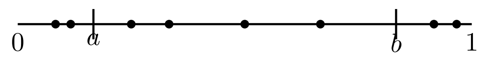
Proposition (1D Case):

Let $S = \{x \in \mathbb{R} \mid 0 < x < 1\}$ be the open interval between 0 and 1. Let X be a set of n numbers such that $X \subset S$, i.e., all elements of X are real numbers satisfying $0 < x < 1$.

Illustration: The following number line represents the interval $(0, 1)$ with two marked points a and b where $b > a$, and an uneven distribution of points in each subinterval:



The set X is said to be **uniformly distributed** if

$$\forall a, b \in S, \quad b > a, \quad \text{the following condition holds:}$$

$$Y = \{y \in X \mid a < y < b\}$$

$$\frac{|Y|}{\Delta} = \frac{b-a}{1-0} = b - a.$$

Proposition (2D Case):

Let

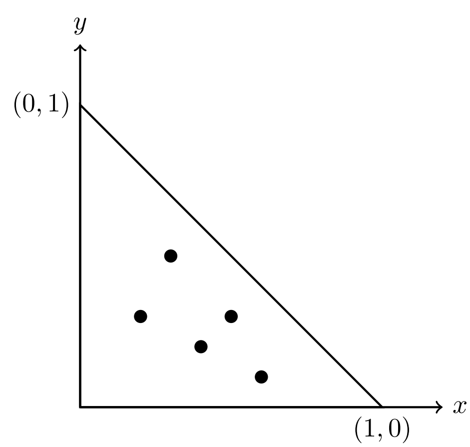
$$S = \{(x, y) \in \mathbb{R}^2 \mid 0 < x, y < 1 \wedge x + y < 1\}$$

be the triangular region bounded by the line $x + y = 1$. Let

$$X = \{(x, y) \in S\}$$

be a set of n points uniformly sampled from S.

Illustration: The following diagram represents the triangular region S with the boundary line $x + y = 1$:



For a small value h, we define:

$$\forall x \in \mathbb{R} \quad 0 < x < 1, \quad \forall y \in \mathbb{R} \quad 0 < y < 1, \quad x + y < 1.$$

Define a subset:

$$P = \{(a, b) \in X \mid x < a < x + h \wedge y < b < y + h\}$$

$$\frac{|P|}{n} = \frac{\text{Area of Interval}}{\text{Area of Shape}}.$$

The area of the interval is computed as:

$$\text{Area of Interval} = \int_{x}^{x+h} \int_{y}^{-x+t+h???} dy\, dx.$$