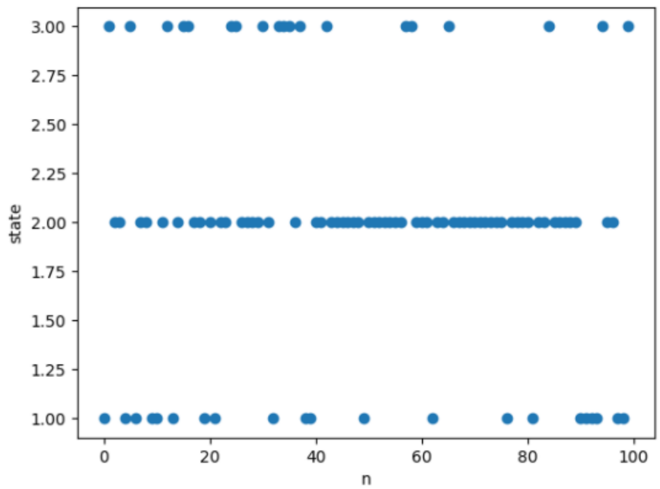

3a.

```
3b.
Average over 1000 simultations: 2.008
Sum using Beta=0.9: 41.627398681746016
3c.
Average of first over 1000 reps of 1000 simulations: 2.0209379999999975
Average of second over 1000 reps of 1000 simulations: 40.88162230654895
```

Code:

```
import numpy as np
import matplotlib.pyplot as plt

def markov(num, matrix, start, fig=''):
    out = []
    current = start
    for _ in range(num):
        out.append(current)
        current = np.random.choice([1,2,3], p=matrix[current-1])
    if fig:
        plt.figure()
        plt.xlabel('n')
        plt.ylabel('state')
        plt.scatter(range(len(out)), out)
        plt.savefig(fig)
    else:
        return out

matrix = [[.4, .38, .22], [.12, .7, .18], [.2, .5, .3]]
markov(100, matrix, 1, fig='3a')
out = markov(1000, matrix, 1)
print("3b.")
print("Average over 1000 simultations: " + str(sum(out) / len(out)))
print("Sum using Beta=0.9: " + str(sum([0.9 ** index * val ** 2 for index, val in enumerate(out)])))

one, two = [], []
for _ in range(1000):
    out = markov(1000, matrix, 1)
    one.append(sum(out) / len(out))
    two.append(sum([0.9 ** index * val ** 2 for index, val in enumerate(out)]))
print("3c.")
print("Average of first over 1000 reps of 1000 simulations: " + str(sum(one) / len(one)))
print("Average of second over 1000 reps of 1000 simulations: " + str(sum(two) / len(two)))
```

Ethan Glaser

MU4 HW3

2a Mean: np Variance: $np(1-p)$

b The Chebyshev inequality states:

$$\mathbb{P}(|Y-E[Y]|\geq a)\leq \frac{Var(Y)}{a^2}$$

in this case, $Y=\frac{X_n}{n}$ and $E[Y]=\frac{np}{n}=p$ with $a\to\epsilon$

The variance of Y is $Var\left(\frac{\sum_{i=1}^{n}X_i}{n}-\mu\right)=Var\left(\frac{\sum X_i}{n_2}\right)=\frac{\sigma^2}{n}$

$$Var(Y)=\frac{p(1-p)}{n}$$ where $\sigma^2=np(1-p)$

$max(p(1-p))$ is when $p=0.5\to p(1-p)=0.25=\frac{1}{4}$

therefore $\mathbb{P}\left(\left|\frac{X_n}{n}-p\right|\geq\epsilon\right)\leq\frac{1}{4n\epsilon^2}$

c. $\mathbb{P}\left(\left|\frac{X_N}{N}-p\right|\leq 0.01\right)\to\mathbb{P}\left(\frac{X_N-Np}{\sigma\sqrt{N}}\leq\frac{0.01}{\sigma\sqrt{n}}\right)\to\mathbb{P}\left(N(0,1)\leq\frac{0.01}{\sqrt{np(1-p)}}\right.$

$\underbrace{1-\mathbb{P}\left(\left|\frac{X_N}{N}\right|\geq 0.01\right)}_{0.95}\to\mathbb{P}\left(\left|\frac{X_N}{N}\right|\geq 0.01\right)<\frac{1}{4n\epsilon^2}=\frac{2500}{n}$

$0.05\leq\frac{2500}{n}\to\boxed{n=50\,000}$

1. $\mathbb{P}(X>x)=exp\left(-\int_0^x t^3dt\right)=exp\left(\frac{-x^4}{4}\right)\to E[X]=\frac{-4}{x^4}, Var[X]=\frac{16}{x^8}$

$X\sim exp\left(\frac{-x^4}{4}\right)\to F(x)=1-e^{\frac{x^5}{4}}\to\frac{x^5}{4}=\ln(1-F(x))\to X=\sqrt[5]{4\ln(1-F(x))}$

Simulation:

$F^{-1}(x)=\sqrt[5]{4\ln(1-x)}$

- Generate $U_1, U_2\ldots U_m\sim Unif[0,1]$
- Compute $\sqrt[5]{4\ln(1-u_1)}, \sqrt[5]{4\ln(1-u_2)},\ldots\sqrt[5]{4\ln(1-u_m)}$

or equivalently: $\sqrt[5]{4\ln(u_1)}, \sqrt[5]{4\ln(u_2)},\ldots\sqrt[5]{4\ln(u_m)}$