Math 115A   Winter 2024   Homework Assignment 3

Due Friday, February 23.

1. Suppose

$$n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$$

where p_1, p_2,...,p_k are distinct prime numbers and $e_1, e_2, ..., e_k > 0$. Prove that

$$\sum_{d|n} \frac{\mu(d)}{d} = \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_k}\right).$$

2. Suppose S is a complete system of residues $(\operatorname{mod} n)$. Prove that for any integers a and b with $(a, n) = 1$, the set

$$\{am + b : \quad m \in S\}$$

is a complete system of residues $(\operatorname{mod} n)$.

3. Suppose X is a reduced system of residues $(\operatorname{mod} n)$. Prove that for any integer a with $(a, n) = 1$, the set

$$\{am : \quad m \in X\}$$

is a reduced system of residues $(\operatorname{mod} n)$.

4. Suppose p is a prime number. Use induction to prove, for any integer n, that

$$n^p \equiv n (\operatorname{mod} p).$$

5. Solve the congruence $256x \equiv 179 (\operatorname{mod} 337)$.

6. Solve the congruence $1215x \equiv 560 (\operatorname{mod} 2755)$.