Sound & Light (1)

1.1 Miscellaneous

$$\%\ \text{error} = \frac{\text{observed} - \text{theoretical}}{\text{theoretical}} * 100\%$$

1.2 Kinematics

$$x = \frac{a}{2}(\Delta t)^2 + v_0\Delta t + x_0 \qquad v = v_0 + a\Delta t$$

$$v^2 = v_0^2 + 2a\Delta x \qquad \Delta x = \frac{v_0 + v}{2} * \Delta t$$

1.3 Simple Harmonic Motion

$$x = A\cos(\omega t+\varphi) \quad v = -\omega A\cos(\omega t+\varphi) \quad a = -\omega^2 A\cos(\omega t+\varphi)$$

$$x_{\max} = A \qquad v_{\max} = \omega A \qquad a_{\max} = \omega^2 A \qquad F_{\max} = m\omega^2 A$$

1.3.1 Springs and Slinkies

$$F_s = kx \qquad F_{s_{\max}} = kx_0 = mg$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}} \qquad \omega = 2\pi f = \sqrt{\frac{m}{k}}$$

$$SPE = \frac{1}{2}kx^2 \qquad KE = \frac{1}{2}mv^2$$

$$TME = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2$$

1.3.2 Pendulums

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

1.4 Waves

$$T = \frac{1}{f} \qquad v = \lambda f \qquad v = \frac{\Delta x}{\Delta t}$$

1.4.1 Slinkies and strings with fixed ends

$$F_T = F_s = kx \qquad \mu = \frac{m}{L} \qquad v = \sqrt{\frac{F_T}{\mu}}$$

Given mass m_T hanging below a pulley, $F_T = m_T g$.

1.5 Standing waves

1.5.1 Open-open, closed-closed

n is the number of antinodes, or the n^{th} harmonic.

$$f_n = f_1 n = \frac{nv}{2L} \qquad f_1 = \frac{v}{2L} \qquad \lambda_n = \frac{2L}{n}$$

1.5.2 Open-closed

$$f_n = f_1 n = \frac{nv}{4L} \qquad f_1 = \frac{v}{4L} \qquad \lambda_n = \frac{2L}{n}$$

1.6 Sound

1.6.1 Speed of sound

$$v = 331\sqrt{\frac{T_{^\circ\mathrm{C}} + 273}{273}} \qquad v \approx 331 + 0.59T$$

1.6.2 Sound intensity

$$I = \frac{\text{Power (W)}}{\text{Area}} = \frac{\text{Power (W)}}{4\pi r^2}$$

$$I_{\mathrm{dB}} = 10\log_{10}\left(\frac{I}{10^{-12}}\right) \qquad I = 10^{\frac{I_{\mathrm{dB}}}{10} - 12}$$

1.6.3 Doppler effect

$$\boxed{O}\rightarrow \ \boxed{S} \quad f_o = f_s\frac{v + v_o}{v} \qquad \leftarrow\boxed{O}\ \boxed{S} \quad f_o = f_s\frac{v - v_o}{v}$$

$$\boxed{O}\ \leftarrow\boxed{S} \quad f_o = f_s\frac{v}{v - v_s} \qquad \boxed{O}\ \boxed{S}\rightarrow \quad f_o = f_s\frac{v}{v + v_s}$$

$$\boxed{O}\rightarrow \ \leftarrow\boxed{S} \quad f_o = f_s\frac{v + v_o}{v - v_s} \qquad \boxed{O}\rightarrow \ \boxed{S}\rightarrow \quad f_o = f_s\frac{v + v_o}{v + v_s}$$

$$\leftarrow\boxed{O}\ \leftarrow\boxed{S} \quad f_o = f_s\frac{v - v_o}{v - v_s} \qquad \leftarrow\boxed{O}\ \boxed{S}\rightarrow \quad f_o = f_s\frac{v - v_o}{v + v_s}$$

1.6.4 Constructive and Destructive Interference (2 dimensions)

For a point on the m^{th} antinodal/nodal line playing the same frequency with the same phase:

$$PD = m\lambda$$

where PD is the path length difference.

1.6.5 Beats

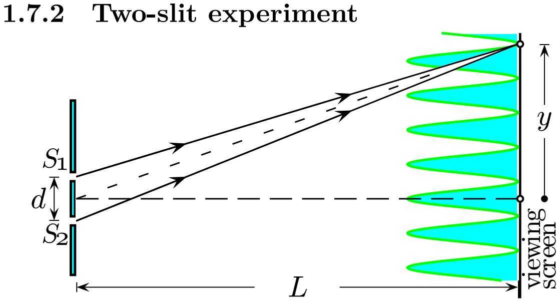
$$f_B = \Delta f$$

1.7 Light

1.7.1 Speed of light

$$c = 299\ 792\ 458\frac{\mathrm{m}}{\mathrm{s}} \approx 3 * 10^8\frac{\mathrm{m}}{\mathrm{s}}$$

1.7.2 Two-slit experiment



$$PD = \frac{dy}{L} = m\lambda$$

1.7.3 Mirror

The **normal line** is the line perpendicular to the mirror surface which touches the intersection of the surface and the light ray.

The **incident angle** is the angle between the ray of light and the normal line.

	Meaning		*)	*|	*(
r	radius	m	+	inf	−
f	focal length	m	+	inf	−
p	object distance	m	+	+	+
q	image distance	m	±	−	−
h_o	object height	m	+	+	+
h_i	image height	m	±	−	−
M	magnification	$\frac{m}{m}$	±	−	−

$$r = 2f \qquad \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad M = \frac{h}{h_o} = \frac{-q}{p}$$

In a plane mirror, $p = -q$.

1.7.4 Refraction / Snell's Law

Refraction occurs when the speed of light in two media are different and light hits the boundary of the two media. The frequency of the light will stay the same, but the speed, wavelength, and direction will change.

$$n = \frac{c}{v} \qquad n_1\sin\theta_1 = n_2\sin\theta_2$$

1.7.5 Lenses

converging () with wider middle ⇒ positive focal length, diverging)(with thinner middle ⇒ negative focal length.

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad M = \frac{h}{h_o} = \frac{-q}{p}$$

Multiple lenses

$$p_2 = \Delta x - q_1$$

Mathematics (2)

2.1 Notation

deg $p(x)$ means the degree of polynomial p.

LC $p(x)$ means the leading coefficient of polynomial p.

2.2 Rational functions

For a rational function $f(x) = \frac{p(x)}{q(x)}$, cancel out any common factors, then:

- When deg $p(x) = $ deg $q(x)$:
  - HA: $y = \frac{\text{LC } p(x)}{\text{LC } q(x)}$
  - VA: roots of $q(x)$
- When deg $p(x) <$ deg $q(x)$:
  - HA: $y = 0$
  - x-intercept: roots of $p(x)$
  - VA: roots of $q(x)$
- When deg $p(x) >$ deg $q(x)$:
  - HA: none
  - slant asymptote: $\frac{p(x)}{q(x)}$ excluding remainder
  - VA: roots of $q(x)$

2.3 Polynomials

2.3.1 Linear equations

$$\text{Slope-intercept form: } y = mx + b$$
$$\text{Point-slope form: } y - y_1 = m(x - x_1) \text{ for point } (x, y)$$
$$\text{Standard form: } ax + by = c$$

2.3.2 Quadratic equations

$$\text{Standard form: } y = ax^2 + bx + c$$
$$\text{Vertex form: } y = a(x-h)^2 + k \text{ for vertex } (h, k)$$
$$\text{Sum of roots: } \frac{-b}{a}$$
$$\text{Product of roots: } \frac{c}{a}$$

2.3.3 Higher-degree polynomials

In a polynomial

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$$

, with roots

$$r_1, r_2, r_3, \ldots, r_n$$

then:

$$r_1 + r_2 + r_3 + \cdots + r_n = \sum_{k=1} nr_k = -\frac{a_{n-1}}{a_n}$$

2.4 Sequences and Series

2.4.1 Explicit formulas

$$\text{Aritmetic sequence: } a_n = a_1 + r(n-1)$$
$$\text{Geometric sequence: } a_n = a_1 * r^{n-1}$$
$$\text{Harmonic sequence: } a_n = \frac{1}{a_1 + r(n-1)}$$

2.4.2 Arithmetic and Geometric Series

$$\sum_{j=1}^{n}(a_1 + r(n-1)) = \frac{n}{2}(2a_1 + (n-1)d)$$
$$\sum_{j=1}^{n}(a_1 * r^{n-1}) = \frac{a_1(1-r^n)}{1-r}$$
$$\sum_{j=1}^{\infty}(a_1 * r^{n-1}) = \frac{a_1}{1-r} \text{ for } r \in [-1, 1]$$

2.4.3 Special Sums

$$\sum_{j=1}^{n} c = nc$$
$$\sum_{j=1}^{n} ca_j = c\sum_{j=1}^{n} a_j$$
$$\sum_{j=1}^{n}(a_j + b_j) = \sum_{j=1}^{n} a_j + \sum_{j=1}^{n} b_j$$
$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$
$$\sum_{j=1}^{n} j^2 = \frac{n(n+\frac{1}{2})(n+1)}{3} = \frac{n(2n+1)(n+1)}{6}$$
$$\sum_{j=1}^{n} j^3 = \frac{n^2(n+1)^2}{4}$$
$$\sum_{j=1}^{n} j^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.5 Trigonometry

$^\circ$	rad	sin	cos	tan
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	*undef*

2.5.1 Double-Angle and Related Identities

Product-to-Sum Formulas

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$
$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$
$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$
$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

Sum-to-Product Formulas

$$\sin\alpha + \sin\beta = 2\sin(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$$
$$\sin\alpha - \sin\beta = 2\sin(\frac{\alpha-\beta}{2})\cos(\frac{\alpha+\beta}{2})$$
$$\cos\alpha - \cos\beta = -2\sin(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})$$
$$\cos\alpha + \cos\beta = 2\cos(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$$

2.6 Geometric Transformations

2.6.1 Vertex matrices

A **vertex matrix** is a matrix in which the columns represent points in a shape and the rows represent the components.

For example, the triangle $\triangle ABC$, with points $A(-8, 7)$, $B(-4, 10)$, and $C(-1, -3)$ is represented by the following vertex matrix:

$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} x \\ y \end{array} & \begin{bmatrix} -8 & -4 & -1 \\ 7 & 10 & -3 \end{bmatrix} \end{array}$$

2.6.2 Translations

To translate a figure h units right and k units up, add an appropriate matrix.

For example, to translate $\triangle ABC$:

$$\begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \end{bmatrix} + \begin{bmatrix} h & h & h \\ k & k & k \end{bmatrix}$$

To translate $f(x)$:

$$f'(x) = f(x - h) + k$$

2.6.3 Dilations

To dilate a figure by a factor of k, multiply the figure's vertex matrix by k.

For example, to dilate $\triangle ABC$:

$$k * \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \end{bmatrix}$$

To dilate a function by a factor of m in the x-direction and n in the y-direction:

$$f'(x) = n * f(\frac{x}{m})$$

2.6.4 Reflections

Over the x-axis