11 - Convolutional Neural Networks

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Intro

A convolutional neural network consists on three types of layers - fully connected layers - pooling layers - convolutional layers

Convolutional Layers

Image example

Since pixels are normally much more strongly correlated to the pixels around it, we can convolute the input image to drastically reduce the number of features. In turn, this reduces the number of weights in the model drastically.

Definition: 1-Dimensional Convolution

$$g * f(t) = \int_{\tau} g(t-\tau)f(t)\,d\tau$$

Definition: Discrete 1-Dimensional Convolution

$$m * f(x) = \sum_{i=0}^{w} m(i)f(x-i)$$

Definition: Discrete 2-Dimensional Convolution

We use a *convolution mask* also known as a *kernel*

$$m * *f(x,y) = \sum_{i=0}^{w}\sum_{j=0}^{w} m(i,j)f(x-i,y-j)$$

This allows us to drastically reduce the number of weights that will enter the first fully connected layer.

Convolutional Layers in Practice

In practice, several filters (sets of weights $w_{x,y}$) to compute several convolved versions of the input. These are called **feature maps**

Example: We go from a `(28, 28)` image to `(3, 24, 24)` which enters the first hidden layer.

Stride Parameter

Reduces and convolves at the same time. It ignores part of the signal. it is normally preceded by layer with `stride=1` to help remedy this.

Pooling Layers

A pooling layer reduces the number of inputs by replacing all of the activations in a neighborhood by a single one. *Can be though of asking if a particular feature is present in some neighborhood while ignoring the exact location.* The output size is reduced by the pooling layers.

Fully Connected Layers

Each neuron in the fully connected layer is connected to all neurons in the preceding one. *This becomes a deep network with many parameters to learn, but many fewer than a fully connected multilayer perceptron.*

Feature Maps

Some convolutional masks are very similar to oriented Gaussian / Gabor filters. Trained networks compute oriented derivatives, which some believe the brain does as well.