Math 132A Assignment 3
Due: Wednesday, January 29th at Midnight on Gradescope.

1. Find the first three terms of the Taylor series for

$$f(x_1, x_2) = 3x_1^4 - 2x_1^3x_2 - 4x_1^2x_2^2 + 10x_1x_2^3 + 2x_2^4$$

at the point $x_0 = (1, -1)^T$. Evaluate the sum of these first three terms at $p = (0.1, 0.01)$. What is the approximate difference of this with the true value of $f(x_0 + p)$?

2. Find the first three terms of the Taylor series for

$$f(x_1, x_2) = \sqrt{x_1^2 + x_2^2}$$

at the point $x_0 = (3, 4)^T$.

3. Determine all possible local minimiza (if any) of the following functions:

   (a) $f(x_1, x_2) = x_1^4 + x_2^4 - 4x_1x_2$.

   (b) $f(x_1, x_2) = x_1^2 - 2x_1x_2^2 + x_2^4 - x_2^5$

   (c) $.f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + 5x_3^2 - 2x_1x_2 - 4x_2x_3 - 2x_3$.

4. Determine whether or not the following matrices are positive definite:

   (a) $\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

   (b) $\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

   (c) $\begin{bmatrix} 5 & 7 & 6 \\ 7 & 10 & 8 \\ 6 & 8 & 10 \end{bmatrix}$