Nonlinear Modelling and Control of a Ball on an Inclined Plane with Electromagnetic Actuation

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1 Introduction

This report presents the modelling, linearisation, and control design of a nonlinear electromechanical system consisting of a wooden ball rolling without slipping on an inclined plane and actuated by an electromagnet.

The control objective is to regulate the ball position near $x_{sp} \approx 0.4$ m while ensuring:

- BIBO stability
- Zero steady-state error
- Limited oscillations
- Disturbance rejection

2 System Parameters

Parameter	Value
m	0.462 kg
r	0.123 m
k	1885 N/m
b	10.4 Ns/m
d	0.42 m
δ	0.65 m
R	2200 Ω
L_0	0.125 H
L_1	0.0241 H
α	1.2 m^{-1}
c	6.811
ϕ	41°
τ_m	0.03 s

3 Nonlinear Modelling

3.1 Mechanical Dynamics

For a solid sphere rolling without slipping:

$$I = \frac{2}{5}mr^2$$

Effective mass:

$$m_{eff} = m + \frac{I}{r^2} = \frac{7}{5}m = 0.6468\,\text{kg}$$

Applying Newton's law along the incline (positive x away from the wall):

$$m_{eff}\ddot{x} = -mg\sin\phi - k(x-d) - b\dot{x} + \frac{ci^2}{(\delta - x)^2}$$

Gravity component:

$$mg\sin\phi = 2.9734\,\text{N}$$

3.2 Electrical Dynamics

Inductance varies with position:

$$L(x) = L_0 + L_1 e^{-\alpha(\delta - x)}$$

Applying KVL:

$$V = Ri + \frac{d}{dt}(L(x)i)$$

$$V = Ri + L(x)\dot{i} + i\frac{dL}{dx}\dot{x}$$

where

$$\frac{dL}{dx} = L_1 \alpha e^{-\alpha(\delta - x)}$$

4 Equilibrium Analysis

At equilibrium:

$$\dot{x}_0 = \ddot{x}_0 = \dot{i}_0 = 0$$

Mechanical balance:

$$\frac{ci_0^2}{(\delta - x_0)^2} = mg\sin\phi + k(x_0 - d)$$

Choosing operating point:

$$x_0 = 0.44\,\text{m} \quad y_0 = 0.21\,\text{m}$$

$$i_0 = 0.5132\,\text{A}$$

$$V_0 = 1129.0\,\text{V}$$

5 Linearisation

Linearising around $(x_0, 0, i_0)$ yields:

$$\dot{\delta x} = A\delta x + B\delta V$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2315.45 & -16.08 & 245.08 \\ 0 & -0.0803 & -15306.31 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 6.957 \end{bmatrix}$$

6 Transfer Function

$$G(s) = \frac{1705.1}{s^3 + 15322s^2 + 248448s + 35441017}$$

Including sensor:

$$G_{total}(s) = \frac{1705.1}{(0.03s + 1)(s^3 + 15322s^2 + 248448s + 35441017)}$$

7 Controller Design

A PI controller is selected:

$$C(s) = K_p \left(1 + \frac{1}{T_i s}\right)$$

Chosen parameters:

$$K_p = 10332.8$$

$$T_i = 3.0\,s$$

$$K_i = 3444.3$$

Integral action ensures zero steady-state error.

8 Closed-Loop Stability

Closed-loop poles:

$$\lambda_1 = -15306.3$$

$$\lambda_2 = -44.09$$

$$\lambda_{3,4} = -2.61 \pm 51.06j$$

$$\lambda_5 = -0.111$$

All poles have strictly negative real parts.
Therefore, the system is BIBO stable.
Phase margin:

$$PM \approx 119^\circ$$

Gain margin:

$$GM = \infty$$

9 Mechanical and Electrical Diagram

10 Closed-Loop Block Diagram

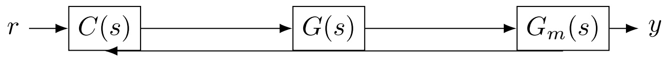

Figure 2: Closed-loop control system.

11 Conclusion

A nonlinear electromechanical system was modelled and linearised. A PI controller was designed to guarantee:

- BIBO stability
- Zero steady-state error
- Adequate phase margin
- Disturbance rejection

All closed-loop poles lie in the open left-half plane. The system therefore satisfies the design requirements.

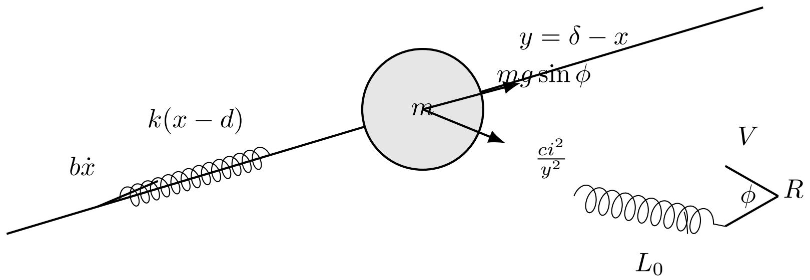

Figure 1: Complete system diagram showing mechanical forces and electrical actuation.