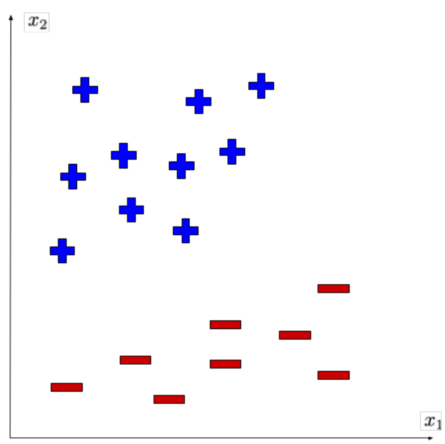
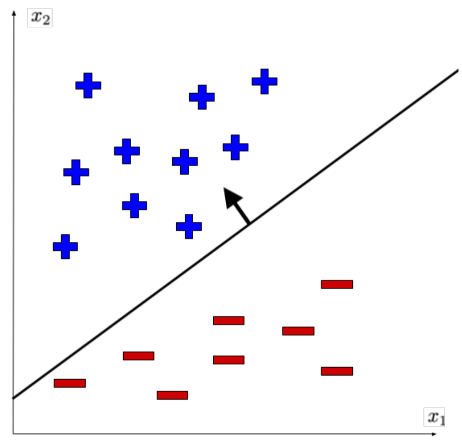
Linear Classifiers

A linear classifier is a type of hypothesis for the supervised learning setup. To visualize how linear classifiers work, one must plot all x values in D_n onto the $\mathbb{R}^d$ space. Each axis of the $\mathbb{R}^d$ space (i.e. x_1, x_2, ... , x_n) corresponds to a dimension of the vector x. The first dimension of x is its first entry, the second dimension is its second entry, and so on. Some of the plotted points are assigned a y value of $+1$ whereas others are assigned to -1.



Once the dataset is visualized on a graph, how to write a classifier becomes really obvious! For the two dimensional space above, one just needs to draw a line that separates $\mathbb{R}^2$ into a $+1$ subspace and -1 subspace. In otherwords, one just needs to find a line such that all points that were assigned $+1$ in the dataset sit on one side while all points that were assigned -1 sit on the other side. The separator that does the job is called a linear classifier.



For a dataset with two-dimensional x vectors, linear classifiers, as described earlier, are just lines. For a dataset with three-dimensional x vectors, linear classifiers are planes. When the dataset contains n-dimensional x vectors, the general term used to describe the separator that classifies them is a hyperplane For the $\mathbb{R}^n$ space, hyperplanes are a space with $n-1$ dimensions. A hyperplane has a normal that points in the direction of the $+1$ subspace.

A linear classifier may be written formally as $h(x;\theta,\theta_0) = sign(\theta^T x + \theta_0) = \begin{cases} +1, \theta^T x + \theta_0 > 0 \\ -1, otherwise \end{cases}$. Visibly, it has two parameters θ and θ_0. Learning algorithms try to find the parameters that will construct a separator that accurately classifies $+1$ from -1. $\theta \in \mathbb{R}^d$ and $\theta_0 \in \mathbb{R}$.

The simplest learning algorithm that one can begin with is the random linear classifier algorithm. The algorithm works by producing random parameters k times. At the end, it returns the parameters with the lowest training set error. k is called a hyperparameter. It is not a parameter of the hypothesis. Rather, it is a parameter of the learning algorithm. It impacts how training occurs.

Input: D_n, k
for $j = 1$ to k **do**
  $\theta^{(j)} \leftarrow \text{random}(\mathbb{R}^d)$
  $\theta_0^{(j)} \leftarrow \text{random}(\mathbb{R})$
end for
$j^* \leftarrow \arg\min_{j\in\{1,\ldots,k\}} E_n(h(\cdot,\theta,\theta_0))$
return $\theta^{(j^*)}, \theta_0^{(j^*)}$
