Sound & Light (1)

1.1 Miscellaneous

$$\%\text{ error} = \frac{\text{observed} - \text{theoretical}}{\text{theoretical}} * 100\%$$

1.2 Kinematics

$$x = \frac{a}{2}(\Delta t)^2 + v_0\Delta t + x_0 \qquad v = v_0 + a\Delta t$$

$$v^2 = v_0^2 + 2a\Delta x \qquad \Delta x = \frac{v_0 + v}{2} * \Delta t$$

1.3 Simple Harmonic Motion

$$x = A\cos(\omega t+\varphi) \quad v = -\omega A\cos(\omega t+\varphi) \quad a = -\omega^2 A\cos(\omega t+\varphi)$$

$$x_{\max} = A \qquad v_{\max} = \omega A \qquad a_{\max} = \omega^2 A \qquad F_{\max} = m\omega^2 A$$

1.3.1 Springs and Slinkies

$$F_s = kx \qquad F_{s_{\max}} = kx_0 = mg$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}} \qquad \omega = 2\pi f = \sqrt{\frac{m}{k}}$$

$$SPE = \frac{1}{2}kx^2 \qquad KE = \frac{1}{2}mv^2$$

$$TME = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2$$

1.3.2 Pendulums

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

1.4 Waves

$$T = \frac{1}{f} \qquad v = \lambda f \qquad v = \frac{\Delta x}{\Delta t}$$

1.4.1 Slinkies and strings with fixed ends

$$F_T = F_s = kx \qquad \mu = \frac{m}{L} \qquad v = \sqrt{\frac{F_T}{\mu}}$$

Given mass m_T hanging below a pulley, $F_T = m_T g$.

1.5 Standing waves

1.5.1 Open-open, closed-closed

n is the number of antinodes, or the n^{th} harmonic.

$$f_n = f_1 n = \frac{nv}{2L} \qquad f_1 = \frac{v}{2L} \qquad \lambda_n = \frac{2L}{n}$$

1.5.2 Open-closed

$$f_n = f_1 n = \frac{nv}{4L} \qquad f_1 = \frac{v}{4L} \qquad \lambda_n = \frac{2L}{n}$$

1.6 Sound

1.6.1 Speed of sound

$$v = 331\sqrt{\frac{T_{^\circ\mathrm{C}} + 273}{273}} \qquad v \approx 331 + 0.59T$$

1.6.2 Sound intensity

$$I = \frac{\text{Power (W)}}{\text{Area}} = \frac{\text{Power (W)}}{4\pi r^2}$$

$$I_{\mathrm{dB}} = 10\log_{10}\left(\frac{I}{10^{-12}}\right) \qquad I = 10^{\frac{I_{\mathrm{dB}}}{10} - 12}$$

1.6.3 Doppler effect

$$\boxed{O}\rightarrow \ \boxed{S} \quad f_o = f_s\frac{v + v_o}{v} \qquad \leftarrow\boxed{O}\ \boxed{S} \quad f_o = f_s\frac{v - v_o}{v}$$

$$\boxed{O}\ \leftarrow\boxed{S} \quad f_o = f_s\frac{v}{v - v_s} \qquad \boxed{O}\ \boxed{S}\rightarrow \quad f_o = f_s\frac{v}{v + v_s}$$

$$\boxed{O}\rightarrow \ \leftarrow\boxed{S} \quad f_o = f_s\frac{v + v_o}{v - v_s} \qquad \boxed{O}\rightarrow \ \boxed{S}\rightarrow \quad f_o = f_s\frac{v + v_o}{v + v_s}$$

$$\leftarrow\boxed{O}\ \leftarrow\boxed{S} \quad f_o = f_s\frac{v - v_o}{v - v_s} \qquad \leftarrow\boxed{O}\ \boxed{S}\rightarrow \quad f_o = f_s\frac{v - v_o}{v + v_s}$$

1.6.4 Constructive and Destructive Interference (2 dimensions)

For a point on the m^{th} antinodal/nodal line playing the same frequency with the same phase:

$$PD = m\lambda$$

where PD is the path length difference.

1.6.5 Beats

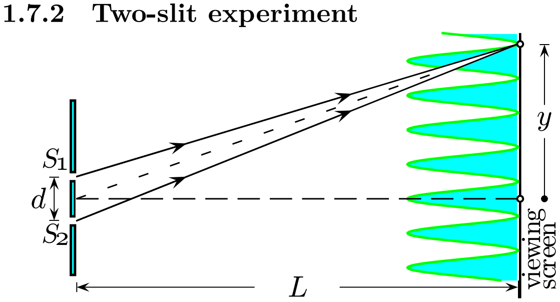
$$f_B = \Delta f$$

1.7 Light

1.7.1 Speed of light

$$c = 299\ 792\ 458\frac{\mathrm{m}}{\mathrm{s}} \approx 3 * 10^8\frac{\mathrm{m}}{\mathrm{s}}$$

1.7.2 Two-slit experiment



$$PD = \frac{dy}{L} = m\lambda$$

1.7.3 Mirror

	Meaning		*)	*|	*(
r	radius	m	+	inf	−
f	focal length	m	+	inf	−
p	object distance	m	+	+	+
q	image distance	m	±	−	−
h_o	object height	m	+	+	+
h_i	image height	m	±	−	−
M	magnification	$\frac{m}{m}$	±	−	−

$$r = 2f \qquad \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad M = \frac{h}{h_o} = \frac{-q}{p}$$

In a plane mirror, $p = -q$.

1.7.4 Lenses

converging () with wider middle ⇒ positive focal length, diverging)(with thinner middle ⇒ negative focal length.

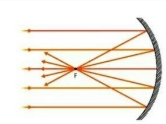
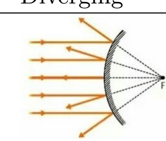
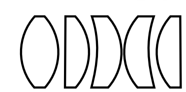
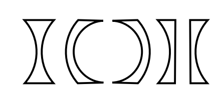
$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad M = \frac{h}{h_o} = \frac{-q}{p}$$

Multiple lenses

$$p_2 = \Delta x - q_1$$

1.7.5 Mirrors and Lenses

Real ⟺ inverted, virtual ⟺ upright.

	Converging	Diverging
Mirror		
Lens		
focal length	+	-
object distance	image	image
∞ to $2f$	Real smaller	Virtual smaller
$2f$ to f	Real larger	
f to 0	Virtual larger	
0 to $-f$	Virtual smaller	Virtual larger
$-f$ to $-2f$		Real larger
$-2f$ to $-\infty$		Real smaller

1.7.6 Refraction / Snell's Law

The **normal line** is the line perpendicular to the surface which touches the intersection of the surface and the light ray.

The **incident angle** is the angle between the ray of light and the normal line.

θ_1 and θ_2 are both measured from the normal line, not the surface.

Refraction occurs when the speed of light in two media are different and light hits the boundary of the two media. The frequency of the light will stay the same, but the speed, wavelength, and direction will change.

$$n = \frac{c}{v} \qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Mathematics (2)

2.1 Sequences and Series

2.1.1 Explicit formulas

$$\text{Aritmetic sequence: } a_n = a_1 + r(n-1)$$
$$\text{Geometric sequence: } a_n = a_1 * r^{n-1}$$
$$\text{Harmonic sequence: } a_n = \frac{1}{a_1 + r(n-1)}$$

2.1.2 Arithmetic and Geometric Series

In the following equations, substituting $j = 1$ with $j = 0$, $j - 1$ with j, and a_1 with a_0 will produce the same result.

$$\sum_{j=1}^{n}(a_1 + r(j-1)) = \frac{n}{2}(2a_1 + (n-1)d)$$
$$\sum_{j=1}^{n}(a_1 * r^{j-1}) = \frac{a_1(1-r^n)}{1-r}$$
$$\sum_{j=1}^{\infty}(a_1 * r^{j-1}) = \frac{a_1}{1-r} \text{ for } r \in [-1, 1]$$

2.1.3 Special Sums

$$\sum_{j=1}^{n} c = nc \qquad \sum_{j=1}^{n} ca_j = c\sum_{j=1}^{n} a_j$$
$$\sum_{j=1}^{n}(a_j + b_j) = \sum_{j=1}^{n} a_j + \sum_{j=1}^{n} b_j \qquad \sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$
$$\sum_{j=1}^{n} j^2 = \frac{n(n+\frac{1}{2})(n+1)}{3} \qquad \sum_{j=1}^{n} j^3 = \frac{n^2(n+1)^2}{4}$$

2.2 Trigonometry

$^\circ$	rad	sin	cos	tan
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	undef

2.2.1 Law of Sines and Cosines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \qquad c^2 = a^2 + b^2 - 2ab\cos(C)$$

2.2.2 Triangle area

$$K = \frac{1}{2}bh \qquad K = \frac{1}{2}bc\sin(A) \qquad K = \sqrt{s(s-a)(s-b)(s-c)}$$

2.2.3 Pythagorean identities

$$(\sin A)^2 + (\cos A)^2 = 1 \qquad (\tan A)^2 + 1 = (\sec A)^2$$
$$(\cot A)^2 + 1 = (\csc A)^2$$

2.2.4 Odd and even functions

$$\cos(-x) = \cos(x) \quad \sin(-x) = \sin(x) \quad \tan(-x) = \tan(x)$$

2.2.5 Co-functions

$$\sin(\frac{\pi}{2} - x) = \cos(x)$$

2.2.6 Slope

Where α is the angle between the line and the x-axis, and m is the slope of the line:

$$\alpha = \tan m$$

2.2.7 Sum and difference formulas

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\tan(A+B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}$$
$$\tan(A-B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$$

2.2.8 Double-angle formulas

$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\cos(2A) = (\cos A)^2 - (\sin A)^2 = 2(\cos A)^2 - 1 = 1 - 2(\sin A)^2$$
$$\tan(2A) = \frac{2\tan(A)}{1 - (\tan A)^2}$$

2.3 Vectors

2.3.1 Vector arithmetic

$$\vec{v} + \vec{w} = \begin{bmatrix} v_x + w_x \\ v_y + w_y \\ v_z + w_z \end{bmatrix} \qquad c * \vec{v} = \begin{bmatrix} c * v_x \\ c * v_y \\ c * v_z \end{bmatrix}$$
$$\vec{v} \cdot \vec{w} = v_x + v_y + v_z = |\vec{v}||\vec{w}|\cos(\theta)$$
$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix} \qquad |\vec{v} \times \vec{w}| = |\vec{v}||\vec{w}|\sin(\theta)$$
$$\vec{v} \times \vec{w} \perp \vec{v} \qquad \vec{v} \times \vec{w} \perp \vec{w} \qquad \vec{v} \parallel \vec{w} \iff \vec{v} \cdot \vec{w} = 0$$
$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} \qquad \text{proj}_{\vec{b}}\vec{v} = \frac{\vec{b} \cdot \vec{v}}{\vec{b} \cdot \vec{b}} * \vec{b} = (|\vec{v}|\cos(\theta))$$

Right-hand rule

To determine the direction of $\vec{v} \times \vec{w}$, put the pinky of the right hand on $\vec{v}$ and curl it toward $\vec{w}$.

2.4 Polar

2.4.1 Polar and Cartesian sytems

With point $(x, y) = (r; \theta) = (r; \beta)$, where θ is CCW from the x-axis and β is a bearing, CW from the y-axis:

$$x = r\cos(\theta) = r\sin(\beta) \qquad y = r\sin(\theta) = r\cos(\beta)$$
$$r = \sqrt{x^2 + y^2} \qquad \theta \equiv \arctan(\frac{y}{x}) \quad \beta \equiv \arctan(\frac{x}{y})$$

2.4.2 Limaçons and Petals

The function $y = A\cos(B(\theta + C)) + D$ is equivalent to $y = A\cos(B\theta) + D$ rotated C degrees/radians clockwise.

When C is 0 and B is 1, the x-intercepts are $A \pm D$ and the y-intercepts are $\pm D$, and it forms a limaçon.

When C is 0, but $B \neq 1$, then this sometimes still holds. The x-intercepts may also be $\pm A \pm D$.

There are B petals, with the axis of the first petal on the positive x-axis.

When B is even and $|D| < 1$, then the number of petals is $2B$.

Using sin instead of cos, limaçons have their axes on the positive y-axis, while for petals, the first petal starts from the positive x-axis and curves upwards.

2.5 Complex

$$\text{cis}(\theta) = e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

To find the n^{th} root of $x_r\,\text{cis}(x_\theta)$, solve the equation $z_r^n\,\text{cis}(nz_\theta) = x_r\,\text{cis}(x_\theta + 360^\circ k)$ for $k \in \mathbb{R}$.