*~~Special~~ the case where it is a convex set contained in Euclidean space

*polytope for a bounded convex shape

<u>proposition.</u> Let $S \in R^3$ be a <u>convex polytope</u> defined by the convex hull of a finite set of vertices $V = \{v_1, v_2, \ldots, v_n\}$, where each $v_i \in R^3$.

*This next part I go pretty strictly to the defenetion of a convex hull (Day #2 notes).
- Do I need to define convex hull?

Define a random point $X \in S$ as a convex combination of these vertices:

$$X = \sum_{i=1}^{n} \alpha_i v_i$$

*So, I define X as a single point in a collection $\{X_1, \ldots, X_n\}$ of N points. To follow this I list the requirements for the defenition to not fall apart (also stated in convex hull def.).

where the weights α_i satisfy

$$\alpha_i \leq 1, \quad \sum_{i=1}^{n} \alpha_i = 1.$$

*Next I give some verbal explanation on the purpose of the weights, in that we're trying to find a method that utilizes them to give us a uniform distribution. I bet there's a more formal ~~th~~ way to write this.

Suppose the weights α_i are sampled from a distribution that ensures that all of the points in S are equally likely to be chosen, meaning the probability density function of X is constant over S.

(PDF) I name-drop 'probability density function' because this is kinda the working def. we have on what it means to be uniformly distributed. ⇒ A function whose value at any given random sample in the sample space can be interpreted as providing a relative liklihood that the value of the random variable would be equal to that sample.

- Do I need to define what it means to be constant over S?
- Note: the wording is clunky in this part.

⇒ continue on next page ☺

So, I just stated the condition for randomly sampling points inside a convex hull where we're using different combinations.

Then, X is uniformly distributed over S if and only if the weights $(\alpha_1, \alpha_2, \dots, \alpha_n)$ are sampled in such a way that every point in S has an equal probability of being selected.

That is the last section. In this I am stating that for the proof, one would need to prove that their chosen weight function produces weights distributed to establish uniformity. I think that this section is entirely too vague.

- Is "if and only if" used correctly?
- How can we de-generalize so that the uniformity condition is more refined?

Extra notes:

- What does "equal probability" mean in a mathematical sense?
- I think it assumes that a method for generating weights $\{\alpha_i\}$ leads to uniform sampling, but it doesn't justify why this is true => methods w/out weights???

And even more notes:

- Does it matter for a complex hull if a certain face of the object is bigger than others? I doubt it.
- Does the prop & proof need to generalize to both 2D & 3D?
- Another option for the proposition I thought of after seeing my finished 1st draft is kinda working in reverse. Instead of going w/ the structure leading to "if and only if", I could first explain the problem in terms of the probability distribution function (PDF) of a random point (X) and THEN derive conditions from said function.