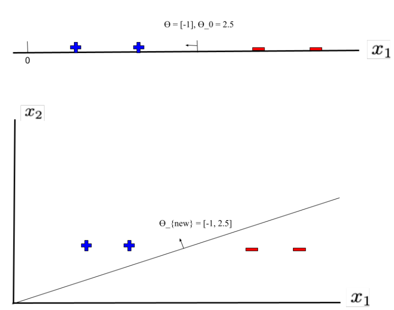
Polynomial Basis

1 Reducing Perceptron-Not-Through-Origin to Perceptron-Through-Origin

As promised earlier, the problem of perceptron-not-through-origin can be reduced to the problem of perceptron-through-origin. The key lies in transforming the dataset used from the $\mathbb{R}^d$ space to a $\mathbb{R}^D$ space when $D > d$.

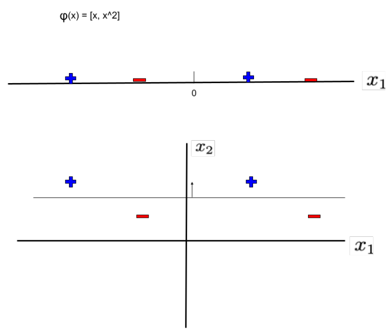
Recall that the parameters of a separator not through the origin are $\theta = [\theta_1, ...\theta_d]$ and θ_0, a scalar. Say that these two parameters are taken to create a single one called θ_{new}: $\theta_{new} = [\theta_1, ...\theta_d, \theta_0]$. θ_{new} is basically just θ with θ_0 put in as the last entry. Now, each x value vector in the dataset must also be modified. Say that $x_{new} = [x_1, ..., x_d, 1]$. x_{new} is basically just x with 1 put in as the last entry. $\theta_{new}^T x_{new} = \theta_1 x_1 + ... + \theta_d x_d + (1)\theta_0 = \theta_1 x_1 + ... + \theta_d x_d + \theta_0 = \theta^T x + \theta_0$. A classifier-not-through-origin in d dimensions can be turned into a classifier-through-origin in $d + 1$ dimensions. Thus, even though the perceptron convergence theorem from before was only proved for perceptron through origin, transformation shows that the theorem also applies to perceptron-not-through-origin.



2 Polynomial Basis

The same concept can be applied to a dataset that is not linearly separable in d dimensions. By moving the dataset into $d + 1$, $d + 2$, or more generally,

$d + n$ dimensions, one can make it linearly separable.



A systematic way of transforming data into higher dimensions exists. It is called polynomial basis.

order	in general
0	$[1]$
1	$[1, x_1, ...x_d]$
2	$[1, x_1, ...x_d, x_1^2, x_1x_2, ...$all two way products$]$
3	$[1, x_1, ...x_d, x_1^2, x_1x_2, ...$all two way products$, x_1^3, ...$all three way products$]$
...	...

For an example, take $[x_1, x_2]$. Using polynomial basis to transform it to the second degree yields $[1, x_1, x_2, x_1^2, x_1x_2, x^2]$.