Math 115A   Winter 2024   Homework Assignment 2

Due Friday, February 2.

1. Let $f(x)$ be a non-negative continuous function defined on the interval $Q \le x \le R$. Prove that the number of lattice points in the region $Q < x \le R$, $0 < y \le f(x)$ is equal to

$$\sum_{Q<n\le R} [f(n)].$$

2. Suppose P and Q are odd numbers, $(P, Q) = 1$. Prove that

$$\sum_{0<m<P/2} \left[\frac{P}{Q}m\right] + \sum_{0<n<P/2} \left[\frac{Q}{P}n\right] = \frac{P-1}{2}\,\frac{Q-1}{2}.$$

3. Suppose α is a real number, c is an integer, $c > 0$. Prove that

$$\left[\frac{[\alpha]}{c}\right] = \left[\frac{\alpha}{c}\right].$$

4. Suppose $X > 1$. Prove that

$$\sum_{n\le X} \tau(n) = \sum_{n\le X} \left[\frac{X}{n}\right].$$

5. Suppose $X > 1$. Prove that

$$\sum_{n\le X} \tau(n) = 2 \sum_{n\le\sqrt{X}} \left[\frac{X}{n}\right] - [\sqrt{X}]^2.$$

6. Suppose $X > 1$. Prove that

$$\sum_{n\le X} \mu(n) \left[\frac{X}{n}\right] = 1.$$

7. Suppose $X > 1$. Prove that

$$\sum_{n\le X} \Lambda(n) \left[\frac{X}{n}\right] = \sum_{l\le X} \log l.$$