Practice Problems for CS 165 B

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1 Probability

1.1 Rolling a Die

Let X and Y be random variables representing the outcome of separate rolls of fair six-sided die.

1. Compute $P(X = 1)$

2. Compute $P(X = 1 \vee X = 2)$

3. Compute $P(X = 1 \wedge X = 2)$

4. Compute $P(X = 1 \vee Y = 1)$

5. Compute $P(X = 3 \wedge Y = 5)$

6. Compute $P(X = 3 | Y = 5)$

1.2 Dependent Variables

Let X and Y be two discrete variables with outcomes $\{a, b, c\}$ with a joint distribution:

$X \backslash Y$	a	b	c
a	1/6	3/12	0
b	1/12	1/4	1/32
c	0	7/32	0

1. Compute $P(X = a \wedge Y = c)$

2. Compute $P(X = b | Y = c)$

3. Compute $P(X = a | Y = a \vee Y = b)$

4. Compute the marginal probability $P(X = a)$

5. Compute the marginal probability $P(X = c)$

6. Compute the marginal probability $P(Y = a)$

7. Compute the marginal probability $P(Y = c)$

8. Compute $P(X = a \vee Y = c)$

9. Compute $P(X = b \vee Y = c)$

1.3 Continuous Random Variables

Let X be a continuous uniform random variable. That is there is an equal chance that it takes on any value from 0 to 1, and $P(X < 0) = 0, P(X > 1) = 0$.

1. Compute $P(X = .5)$

2. Compute $P(X \leq .5)$

3. Find the Probability $P(X \leq x)$ Given $0 \leq x \leq 1$

4. Find the density function $p(x)$ which gives the density for any value of x

1.4 Joint Distributions

Consider a joint continuous distribution over two uniform random variables, X and Y.

1. Compute the probability $P(X + Y \leq .5)$

2. Compute the marginal probability $P(X \leq .25)$

3. Compute the marginal probability $P(X \leq .25 | Y \leq .5)$

Now consider a variation on this problem wherein the density fucntion $p(x, y) = x + y$.

1. Compute the probability $P(X + Y \leq .5)$

2. Compute the marginal probability $P(X \leq .25)$

3. Compute the marginal probability $P(X \leq .25 | Y \leq .5)$

1.5 Bayes' Rule

Given two random events X and Y, and the facts that $P(X) = .25$, $P(Y|X) = .15$ and $P(\neg Y | \neg X) = .05$ compute $P(X|Y)$.

2   Linear Algebra

2.1   Matrix operations

Let A, B, C, and D be matrices:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 5 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & -2 & 3 \\ 9 & 2 & 8 \\ -6 & 7 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -1 \\ 5 & 2 \\ 4 & 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

1. Compute A^t

2. Compute $A \times I_3$

3. Compute $A + B$

4. Compute $A \times B$

5. Compute $A \times C$

6. Compute $C \cdot C^t$

7. Compute $\sum_{i=1}^{|C_1|}(C_{1,i})^2$

3   Probability and Linear Algebra

If we have a a weighted die with values:

$$X = \{1, 2, 3, 4, 5, 6\}$$

Which have probabilities

$$P = \{1/2, 1/20, 1/10, 1/5, 1/20, 1/10\}$$

Compute the Expected value as $X \cdot P^t$
Compute the variance.