Sound & Light (1)

1.1 Miscellaneous

$$\% \text{ error} = \frac{\text{observed} - \text{theoretical}}{\text{theoretical}} * 100\%$$

1.2 Kinematics

$$x = \frac{a}{2}(\Delta t)^2 + v_0\Delta t + x_0 \qquad v = v_0 + a\Delta t$$

$$v^2 = v_0^2 + 2a\Delta x \qquad \Delta x = \frac{v_0 + v}{2} * \Delta t$$

1.3 Simple Harmonic Motion

$$x = A\cos(\omega t + \varphi) \qquad v = -\omega A\cos(\omega t + \varphi) \qquad a = -\omega^2 A\cos(\omega t + \varphi)$$

$$x_{\max} = A \qquad v_{\max} = \omega A \qquad a_{\max} = \omega^2 A \qquad F_{\max} = m\omega^2 A$$

1.3.1 Springs and Slinkies

$$F_s = kx \qquad F_{s_{\max}} = kx_0 = mg$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}} \qquad \omega = 2\pi f = \sqrt{\frac{m}{k}}$$

$$SPE = \frac{1}{2}kx^2 \qquad KE = \frac{1}{2}mv^2$$

$$TME = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2$$

1.3.2 Pendulums

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

1.4 Waves

$$T = \frac{1}{f} \qquad v = \lambda f \qquad v = \frac{\Delta x}{\Delta t}$$

1.4.1 Slinkies and strings with fixed ends

$$F_T = F_s = kx \qquad \mu = \frac{m}{L} \qquad v = \sqrt{\frac{F_T}{\mu}}$$

Given mass m_T hanging below a pulley, $F_T = m_T g$.

1.5 Standing waves

1.5.1 Open-open, closed-closed

n is the number of antinodes, or the n^{th} harmonic.

$$f_n = f_1 n = \frac{nv}{2L} \qquad f_1 = \frac{v}{2L} \qquad \lambda_n = \frac{2L}{n}$$

1.5.2 Open-closed

$$f_n = f_1 n = \frac{nv}{4L} \qquad f_1 = \frac{v}{4L} \qquad \lambda_n = \frac{2L}{n}$$

1.5.3 End correction

Although a theoretical air column will have a frequency only dependent on the velocity and length of the column, the diameter of the air column plays an effect in the real world.

In an open-open pipe with diameter d:

$$f_n = f_1 n = \frac{nv}{2(L + 0.6d)} \qquad f_1 = \frac{v}{2(L + 0.6d)} \qquad \lambda_n = \frac{2(L + 0.6d)}{n}$$

In an open-closed pipe with diameter d:

$$f_n = f_1 n = \frac{nv}{4(L + 0.6d)} \qquad f_1 = \frac{v}{4(L + 0.6d)} \qquad \lambda_n = \frac{4(L + 0.6d)}{n}$$

1.6 Sound

1.6.1 Speed of sound

$$v = 331\sqrt{\frac{T_{^\circ\mathrm{C}} + 273}{273}} \qquad v \approx 331 + 0.59T$$

1.6.2 Sound intensity

$$I = \frac{\text{Power (W)}}{\text{Area}} = \frac{\text{Power (W)}}{4\pi r^2}$$

$$I_{\mathrm{dB}} = 10\log_{10}\left(\frac{I}{10^{-12}}\right) \qquad I = 10^{\frac{I_{\mathrm{dB}}}{10} - 12}$$

1.6.3 Doppler effect

$$\boxed{O}\rightarrow \quad \boxed{S} \qquad f_o = f_s\frac{v + v_o}{v} \qquad\qquad \leftarrow\boxed{O} \quad \boxed{S} \qquad f_o = f_s\frac{v - v_o}{v}$$

$$\boxed{O} \quad \leftarrow\boxed{S} \qquad f_o = f_s\frac{v}{v - v_s} \qquad\qquad \boxed{O} \quad \boxed{S}\rightarrow \qquad f_o = f_s\frac{v}{v + v_s}$$

$$\boxed{O}\rightarrow \quad \leftarrow\boxed{S} \qquad f_o = f_s\frac{v + v_o}{v - v_s} \qquad\qquad \boxed{O}\rightarrow \quad \boxed{S}\rightarrow \qquad f_o = f_s\frac{v + v_o}{v + v_s}$$

$$\leftarrow\boxed{O} \quad \leftarrow\boxed{S} \qquad f_o = f_s\frac{v - v_o}{v - v_s} \qquad\qquad \leftarrow\boxed{O} \quad \boxed{S}\rightarrow \qquad f_o = f_s\frac{v - v_o}{v + v_s}$$

1.6.4 Constructive and Destructive Interference (2 dimensions)

For a point on the m^{th} antinodal/nodal line playing the same frequency with the same phase:

$$PD = m\lambda$$

where PD is the path length difference.

1.6.5 Beats

$$f_B = \Delta f$$

Mathematics (2)

2.1 Notation

deg $p(x)$ means the degree of polynomial p.

LC $p(x)$ means the leading coefficient of polynomial p.

2.2 Rational functions

For a rational function $f(x) = \frac{p(x)}{q(x)}$, cancel out any common factors, then:

- When deg $p(x) =$ deg $q(x)$:
  - HA: $y = \frac{\text{LC } p(x)}{\text{LC } q(x)}$
  - VA: roots of $q(x)$
- When deg $p(x) <$ deg $q(x)$:
  - HA: $y = 0$
  - x-intercept: roots of $p(x)$
  - VA: roots of $q(x)$
- When deg $p(x) >$ deg $q(x)$:
  - HA: none
  - slant asymptote: $\frac{p(x)}{q(x)}$ excluding remainder
  - VA: roots of $q(x)$

2.3 Common functions

2.3.1 Linear equations

$$\text{Slope-intercept form: } y = mx + b$$
$$\text{Point-slope form: } y - y_1 = m(x - x_1) \text{ for point } (x, y)$$
$$\text{Standard form: } ax + by = c$$

2.3.2 Quadratic equations

$$\text{Standard form: } y = ax^2 + bx + c$$
$$\text{Vertex form: } y = a(x - h)^2 + k \text{ for vertex } (h, k)$$

2.4 Sequences and Series

2.4.1 Explicit formulas

$$\text{Aritmetic sequence: } a_n = a_1 + r(n - 1)$$
$$\text{Geometric sequence: } a_n = a_1 * r^{n-1}$$
$$\text{Harmonic sequence: } a_n = \frac{1}{a_1 + r(n - 1)}$$

2.4.2 Arithmetic and Geometric Series

$$\sum_{j=1}^{n}(a_1 + r(n-1)) = \frac{n}{2}(2a_1 + (n-1)d)$$
$$\sum_{j=1}^{n}(a_1 * r^{n-1}) = \frac{a_1(1 - r^n)}{1 - r}$$
$$\sum_{j=1}^{\infty}(a_1 * r^{n-1}) = \frac{a_1}{1 - r} \text{ for } r \in [-1, 1]$$

2.4.3 Special Sums

$$\sum_{j=1}^{n} c = nc$$
$$\sum_{j=1}^{n} ca_j = c\sum_{j=1}^{n} a_j$$
$$\sum_{j=1}^{n}(a_j + b_j) = \sum_{j=1}^{n} a_j + \sum_{j=1}^{n} b_j$$
$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$
$$\sum_{j=1}^{n} j^2 = \frac{n(n+\frac{1}{2})(n+1)}{3} = \frac{n(2n+1)(n+1)}{6}$$
$$\sum_{j=1}^{n} j^3 = \frac{n^2(n+1)^2}{4}$$
$$\sum_{j=1}^{n} j^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$