OPTIMIZATION I

PROJECT 3 – NON-LINEAR PROGRAMMING

Newsvendor Optimization with Rush & Disposal Costs

$$\max_q E[p\min(q,D) - qc]$$

$$\max_q \frac{1}{n}\sum_{i=1}^{n}(p\min(q,D_i) - qc).$$

$$\max_{q,h} \frac{1}{n}\sum_{i=1}^{n} h_i$$

s.t.

$$h_i \le pD_i - qc$$
$$h_i \le pq - qc$$
$$h_i \ge -\infty$$

$$\max_q \frac{1}{n}\sum_{i=1}^{n}(pD_i - qc - g(D_i - q)^+ - t(q - D_i)^+)$$

Team G4

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I. Executive Summary

This report applies optimization techniques to determine profit-maximizing printing and pricing decisions for a publishing company. Historically, our organization has relied on a **classic Newsvendor model** to set the print quantity for a title with uncertain demand. While effective in simple cases, this traditional NV approach:

- Assumes **price is fixed**
- Ignores **price elasticity of demand**
- Does not explicitly model **rush printing** or **disposal costs**

In this project, we have built a more realistic and powerful optimization framework:

1. A **regression-based demand model** that captures how price changes affect demand
2. An extended **linear program (LP)** for fixed-price decisions with rush/disposal
3. A joint **quadratic program (QP)** that jointly optimizes **price and quantity**
4. A **bootstrap analysis** with 4,000 data resamples, drawn from the existing 99 historical price-demand observations used throughout the analysis
5. A comprehensive **managerial comparison** between the legacy NV model and the extended model

Key Outcomes

- **Standard NV (p = 1):**
    - $(q^* \approx 472)$
    - Expected Profit ≈ **231.48**
- **Joint Price–Quantity Optimization:**
    - $(p^* \approx 0.954)$
    - $(q^* \approx 535.29)$
    - Expected Profit ≈ **234.42**
- **Profit Improvement:**
    - $[234.42 - 231.48 = \mathbf{2.94}\ \text{units}]$

      ≈ **1.2–1.5% improvement**
- **Bootstrap Analysis (4,000 replications):** The optimal pricing, quantity, and profit distributions are **tight and stable**, confirming high model robustness.

The mentioned profit values represent the **expected profit per demand scenario**, meaning the average profit is computed across all simulated demand outcomes.

This document summarizes each model, interprets results, and visualizes key analyses to provide actionable managerial recommendations for a robust newsvendor optimization strategy.

II. Introduction and Problem Context

The publishing company needs to decide how many units of a title to print before demand is realized. Traditionally, managers use:

- Price fixed at **p = 1**
- A **standard NV model** that considers demand uncertainty but *ignores price effects*

This project is motivated by several real-world realities:

a. Price affects Demand

Demand clearly changes with pricing - readers are sensitive to price variations, especially for non-essential content. Failing to incorporate elasticity may lead to under or over-production.

b. Operational Costs Are non-linear

- **Rush printing** is significantly more expensive
- **Excess inventory** gives rise to disposal costs
- Missed sales due to under-printing affects reputation and market share

c. Data-Driven Decisions

With access to historical data (price + demand), we can build:

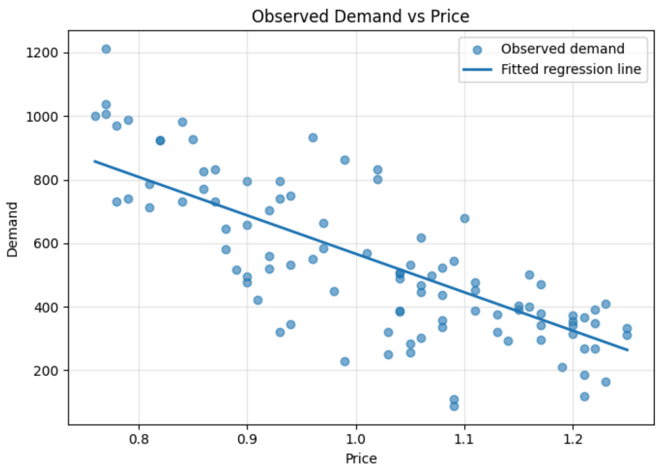
- A more accurate statistical view of demand
- A more realistic profit function
- A better optimization model that respects all operational realities

This report builds such a model.

III. Data Description & Regression Model

Having established why the classic Newsvendor framework falls short in our price-sensitive environment, we now turn to the data itself.

Understanding how customers historically responded to price changes is the foundation for any



improved decision model. This section begins our analytical journey by uncovering the relationship between price and demand - setting the stage for everything that follows.

The dataset includes daily records of:

- Selling **price**
- Corresponding **demand**

We fit the regression:

$[D_i = a + bp_i + \varepsilon_i]$

This equation represents the **ordinary least squares (OLS)** model we fit, capturing how demand changes as a linear function of price (demand = a + b·price + residual).

Using the following code:

```
X = df[["price"]].to_numpy()
y = df["demand"].to_numpy()

ols = LinearRegression()
ols.fit(X, y)

a = float(ols.intercept_)
b = float(ols.coef_[0])
r2 = float(ols.score(X, y))
residuals = y - (a + b*X[:,0])
```

a. Regression Findings

- **Intercept (a)** ≈ 1851
- **Slope (b)** ≈ −895
- **Interpretation:** For every $1 increase in price, demand decreases by roughly **895 units**, indicating strong price elasticity.
- Additionally, the intercept represents the predicted demand when price is set to $0, which in this case corresponds to approximately **1,851 units**.
- **R^2 ≈ 0.78**, meaning 78% of demand variation is explained by price alone.

b. Residuals as Demand Scenarios

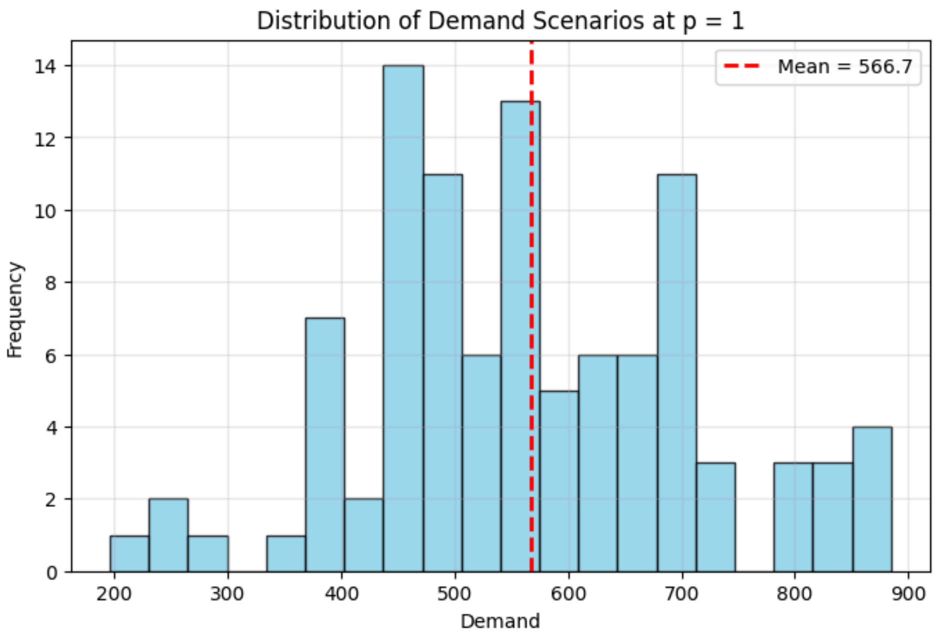
Residuals represent historical demand shocks and form the basis for our scenario-based optimization in later sections.

The regression residuals are centered around zero, as expected for an OLS model, and therefore serve as a reasonable empirical distribution for demand shocks. Using these residuals preserves both the magnitude and variability of historical demand deviations when constructing the scenario-based demand values for optimization.

IV. Fixed Price Newsvendor with Rush & Disposal

With a clear view of how demand responds to price, we next explore how the firm currently operates: fixing price at p = 1 and adjusting only print quantities.

Building on the demand scenarios developed in the previous section, we evaluate how well the traditional Newsvendor approach performs once rush and disposal costs are included. This section provides a direct baseline against which our enhanced model will be compared.



At the fixed price (p = 1), demand scenarios become:

$$[D_i(1) = a + b \cdot 1 + \varepsilon_i]$$

Using this:

```
d1 = a + b*p_fixed + residuals
n = d1.size
```

Where:

- ($c = 0.5$): printing cost
- ($g = 0.75$): rush cost
- ($t = 0.15$): disposal cost

a. Extended LP Model

To make the structure of this model transparent, we first lay out the decision variables and the LP formulation that drives the extended model.

Decision Variables

- (q): total number of units printed in the initial production run
- (s_i): units sold in scenario (i) (effective sales, capped by both (q) and demand)
- (r_i): units rushed in scenario (i) when demand exceeds the initial print quantity
- (d_i): units disposed of in scenario (i) when the print quantity exceeds demand

Mathematical Formulation

$$\max_{q,s_i,r_i,d_i} \frac{1}{n}\left[p\sum_{i=1}^{n} s_i - cnq - g\sum_{i=1}^{n} r_i - t\sum_{i=1}^{n} d_i\right]$$

subject to the constraints:

$$[s_i \leq q, \quad \forall i],$$

$$[s_i \leq D_i(1), \quad \forall i],$$

$$[r_i \geq D_i(1) - q, \quad r_i \geq 0, \quad \forall i],$$

$$[d_i \geq q - D_i(1), \quad d_i \geq 0, \quad \forall i],$$

$$[q \geq 0]$$

This formulation captures the operational logic of the fixed-price setting - sales are capped by both production quantity and realized demand, rush printing occurs when demand exceeds the initial run, and disposal costs apply when production exceeds realized demand.

Implementation:

```
m = gp.Model()
q = m.addMVar(1, lb=0.0, name="q")
s = m.addMVar(n, lb=0.0, name="s")
r = m.addMVar(n, lb=0.0, name="r")
d = m.addMVar(n, lb=0.0, name="d")

m.addConstr(s <= q[0])
m.addConstr(s <= d1)
m.addConstr(r >= d1 - q[0])
m.addConstr(d >= q[0] - d1)
```

Objective:

```
total_revenue = p_fixed * np.sum(d1)
cost_expr = c * n * q[0] + g * quicksum(r) + t * quicksum(d)
obj = (total_revenue - cost_expr) / n
```

$$[\text{obj} = \frac{\text{Total Revenue} - \text{Cost Expression}}{n}]$$

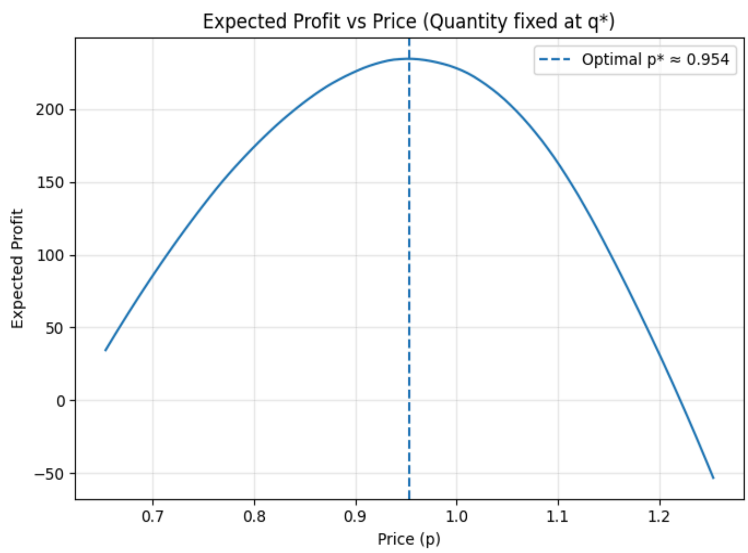
We divide the objective by the number of demand scenarios (n) so that the result reflects **expected profit** rather than the total profit summed across all scenarios.

b. LP Results

Metric	Value
Optimal Quantity	**471.865**
Expected Profit	**231.484**

V. Joint Price-Quantity Optimization

While the fixed-price model reveals important operational tradeoffs, it still ignores the key driver of demand: **price**. This section moves the analysis forward by allowing price itself to be optimized.



Building upon the regression insights from Section 3 and the operational cost structure from Section 4, we now solve a full quadratic program that jointly determines both price and print quantity. This transition marks the moment where our model becomes truly data-driven.

The profit function becomes:

$$[\pi_i(p, q) = pD_i(p) - cq - g(D_i(p) - q)^+ - t(q - D_i(p))^+]$$

where:

$$[D_i(p) = a + bp + \varepsilon_i]$$

a. Joint QP Structure

To see how pricing and production interact in this expanded setting, we begin by clearly defining the decision variables and writing out the full QP formulation that guides the joint optimization.

Decision Variables

- (p): selling price
- (q): initial production quantity
- (s_i): units sold in scenario (i) (capped by both (q) and demand)
- (r_i): units rushed in scenario (i) when demand exceeds initial production
- (d_i): units disposed in scenario (i) when production exceeds demand

Price-Dependent Demand

For each scenario (i), demand depends on price as:

$[D_i(p) = a + bp + \varepsilon_i]$

Mathematical Formulation

$$\max_{p,q,s_i,r_i,d_i} \frac{1}{n}\left[p\sum_{i=1}^{n} s_i - cnq - g\sum_{i=1}^{n} r_i - t\sum_{i=1}^{n} d_i\right]$$

subject to:

$[s_i \le q, \quad \forall i]$,

$[s_i \le D_i(p), \quad \forall i]$,

$[r_i \ge D_i(p) - q, \quad r_i \ge 0, \quad \forall i]$,

$[d_i \ge q - D_i(p), \quad d_i \ge 0, \quad \forall i]$,

$[p \ge 0, \quad q \ge 0]$

This formulation directly links price to demand, allowing the model to jointly optimize both decisions while incorporating rush and disposal penalties across all demand scenarios.

```
mq = gp.Model()

p = mq.addMVar(1, lb=0.0, name="p")
q = mq.addMVar(1, lb=0.0, name="q")
```

```
s = mq.addMVar(n, lb=0.0, name="s")
r = mq.addMVar(n, lb=0.0, name="r")
d = mq.addMVar(n, lb=0.0, name="d")

d2 = a + b * p[0] + residuals

mq.addConstr(s <= q[0])
mq.addConstr(s <= d2)
mq.addConstr(r >= d2 - q[0])
mq.addConstr(d >= q[0] - d2)

revenue_qp = p[0] * gp.quicksum(d2)
cost_qp = c * n * q[0] + g * quicksum(r) + t * quicksum(d)
obj_qp = (revenue_qp - cost_qp) / n
```

b. QP Results

Metric	Value
Optimal Price	**0.9536**
Optimal Quantity	**535.29**
Expected Profit	**234.42**

Interpretation

- Lowering price slightly increases demand significantly (due to steep slope *(b)*).
- Printing more reduces expected rush costs.
- Profit increases relative to the fixed-price model.

VI. Bootstrap Analysis (4000 Iterations)

Once we obtain optimal price and quantity decisions, the natural question becomes: **How reliable are they?** Optimization based on a single sample may overlook uncertainty in the data-generating process.

Bootstrapping allows us to approximate how **sensitive** the optimal price and quantity are to sampling variation in the underlying data. By repeatedly refitting the regression model and re-solving the QP on resampled datasets, we can observe **how much the optimal decisions would change if the historical data had been even slightly**

different. This provides a way to quantify the stability of our decisions and understand the robustness of our pricing and production recommendations.

Here, we build directly on the QP result from Section 5 and use bootstrap resampling to evaluate the stability of our decisions under many plausible variations of the historical data. The goal is to quantify risk - not just return.

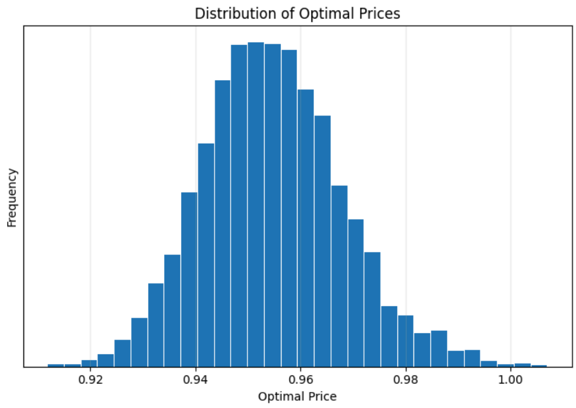
The bootstrap function:

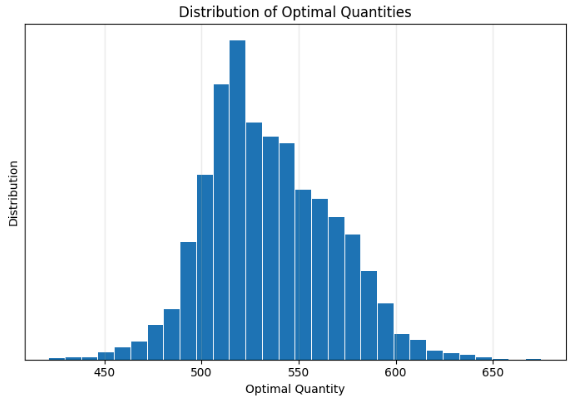
```
def boot(n_iter, df):
    prices = []
    quantities = []
    profits = []

    for i in range(n_iter):
        df_boot = resample(df, replace=True, n_samples=len(df))
        # df_boot.head()

        X = df_boot[["price"]].to_numpy()
        y = df_boot["demand"].to_numpy()

       ols = LinearRegression()
        ols.fit(X, y)

        a = float(ols.intercept_)
        b = float(ols.coef_[0])
        # r2 = float(ols.score(X, y))
        ......

    })

    print("Bootstrap complete!")
    print(f"Mean price:     {np.mean(prices):.4f}")
    print(f"Mean quantity: {np.mean(quantities):.4f}")
    print(f"Mean profit:    {np.mean(profits):.4f}")

    return results

  results = boot(4000, df)
```

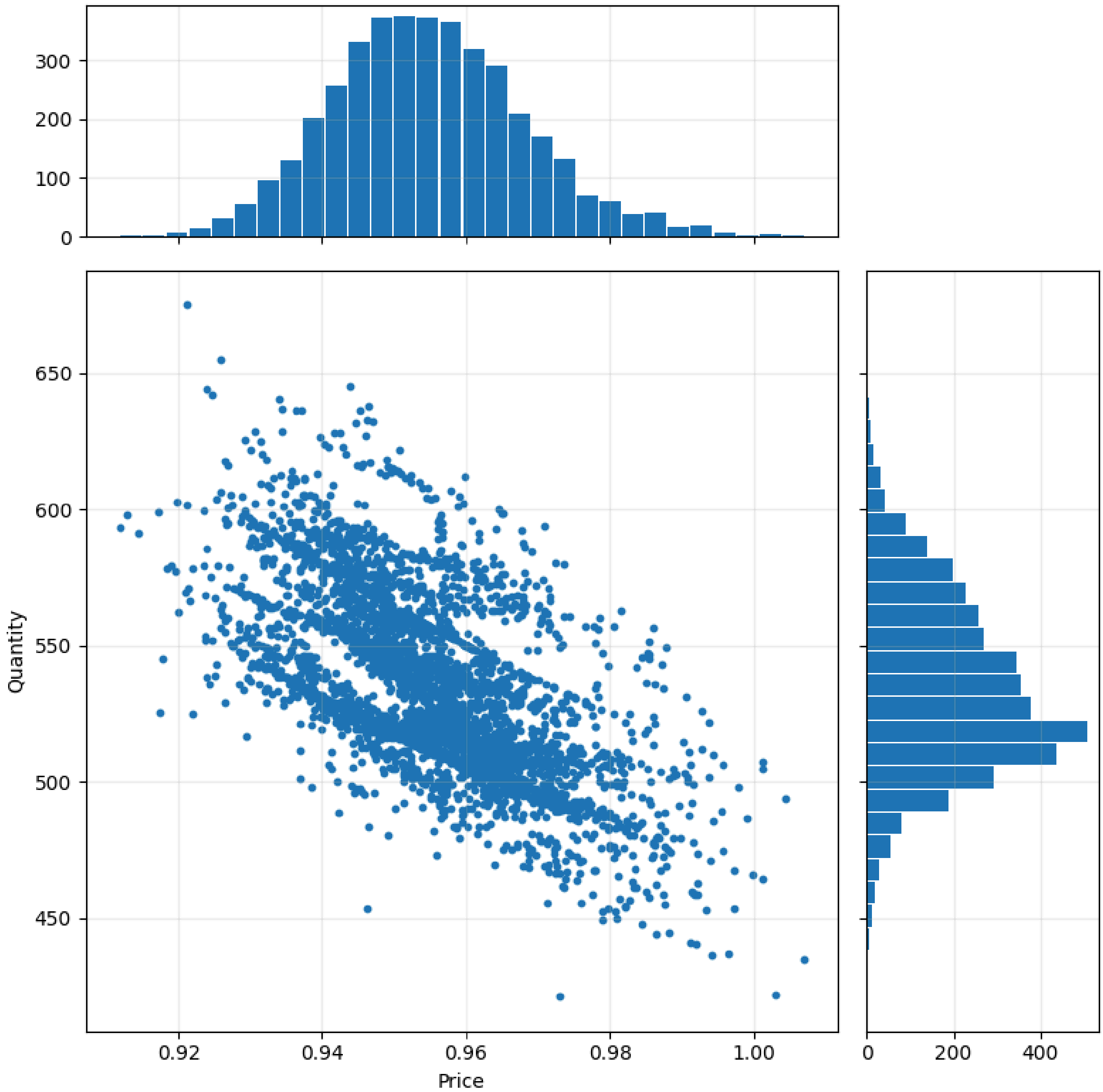
The function:

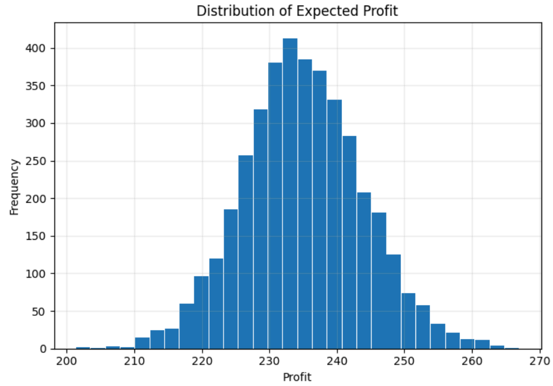
- Resamples the dataset
- Refits regression

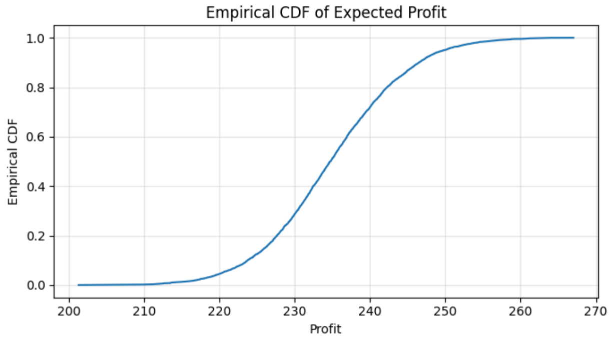
- Rebuilds QP
- Re-solves
- Stores optimal p, q, profit











a. Bootstrap Summary Statistics

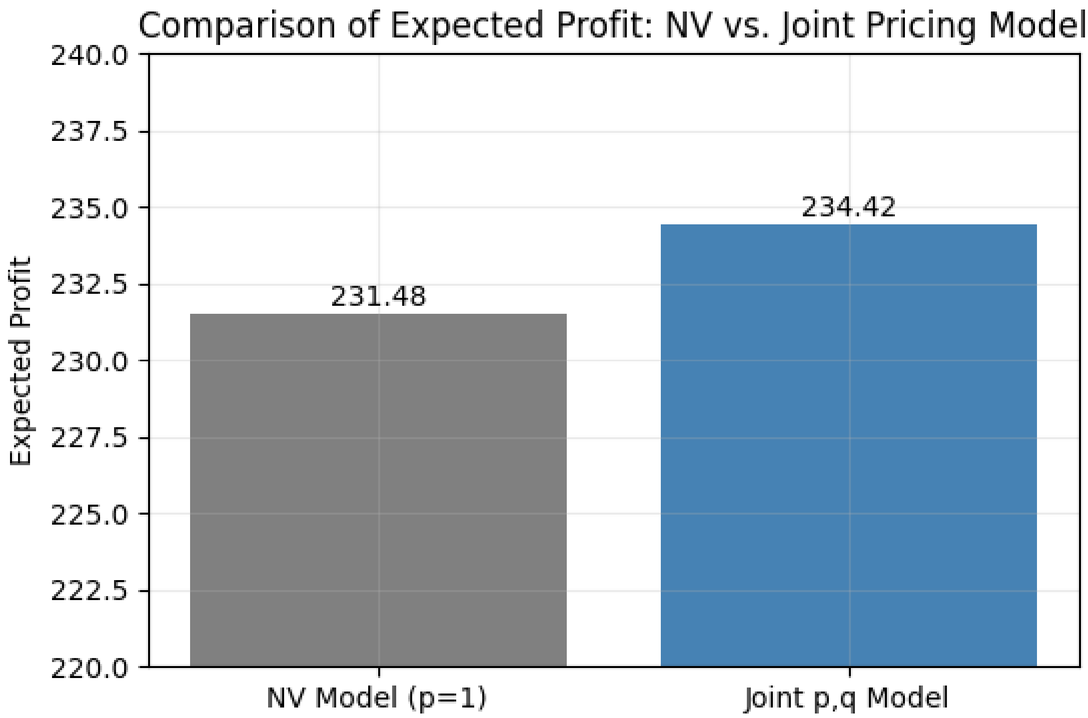
Metric	Mean	Comment
Price	**0.9547**	Very stable
Quantity	**534.49**	Very stable
Profit	**234.66**	Matches QP estimate

b. Risk & Robustness Insight

- The distributions of (p*) and (q*) are **tight**, meaning the model does not fluctuate wildly between bootstrap datasets.
- The profit distribution is similarly concentrated around ≈ 234–236.
- This ensures that the model recommendations will remain **consistent** even if underlying data vary.

VII. Managerial Comparison & Discussion

Armed with both optimized decisions and an understanding of their robustness, we now compare outcomes across the two competing approaches: the classic NV method from Section 4 and the data-driven joint model developed in Section 5. This comparison brings the analysis full circle, connecting statistical insight, optimization, and managerial implications into a unified narrative.



a. Is the NV Model Good Enough?

The NV model works when:

- Price is fixed
- Operational complexity is low
- Data is sparse

But here:

- Price elasticity is **strong**
- Rush and disposal costs matter
- We have enough data for statistical modeling

So the NV model leaves **profit on the table**.

b. Profit Improvement from Switching

$$[\text{QP Profit} - \text{NV Profit} = 234.42 - 231.48 = 2.94]$$

For a single title, this may seem small.

But across **hundreds** of titles and **multiple print cycles per year**, the improvement compounds substantially.

c. Advantages & Disadvantages

Standard NV Model

Pros:

- Simple
- Fast
- Easy to explain
- Works when price fixed

Cons:

- Ignores price elasticity
- Doesn't model rush/disposal inherently
- Lower profitability
- Less data-driven

Extended Price-Sensitive QP Model

Pros:

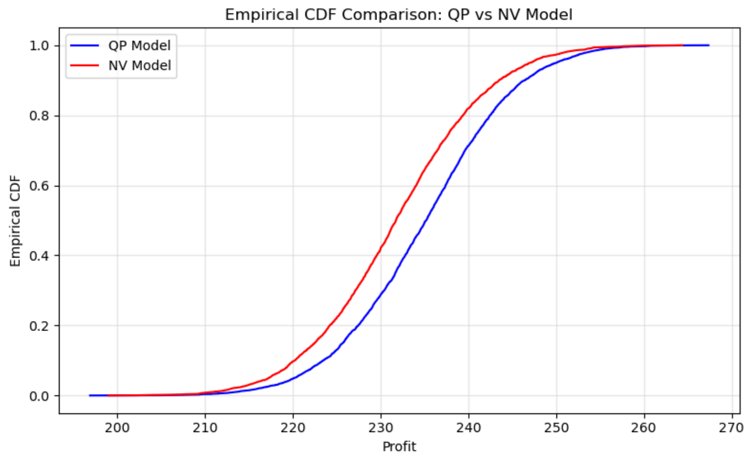
- Captures real demand behavior
- Models operational constraints
- Higher profits
- Robust to sampling variability
- Better strategic insight

Cons:

- More complex
- Requires regression + optimization
- Needs more computational time
- Requires ongoing data maintenance

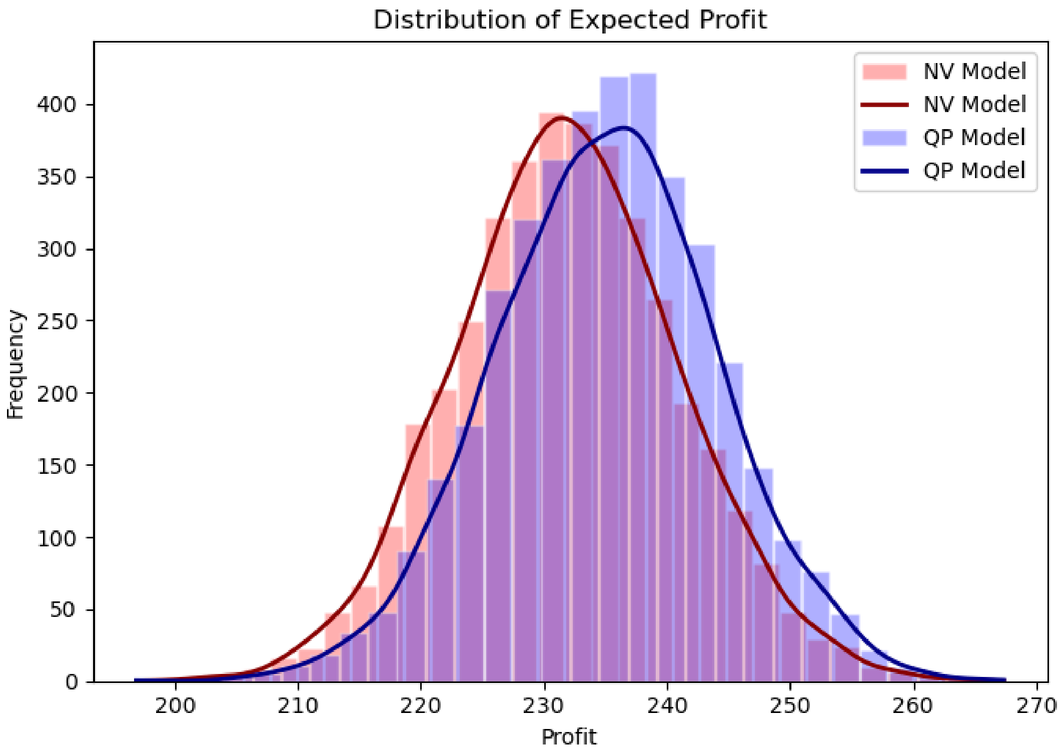
Taken together, the progression from the Standard NV model to the Extended LP model and finally to the Joint QP framework makes the comparison much clearer.

The basic NV approach works well when price is fixed and when the firm faces no meaningful shortage or excess penalties, but it becomes structurally misaligned with our environment as soon as rush printing and disposal costs are introduced.



The LP resolves this by modeling these penalties explicitly, and the QP builds on that by incorporating the strong price elasticity observed in the data.

In effect, each model corrects a specific limitation of the prior one: **NV ignores operational frictions, LP incorporates them, and QP adds the behavioral component of demand**. This stepwise improvement explains why the QP achieves higher expected profit and why, in this context, the simpler NV model systematically leaves money on the table.



VIII. Final Recommendations

Based on the details of both the models, it is suggested to adopt the **price-sensitive QP model** for **high-volume or revenue-critical titles** as a better alternative. It provides:

- Higher expected profit
- Better alignment with true demand behavior
- More realistic treatment of operational costs
- Strong empirical robustness through bootstrap testing

The NV model should be retained only for:

- Low-volume titles
- Cases where price must be externally fixed
- Quick heuristic decisions with incomplete data

Recommended Action:

Adopt the **price-sensitive model** as the default decision-making framework for high-volume titles. Although gains seem modest per title, they scale substantially over thousands of print runs annually.

The price-sensitive model is an overall **superior and future-proof decision tool**.

It reflects real market and operational dynamics, leads to higher profit, and maintains robust performance under uncertainty.

END OF REPORT