Jul16

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Exponential Weights

Let the symbol ϕ be defined by $\phi(z) = z^a \overline{z}^b$, and let the function f be defined by $f(z) = z^n \overline{z}^m$. We compute

$$T_\phi(f) = P(\phi f) = \int_{\mathbb{D}} B_\lambda(z,w) f(w) \phi(w) \lambda(w) dA(w)$$

where

$$\lambda(w) = \exp\left(- \frac{1}{1 - |w|^2} \right)$$

We write

$$B_\lambda(z,w) = \sum_{j=0}^{\infty} \frac{z^j \overline{w}^j}{\|z^j\|^2}$$

so that we obtain

$$\int_{\mathbb{D}} B_\lambda(z,w) f(w) \phi(w) \lambda(w) dA(w) = \sum_{j=0}^{\infty} \frac{z^j}{\|z^j\|^2} \int_{\mathbb{D}} w^{j+a+n} \overline{w}^{b+m} \lambda(w) dA(w)$$

Because the weight λ is radial, we note that the integral is only nonzero when $j+a+n = b+m$, or when $j = b+m-(a+n)$. In order for this to occur, we note that $b+m-(a+n)$ must be nonnegative. So we have

$$\frac{z^{b+m-a-n}}{\|z^{b+m-a-n}\|^2} \int_{\mathbb{D}} w^{b+m} \overline{w}^{b+m} \lambda(w) dA(w) = z^{b+m-a-n} \frac{\|z^{b+m}\|^2}{\|z^{b+m-a-n}\|^2}$$

where the norm is taken in the weighted Bergman space.