HW 11 7.4 7, 10, 19, 23

(7) $\int \frac{x^4}{x-1}\,dx$

$x-1 \overline{)\,x^4+0x^3+0x^2+0x+0}$

x^3+x^2+x+1 R 1

$-(x^4-x^3)$

x^3+0x^2

$-(x^3-x^2)$

x^2+0x

$-(x^2-x)$

$x+0$

$-(x-1)$

1

$= \int x^3+x^2+x+1+\frac{1}{x-1}\,dx$

$= \frac{1}{4}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2+x+\ln|x-1|+C$

(10) $\int \frac{y}{(y+4)(2y-1)}\,dy = \left[\frac{A}{y+4}+\frac{B}{2y-1}\right]$

$y = A(2y-1)+B(y+4)$

$-4 = A(-8-1)+$

$-4 = -9A$

$\frac{4}{9} = A$

$\frac{1}{2} = B\left(\frac{1}{2}+\frac{8}{2}\right)$

$\frac{2}{9}\left(\frac{1}{2} = \frac{9}{2}B\right)$

$\frac{1}{9} = B$

$\int \frac{\frac{4}{9}}{y+4}+\frac{\frac{1}{9}}{2y-1}\,dy =$

$\frac{4}{9}\ln|y+4|+\frac{1}{9}\ln|2y-1|+C$

(19) $\int \frac{x^2+x+1}{(x+1)^2(x+2)}\,dx$

$\int \frac{A}{x+1} + \int \frac{B}{(x+1)^2} + \int \frac{C}{x+2}$

$\ln|x+1| + \ln(x+1)^2 + \ln|x+2| + C$?

(23) $\int \frac{10}{(x-1)(x^2+9)}\,dx = \int \frac{1}{x-1} + \frac{-x-1}{x^2+9}\,dx$

$\int \frac{1}{x-1}\,dx - \int \frac{x}{x^2+9}\,dx - \int \frac{1}{x^2+9}\,dx$ $\quad 3^2$ $\quad \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

$= \ln|x-1| - \frac{1}{2}\ln(x^2+9) - \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

Math 7.8 5 6 29 30

⑤ $\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx$ $\lim_{b\to\infty} \int_3^b (x-2)^{-3/2}\,dx$ $u=x-2$ $du=dx$ $u^{-3/2}$ $\frac{u^{-1/2}}{-1/2}$ $-2u^{-1/2}$

$\lim_{b\to\infty} \left[-2(x-2)^{-1/2}\right]_3^b$

$\lim_{b\to\infty} \left(\frac{-2}{\sqrt{b-2}} + \frac{2}{\sqrt{3-2}}\right)$ $0+\frac{2}{1}$ $\boxed{2}$

⑥ $\int_0^\infty \frac{1}{\sqrt[4]{1+x}}\,dx$ $\lim_{b\to\infty} \int_0^b (1+x)^{-1/4}\,dx$ $u=1+x$ $du=dx$ $\int u^{-1/4}\,du$ $\frac{4}{3}u^{3/4}+C$

$\lim_{b\to\infty} \left[\frac{4}{3}(1+x)^{3/4}\right]_0^b$

$= \lim_{b\to\infty} \left[\frac{4}{3}(1+b)^{3/4} - \frac{4}{3}\right]$ $\infty - \frac{4}{3} = \infty$ diverges

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(29) $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}}$ $\lim_{t \to -2} \int_{t}^{14} (x+2)^{-1/4} dx$

$\lim_{t \to -2} \left[\frac{4}{3}(x+2)^{3/4} \right]_{t}^{14}$

$\frac{4}{3} \lim_{t \to -2} + \left(16^{3/4} - (t+2)^{3/4} \right)$

$\frac{4}{3}(8-0) = \frac{32}{3}$ converges

(30) $\int_{-1}^{2} \frac{x}{(x+1)^2} dx$

$\lim_{t \to -1^+} \int_{t}^{2} \frac{x}{(x+1)^2} dx$ $x(x^2+2x+1)^{-1}$ $\int_{t}^{2} \frac{x}{(x+1)^2} dx =$ $u = x+1$, $du = dx$, $x = u-1$

$\int_{t+1}^{3} \frac{u-1}{u^2} du = \int_{t+1}^{3} \left(\frac{1}{u} - \frac{1}{u^2} \right) du = \ln|u| + \frac{1}{u} \Big|_{t+1}^{3}$

$\lim_{t \to -1^+} \ln|t+1| + \frac{1}{t+1} = -\infty$