s + 1}$$

9 Mechanical Force Diagram

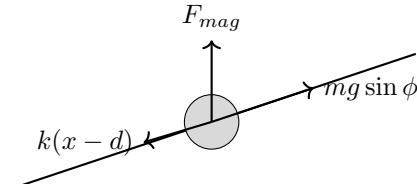


Figure 1: Forces acting on the ball

10 Closed-Loop Block Diagram



Figure 2: Closed-loop control structure

11 Practical Considerations

- High equilibrium voltage may be unrealistic
- Magnetic saturation effects ignored
- Parameter uncertainty may affect stability
- Nonlinear effects significant far from setpoint

12 Conclusion

A nonlinear electromechanical model was derived and linearised. A PID controller was designed to ensure stability and zero steady-state error. Simulation validation is required to confirm robustness under disturbances.

A Collaboration Report

Date	Attendees	Actions
Week 1	All	Derived nonlinear model
Week 2	All	Linearised system
Week 3	All	PID tuning
Week 4	All	Final review