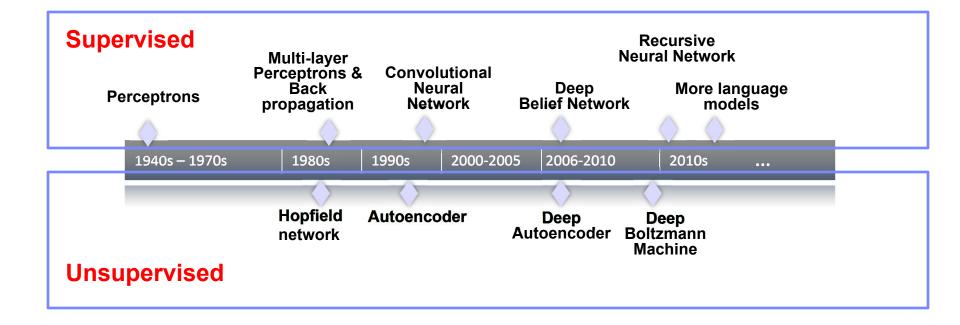
CS291K - Advanced Data Mining

Instructor: Xifeng Yan
Computer Science
University of California at Santa Barbara

Training Neural Networks

Lecturer: Fangqiu Han Computer Science University of California at Santa Barbara

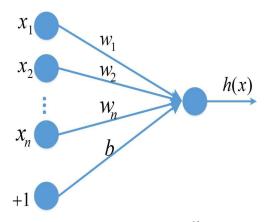
Neural network timeline



Outline

- □ Trainging Perceptron
- ☐ Trainging feed forward nueral networks (MLP)
 - Backpropagation algorithm
 - Gradient checking
 - Activation functions
 - Preprocessing
 - Weight initialization
 - Regularization
 - Parameter updates

Perceptron: the simplest neural network



x: n-dimension input

w: combination weights

b: bias

$$h(x) = f(\sum_{i=1}^{n} w_i x_i + b) = f(w^T x + b)$$

 $f(\cdot)$ is called Activation function, e.g.,

Step function:
$$f(z) = \begin{cases} +1 & \text{if } z > 0 \\ -1 & \text{otherwise} \end{cases}$$

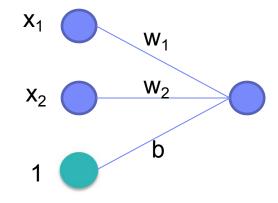
Training Perceptron

☐ Initialize all parameters to 0.

□ Pick training data using any policy that ensures that all training data will keep getting picked.

Perceptron Training Example

- □ Task: Train a classifier using perceptron on two data points:
 - ➤ (1,1) with label 1; (2,-3) with label -1.
- □ Initialize $w_1=w_2=b=0$.
- □ Apply a round-robin order on data points.



$$h(x) = f(w^T x + b)$$

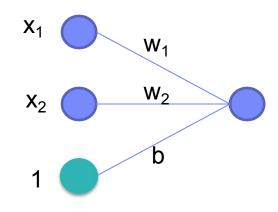
$$f(z) = \begin{cases} +1 & \text{if } z > 0 \\ -1 & \text{otherwise} \end{cases}$$

Training Perceptron

- ☐ Initialize all parameters to 0.
- □ Pick training cases using any policy that ensures that every training case will keep getting picked.
 - If the output unit is correct, leave its weights alone.
 - If the output unit incorrectly outputs a -1, add the input vector to the weight vector: w += x
 - ➤ If the output unit incorrectly outputs an 1, subtract the input vector from the weight vector: w -= x

Perceptron Training Example

- □ Task: Train a classifier using perceptron on two data points:
- □ Initialize $w_1=w_2=b=0$.
- □ Apply a round-robin order on data points:
 - > 1. data: (1,1), $f(w^Tx+b)=-1$. Update $w_1=w_2=b=1$.
 - \triangleright 2. data: (3,-2), f(w^Tx+b)=1. Update w₁=-1, w₂=3, b=0.
 - > 3. data: (1,1), $f(w^Tx+b)=1$. Do nothing.
 - \triangleright 4. data: (3,-2), f(w^Tx+b)=-1. Do nothing.
 - Converged.



$$h(x) = f(w^T x + b)$$

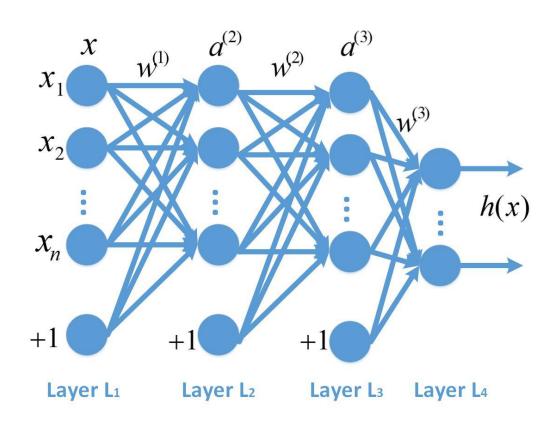
$$f(z) = \begin{cases} +1 & \text{if } z > 0 \\ -1 & \text{otherwise} \end{cases}$$

Training Perceptron

- ☐ Initialize all parameters to 0.
- □ Pick training cases using any policy that ensures that every training case will keep getting picked.
 - If the output unit is correct, leave its weights alone.
 - If the output unit incorrectly outputs a -1, add the input vector to the weight vector: w += x
 - ➤ If the output unit incorrectly outputs an 1, subtract the input vector from the weight vector: w -= x
- ☐ This is guaranteed to find a set of weights that gets the right answer for all the training cases if any such set exists.

Multi-layer Perceptrons

☐ Second generation (1980s)



Input and output of 2nd layer:

$$z^{(2)} = w^{(1)}x + b^{(1)}$$
$$a^{(2)} = f(z^{(2)})$$

Input and output of 3rd layer:

$$z^{(3)} = w^{(2)}a^{(2)} + b^{(2)}$$
$$a^{(3)} = f(z^{(3)})$$

Output layer:

$$h(x) = f(w^{(3)}a^{(3)} + b^{(3)})$$

Parameters { $w^{(1)}$, $w^{(2)}$, $w^{(3)}$, $b^{(1)}$, $b^{(2)}$, $b^{(3)}$ } to be learnt.

Parameter training

Preprocess the data?

- \Box A training set of m data points, $\{(x^{(1)}, y^{(1)}), ..., (x^{(m)}, y^{(m)})\}$
- □ Objective function

where,

$$\frac{1}{2m} \sum_{i=1 \atop l=1}^m \left\| h(x^{(i)}) - y^{(i)} \right\|^2 : \text{ average sum-of-squares error term}$$

$$\frac{\lambda}{2} \sum_{l=1}^m \left\| w^{(l)} \right\|_F : \text{ weight decay term; } L: \text{ the number of layers}$$

Parameter training

□ Task:

Find w, b minimize $E = [t - h(x)]^2$

- ☐ Algorithm:
 - 1. Initialize: w, b How to initialize?
 - 2. For data x and label t

Predict the label of x: $y = f(w^Tx + b)$

Update the parameters by gradient descent:

$$w \leftarrow w - \eta \left(\nabla_w E\right) \text{ and } b \leftarrow b - \eta \left(\nabla_b E\right)$$

where
$$E = [t - h(x)]^2$$

How to compute gradient?

3. Repeat until convergence

How to update parameters?

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Optimization algorithm

□ Gradient descent

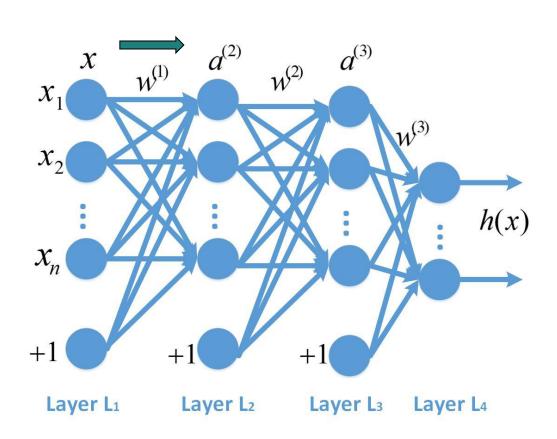
$$w_{ij}^{(l)} := w_{ij}^{(l)} - \alpha \frac{\partial H}{\partial w_{ij}^{(l)}}$$

$$b_i^{(l)} := b_i^{(l)} - \alpha \frac{\partial H}{\partial b_i^{(l)}}$$

Backpropagation algorithm: a systematic way

to compute
$$\frac{\partial H}{\partial w_{ij}^{(l)}}$$
 and $\frac{\partial H}{\partial b_i^{(l)}}$

 \square Perform a **feedforward pass**, computing the activations for layers L2, L3, and so on up to the output layer h(x).

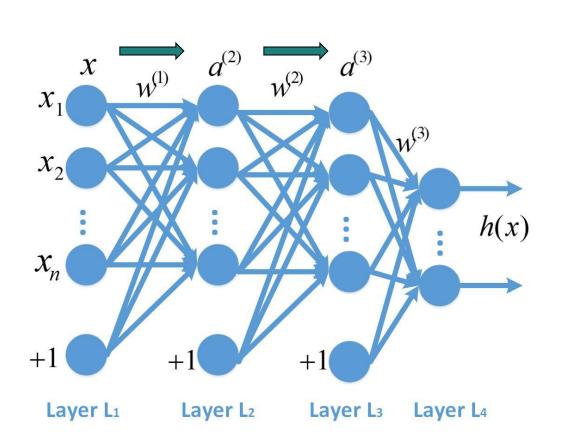


Input and output of 2nd layer:

$$z^{(2)} = w^{(1)}x + b^{(1)}$$

$$a^{(2)} = f(z^{(2)})$$

 \square Perform a **feedforward pass**, computing the activations for layers $\angle 2$, $\angle 3$, and so on up to the output layer h(x).



Input and output of 2nd layer:

$$z^{(2)} = w^{(1)}x + b^{(1)}$$

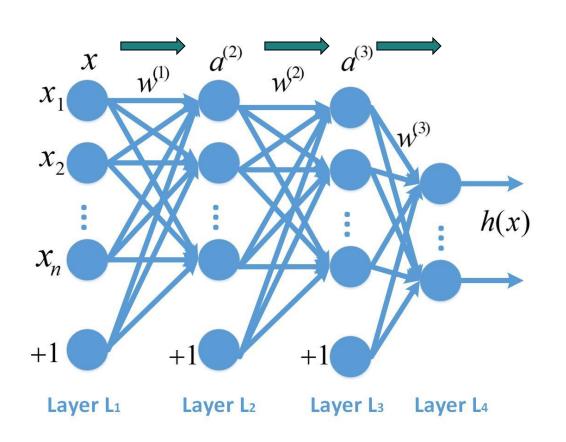
$$a^{(2)} = f(z^{(2)})$$

Input and output of 3rd layer:

$$z^{(3)} = w^{(2)}a^{(2)} + b^{(2)}$$

$$a^{(3)} = f(z^{(3)})$$

 \square Perform a **feedforward pass**, computing the activations for layers $\angle 2$, $\angle 3$, and so on up to the output layer h(x).



Input and output of 2nd layer:

$$z^{(2)} = w^{(1)}x + b^{(1)}$$

$$a^{(2)} = f(z^{(2)})$$

Input and output of 3rd layer:

$$z^{(3)} = w^{(2)}a^{(2)} + b^{(2)}$$

$$a^{(3)} = f(z^{(3)})$$

Output layer:

$$h(x) = f(w^{(3)}a^{(3)} + b^{(3)})$$

- \square Perform a feedforward pass, computing the activations for layers L_2 , L_3 , and so on up to the output layer h(x).
- ☐ For each output unit i in the output layer, set

$$\delta_{i}^{(L)} = \frac{\partial}{\partial z_{i}^{(L)}} \frac{1}{2} \|y - h(x)\|^{2} = -(y_{i} - a_{i}^{(L)}) \cdot f'(z_{i}^{(L)})$$

$$\sum_{x_{1} = x_{2} = x_{3} = x_{4}}^{x_{1}} \frac{\delta_{i}^{(L)}}{\lambda_{2}} \frac{\delta_{i}^{(L)}}{\lambda_{3}} \frac{\delta_{i}^{(L)}}{\lambda_{4}} \frac{\delta_{i}^$$

Chain rule

$$\delta_i^{(L)} = \frac{\partial}{\partial z_i^{(L)}} \frac{1}{2} \| y - h(x) \|^2 = -(y_i - a_i^{(L)}) \cdot f'(z_i^{(L)})$$

$$\delta_{i}^{L} = \frac{\partial ||y - h(x)||^{2}}{2 \partial z_{i}^{L}} = \frac{\partial ||y - h(x)||^{2}}{2 \partial a_{i}^{L}} \frac{\partial a_{i}^{L}}{\partial z_{i}^{L}}$$

- \square Perform a feedforward pass, computing the activations for layers L_2 , L_3 , and so on up to the output layer h(x).
- ☐ For each output unit /in the output layer, set

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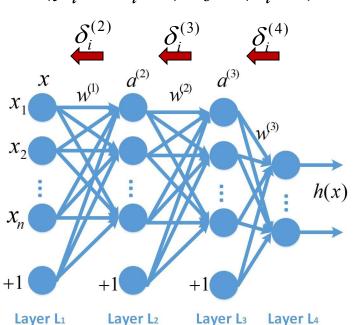
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- \square Perform a feedforward pass, computing the activations for layers L_2 , L_3 , and so on up to the output layer h(x).
- ☐ For each output unit /in the output layer, set

$$\delta_i^{(n_l)} = \frac{\partial}{\partial z_i^{(n_l)}} \frac{1}{2} \| y - h(x) \|^2 = -(y_i - a_i^{(n_l)}) \cdot f'(z_i^{(n_l)})$$

□ For $l = n_l - 1, n_l - 2, ..., 2$ For each node / in layer /, set

$$\delta_{i}^{(l)} = (\sum_{j=1}^{s_{l}+1} w_{ji}^{(l)} \delta_{j}^{(l+1)}) f'(z_{i}^{(l)})$$



Chain rule

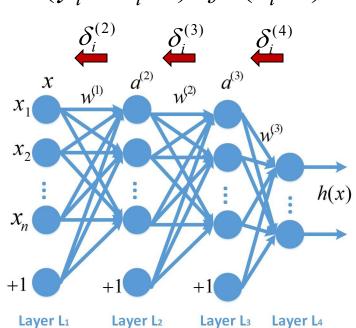
$$\begin{split} & \delta_{i} = \frac{\partial E}{\partial z_{i}} \\ & z_{i+1} = w_{i} f(z_{i}) + b_{i} \\ & \delta_{i} = \frac{\partial E}{\partial z_{i}} = \frac{\partial E}{\partial z_{i}} \frac{\partial z_{i+1}}{\partial z_{i}} = w_{i} \delta_{i+1} f'(z_{i}) \end{split}$$

- \square Perform a feedforward pass, computing the activations for layers L_2 , L_3 , and so on up to the output layer h(x).
- ☐ For each output unit /in the output layer, set

$$\delta_i^{(n_l)} = \frac{\partial}{\partial z_i^{(n_l)}} \frac{1}{2} \| y - h(x) \|^2 = -(y_i - a_i^{(n_l)}) \cdot f'(z_i^{(n_l)})$$

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 \square For $l = n_l - 1, n_l - 2, ..., 2$

For each node /in layer /, set

$$\delta_i^{(l)} = (\sum_{j=1}^{s_l+1} w_{ji}^{(l)} \delta_j^{(l+1)}) f'(z_i^{(l)})$$

 \square Compute the partial derivatives in each layer,

$$\frac{\partial H}{\partial w_{ii}^{(l)}} = a_j^{(l)} \delta_i^{(l+1)} + \lambda \cdot w_{ij}^{(l)} \quad ; \quad \frac{\partial H}{\partial b_i^{(l)}} = \delta_i^{(l+1)}$$

Gradient Checking (important!)

□ Definition of derivative For function $J(\theta)$ with parameter θ

$$\frac{d}{d\theta}J(\theta) = \lim_{\varepsilon \to 0} \frac{J(\theta + \varepsilon) - J(\theta - \varepsilon)}{2\varepsilon}$$

□ Comparison

$$\frac{\left\|A - B\right\|_F}{\left\|A + B\right\|_F} \le \delta$$

Where, A are the derivatives obtained by backpropagation; B are those obtained by definition;

$$\delta$$
, usually, $\leq 10^{-9}$

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Activation functions

□ Step function:

$$f(z) = \begin{cases} +1, z > 0 \\ 0, z \le 0 \end{cases}$$

□ Rectifier function:

$$f(z) = \max \{0, z\}$$

□ Sigmoid function

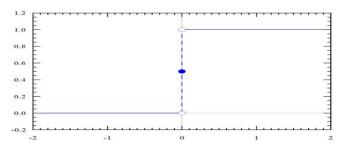
$$f(z) = \frac{1}{1 + e^{-z}}$$

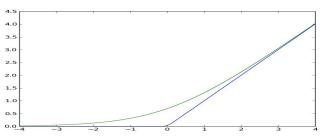
□ Hyperbolic tan function

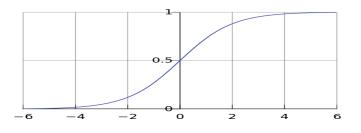
$$f(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

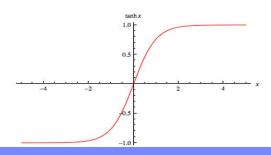
Stochastic binary neural

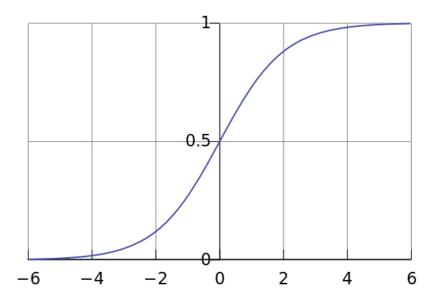
$$P(f(z) = 1) = \frac{1}{1 + e^{-z}}$$











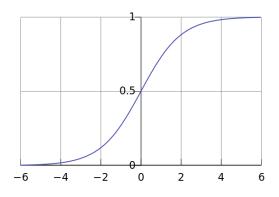
$$f(x) = \frac{1}{1 + e^{-x}}$$

Pros:

- ➤ Squashes numbers to range [0,1].
- ➤ Historically popular since they have nice interpretation as a "firing rate" of a neuron.

Cons:

- > exp() is a bit compute expensive.
- Saturated neurons "kill" the gradients.



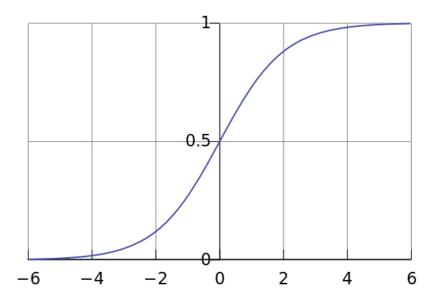
Forward pass:

$$x \longrightarrow f(x) = \frac{1}{1 + e^{-x}} \longrightarrow f(x)$$

Backward pass:

What happens when
$$x = -10$$
?
What happens when $x = 0$?
What happens when $x = 10$?

$$\frac{\partial H(x)}{\partial f(x)} \frac{\partial f(x)}{\partial x} \leftarrow f(x) = \frac{1}{1 + e^{-x}} \leftarrow \frac{\partial H(x)}{\partial f(x)}$$



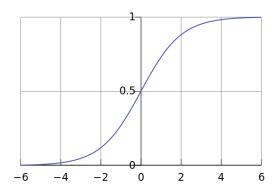
$$f(x) = \frac{1}{1 + e^{-x}}$$

Pros:

- ➤ Squashes numbers to range [0,1].
- ➤ Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron.

Cons:

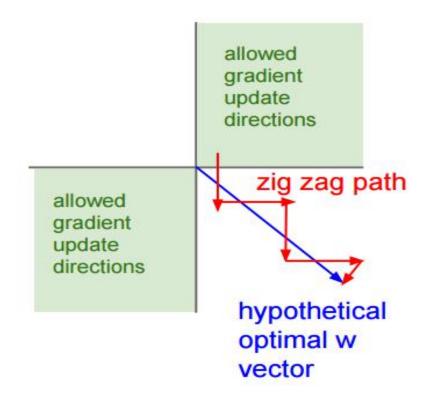
- > exp() is a bit compute expensive.
- Saturated neurons "kill" the gradients.
- Sigmoid outputs are not zerocentered.



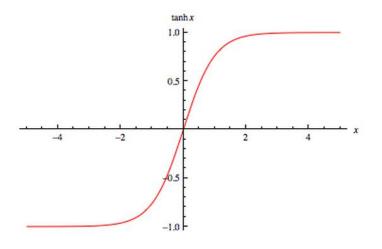
$$f(\sum_{i} w_{i} x_{i} + b)$$

Consider what happens when the input to a neuron x is always positive:

Always all positive or all negative!



Activation Function: tanh



$$f(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$$

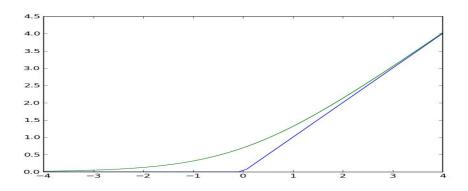
Pros:

- ➤ Squashes numbers to range [-1,1].
- Outputs are zerocentered.

Cons:

- > exp() is a bit compute expensive.
- Saturated neurons "kill" the gradients.

Activation Function: Rectified Linear Unit (ReLU)



$$f(x) = \max(x,0)$$

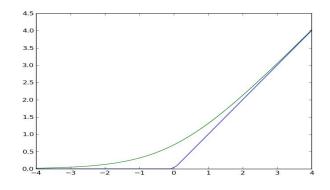
Pros:

- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

Cons:

- ➤ Not zero-centered output.
- An annoyance...

ReLU Function



What happens when x = -10? What happens when x = 0? What happens when x = 10? Forward pass:

$$x \longrightarrow f(x) = \max(0, x) \longrightarrow f(x)$$

Backward pass:

$$\frac{\partial H(x)}{\partial f(x)} \frac{\partial f(x)}{\partial x} \leftarrow f(x) = \max(0, x) \leftarrow \frac{\partial H(x)}{\partial f(x)}$$

- \square Perform a feedforward pass, computing the activations for layers $\angle 2$, $\angle 3$, and so on up to the output layer h(x).
- ☐ For each output unit /in the output layer, set

$$\delta_i^{(n_l)} = \frac{\partial}{\partial z_i^{(n_l)}} \frac{1}{2} \| y - h(x) \|^2 = -(y_i - a_i^{(n_l)}) \cdot f'(z_i^{(n_l)})$$

 \square For $l = n_l - 1, n_l - 2, ..., 2$

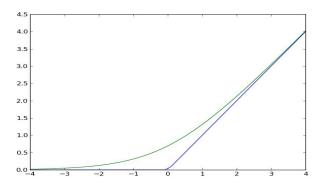
For each node /in layer /, set

$$\delta_i^{(l)} = (\sum_{j=1}^{s_l+1} w_{ji}^{(l)} \delta_j^{(l+1)}) f'(z_i^{(l)})$$

☐ Compute the partial derivatives in each layer,

$$\frac{\partial H}{\partial w_{ij}^{(l)}} = a_j^{(l)} \delta_i^{(l+1)} + \lambda \cdot w_{ij}^{(l)} \quad ; \qquad \frac{\partial H}{\partial b_i^{(l)}} = \delta_i^{(l+1)}$$

ReLU Function



What happens when x = -10? What happens when x = 0?

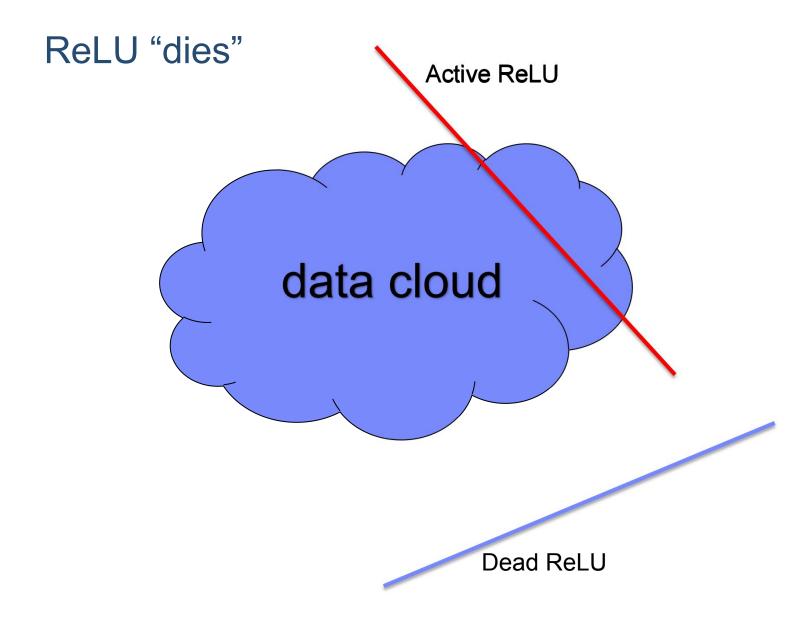
What happens when x = 10?

Forward pass:

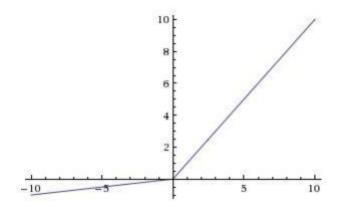
$$x \longrightarrow f(x) = \max(0, x) \longrightarrow f(x)$$

Backward pass:

$$\frac{\partial H(x)}{\partial f(x)} \frac{\partial f(x)}{\partial x} \leftarrow f(x) = \max(0, x) \leftarrow \frac{\partial H(x)}{\partial f(x)}$$



Alternatives



Leak ReLU:

$$f(x) = \max(0.01x, x)$$

Pros:

- > Does not saturate
- Very computationally efficient
- ➤ Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- > Will not die

Cons:

➤ Not zero-centered output.

Parametric ReLU (PReLU):

$$f(x) = \max(\alpha x, x)$$

Alternatives: Maxout

$$f(x) = \max(w_1^T x + b_1, w_2^T x + b_2)$$

Pros:

- > Does not saturate
- Very computationally efficient.
- Converges much faster than sigmoid/tanh in practice (e.g. 6x).
- ➤ Will not die.

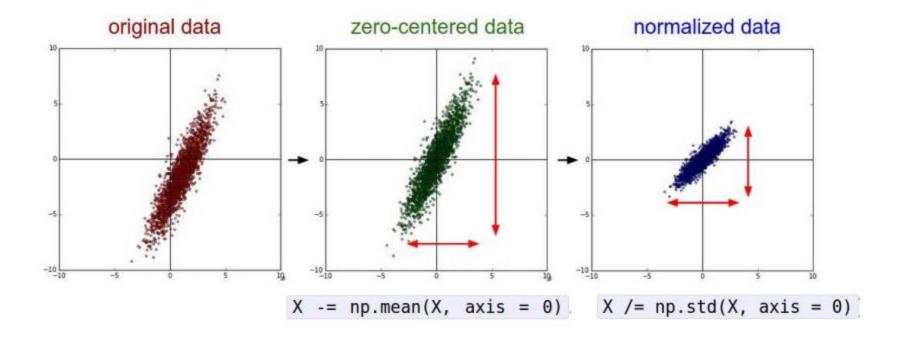
Cons:

- > Does not have the basic form of dot product nonlinearity.
- Not zero-centered output.
- ➤ Doubles the number of parameters.

Activation Function: in practise

- ➤ Use ReLU. Be careful with your learning rates.
- ➤ Try out Leaky ReLU/PReLU/Maxout.
- Try out tanh but don't expect much.
- > Don't use sigmoid.

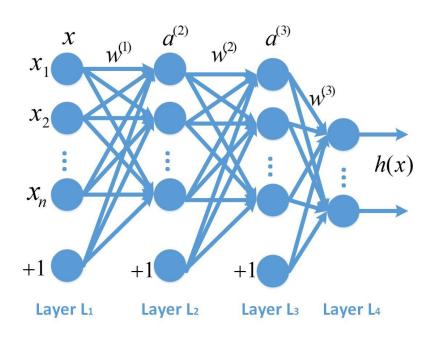
Data Preprocessing



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Weight initialization



1. Can we initialize W=0?



$$\delta_i^{(l)} = (\sum_{j=1}^{s_l+1} w_{ji}^{(l)} \delta_j^{(l+1)}) f'(z_i^{(l)})$$

2. Can we initialize W=const?



All nuerons will learn the same thing.

3. Can we initialize W=small random?



$$\delta_i^{(l)} = (\sum_{j=1}^{s_l+1} w_{ji}^{(l)} \delta_j^{(l+1)}) f'(z_i^{(l)})$$

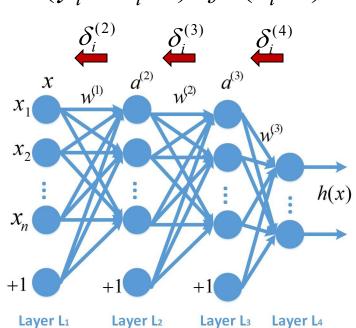
Backpropagation

- \square Perform a feedforward pass, computing the activations for layers L_2 , L_3 , and so on up to the output layer h(x).
- ☐ For each output unit /in the output layer, set

$$\delta_i^{(n_l)} = \frac{\partial}{\partial z_i^{(n_l)}} \frac{1}{2} \| y - h(x) \|^2 = -(y_i - a_i^{(n_l)}) \cdot f'(z_i^{(n_l)})$$

□ For $l = n_l - 1, n_l - 2, ..., 2$ For each node / in layer /, set

$$\delta_i^{(l)} = (\sum_{j=1}^{s_l+1} w_{ji}^{(l)} \delta_j^{(l+1)}) f'(z_i^{(l)})$$



Proper initialization is an active area of research...

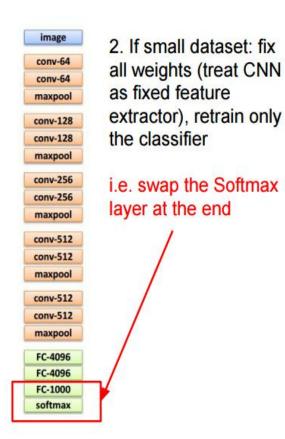
- □ Understanding the difficulty of training deep feedforward neural networks. Glorot and Bengio, 2010
- □ Exact solutions to the nonlinear dynamics of learning in deep linear neural networks. Saxe et al, 2013
- □ Random walk initialization for training very deep feedforward networks. Sussillo and Abbott, 2014
- □ Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification. He et al., 2015
- □ Data-dependent Initializations of Convolutional Neural Networks. Krähenbühl et al., 2015
- ☐ All you need is a good init. Mishkin and Matas, 2015

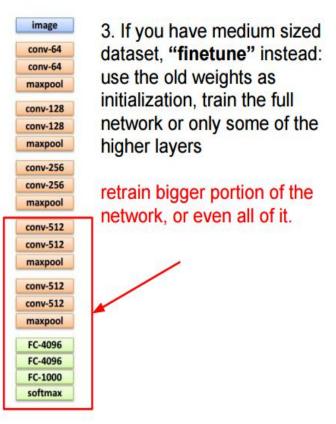
Why should we do?

- ☐ Try initialize initializate uniformly from [-1,1]
- ☐ Try initialize initializate from 0-mean normal distribution
- ☐ Try initialize parameters from well-trained models

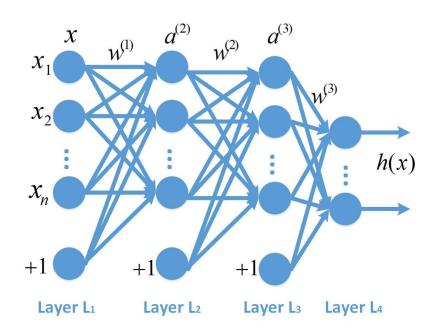
Transfer Learning with CNNs

image 1. Train on conv-64 **ImageNet** conv-64 maxpool conv-128 conv-128 maxpool conv-256 conv-256 maxpool conv-512 conv-512 maxpool conv-512 conv-512 maxpool FC-4096 FC-4096 FC-1000 softmax





Regularization



- □ Why need regularization?
 Overfitting!
- □ L2/L1 Regularization
- □ Dropout

min
$$H = \frac{1}{2m} \sum_{i=1}^{m} \|h(x^{(i)}) - y^{(i)}\|^2 + \frac{\lambda}{2} \sum_{l=1}^{L} \|w^{(l)}\|_F^2$$

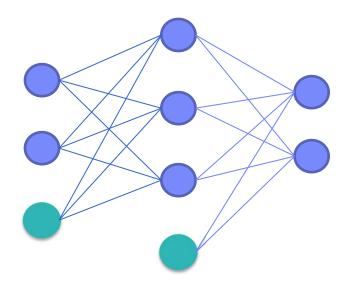
L2/L1 Regularization example

Demo

Dropout

- □ It appears to be hard to do massive model averaging in deep neural networks:
 - Each net takes a long time to learn.
 - At test time we don't want to run lots of different large neural nets.
- □ You need to use many different types of model and then combine them to make predictions at test time.
 - Decision Tree V.S. Random Forest

Dropout



Input hidden output

- Consider a neural net with one hidden layer.
- □ Each time we present a training example, we randomly omit each hidden unit with probability 0.5.
- ☐ So we are randomly sampling from 2^h different architectures.
 - All architectures share weights

Dropout Training

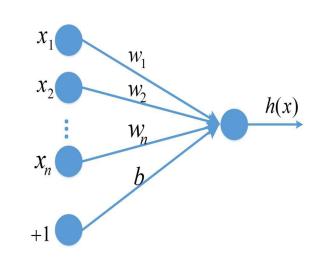
- □ We sample from 2^h models. So only a few of the models ever get trained, and they only get one training example.
- □ The sharing of the weights means that every model is very strongly regularized.
 - It's a much better regularizer than L2 or L1 penalties that pull the weights towards zero.
 - It pulls the weights towards what other models want.

Dropout Testing

- □ We could sample many different architectures and take the geometric mean of their output distributions.
- □ It is faster to use all of the hidden units, but to halve their outgoing weights.
 - This exactly computes the geometric mean of the predictions of all 2^h models.

A familiar example of dropout

- □ Do logistic regression, but for each training case, dropout all but one of the inputs.
- ☐ At test time, use all of the inputs.
- □ This is called "Naïve Bayes".

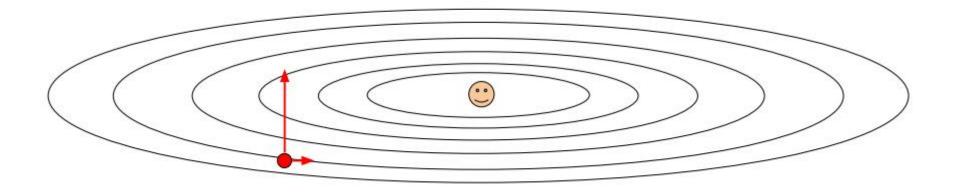


Outline

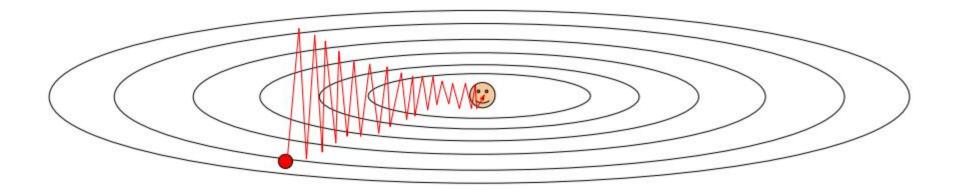
- □ Trainging Perceptron
- ☐ Trainging feed forward nueral networks (MLP)
 - Backpropagation algorithm
 - Gradient checking
 - Activation functions
 - Preprocessing
 - Weight initialization
 - Regularization
 - Parameter updates

Parameter updates: mini-batch gradient descent

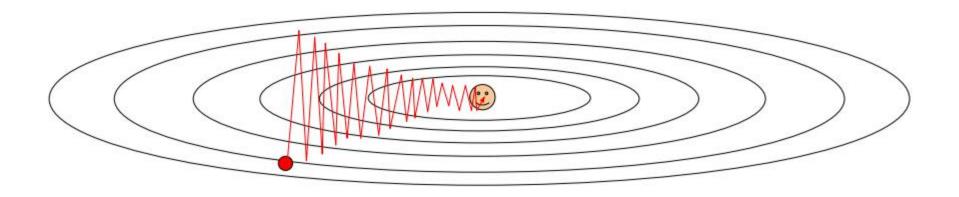
- □ Sample a batch of data
- ☐ Forward prop it through the network, get loss
- Backprop to calculate the gradients
- ☐ Update the parameters using the gradients
- □ Repeat until convergence



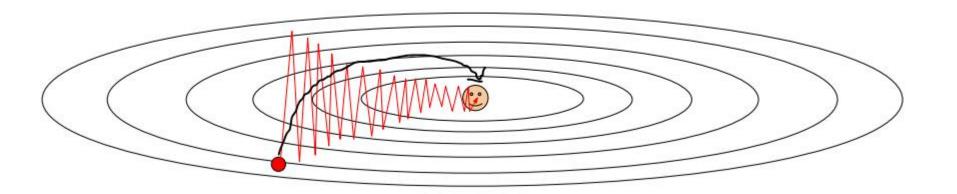
Suppose loss function is steep vertically but shallow horizontally



Suppose loss function is steep vertically but shallow horizontally



$$w = w - \eta \frac{\partial E}{\partial w}$$



$$v = \mu v - \eta \frac{\partial E}{\partial w}$$

$$w = w + v$$

where, $\mu = 0.5/0.9/0.99$

Parameter updates: 2nd order method

□ Second-order Taylor expansion:

$$f_T(x) = f_T(x_n + \Delta x) \approx f(x_n) + f'(x_n)\Delta x + \frac{1}{2}f''(x_n)\Delta x^2$$

 \square Search Δx that minimize $f(x+\Delta x)$:

$$0 = \frac{d}{d\Delta x} \left(f(x_n) + f'(x_n) \Delta x + \frac{1}{2} f''(x_n) \Delta x^2 \right) = f'(x_n) + f''(x_n) \Delta x$$
$$\Delta x = \frac{f'(x)}{f''(x)}, \quad x = x + \frac{f'(x)}{f''(x)}$$

□ Generalize to higher dimension:

$$\mathbf{x}_{n+1} = \mathbf{x}_n - [\mathbf{H}f(\mathbf{x}_n)]^{-1} \nabla f(\mathbf{x}_n), \ n \ge 0.$$

Parameter updates: L-BFGS

- □ Quasi-Newton methods (BGFS most popular): instead of inverting the Hessian (O(n^3)), approximate inverse Hessian with rank 1 updates over time (O(n^2) each).
- □ L-BFGS (Limited memory BFGS): Does not form/store the full inverse Hessian
- ☐ Usually works very well in full batch, but not for mini-batch
- □ No learning ratio

Outline

- □ Trainging Perceptron
- ☐ Trainging feed forward nueral networks (MLP)
 - Backpropagation algorithm
 - Gradient checking
 - Activation functions
 - Preprocessing
 - Weight initialization
 - Regularization
 - Parameter updates

Questions?

All together!

- ☐ Step 1: Preprocess the data
- ☐ Step 2: Choose the architecture
- □ Step 3: Implement the Backpropagation algorithm
 - Gradient check
- ☐ Step 3: Turn off regularization, have a try
 - Make sure that you can overfit small portion of the training data.
- ☐ Step 4: Start with small regularization, have a try
 - Make sure that you see the loss go down.
 - If not, adjust learning rate.