Math 332 A - Mathematical Statistics

Ethan Jensen

January 31, 2020

HW p.254 #48,49,50 p.257 #88

1 p. 254 #48

Find the mean and the variance of the sampling distribution of Y_1 for random samples of size n from a continous uniform population with $\alpha = 0$ and $\beta = 1$.

By Theorem 8.16 we can find the density of Y_1.

$$g_1(y_1) = nf(y_1)\left[\int_{y_1}^{\infty} f(x)dx\right]^{n-1}$$

Since Y_1 comes from a continuous uniform population with $\alpha = 0$ and $\beta = 1$,

$$f(x) = 1,\ 0 \le x \le 1$$

Thus, computing $g_1(y_1)$ we have

$$g_1(y_1) = n \cdot 1 \cdot \left[\int_{y_1}^{1} 1dx\right]^{n-1} = n(1-y_1)^{n-1},\ 0 \le y_1 \le 1$$

Now we compute the first and second moments about the origin.

$$\mu_1' = \int_0^1 n(y_1)(1-y_1)^{n-1}dy_1 = \int_0^1 n(1-x)x^{n-1}dx = \frac{1}{n+1}$$

$$\mu_2' = \int_0^1 n(y_1)^2(1-y_1)^{n-1}dy_1 = \int_0^1 n(1-x)^2x^{n-1}dx = \frac{2}{(n+1)(n+2)}$$

By Theorem 4.6

$$\sigma_{y_1}^2 = \mu_2' - {\mu_1'}^2$$

$$\sigma_{y_1}^2 = \frac{2}{(n+1)(n+2)} - \frac{1}{(n+1)(n+1)} = \frac{n}{(n+1)^2(n+2)}$$

$$\boxed{\begin{aligned} \mu_{y_1} &= \tfrac{1}{n+1} \\ \sigma_{y_1}^2 &= \tfrac{n}{(n+1)^2(n+2)} \end{aligned}}$$

HW p.254 #48,49,50 p.257 #88

2 p. 254 #49

Find the sampling distributions of Y_1 and Y_n for random samples of size n from a population having the beta distribution with $\alpha = 3$ and $\beta = 2$.

The density of a beta R.V. with $\alpha = 3$ and $\beta = 2$ is given by

$$f(x) = \frac{\Gamma(3+2)}{\Gamma(2)\Gamma(3)} x^2(1-x)^1 = 12x^2(1-x),\ 0 < x < 1$$

By Theorem 8.16 we have

$$g_1(y_1) = nf(y_1)\left[\int_{y_1}^{\infty} f(x)dx\right]^{n-1}$$

$$g_n(y_n) = nf(y_2)\left[\int_{-\infty}^{y_2} f(x)dx\right]^{n-1}$$

Plugging in our density function and its range we have

$$g_1(y_1) = n12y_1^2(1-y_1)\left[\int_{y_1}^{1} 12x^2(1-x)dx\right]^{n-1}$$

$$\boxed{g_1(y_1) = 12^n n y_1^2(1-y_1)^{1+2(n-1)}(3y_1^2+2y_1+1)^{n-1},\ 0 < y_1 < 1}$$

$$g_n(y_n) = n12y_2^2(1-y_2)\left[\int_{0}^{y_2} 12x^2(1-x)dx\right]^{n-1}$$

$$\boxed{g_n(y_n) = 12^n n y_n^{2+3(n-1)}(y_n-1)(3y_n-4)^{n-1},\ 0 < y_n < 1}$$

HW p.254 #48,49,50 p.257 #88

3 p. 254 #50

Find the sampling distribution of the median for random samples of size $2m+1$ for the population from Exercise 8.49.

From Exercise 8.49 the density of the population is given by

$$f(x) = 12x^2(1-x),\ 0 < x < 1$$

By Theorem 8.16 we have

$$h(\tilde{x}) = \frac{(2m+1)!}{m!m!}\left[\int_{-\infty}^{\tilde{x}} f(x)dx\right]^m f(\tilde{x})\left[\int_{\tilde{x}}^{\infty} f(x)dx\right]^m$$

Plugging in our density function and its range we have

$$h(\tilde{x}) = \frac{(2m+1)!}{m!m!}\left[\int_{0}^{\tilde{x}} 12x^2(1-x)dx\right]^m 12\tilde{x}^2(1-\tilde{x})\left[\int_{\tilde{x}}^{1} 12x^2(1-x)dx\right]^m$$

$$\boxed{h(\tilde{x}) = \frac{12^{2m+1}(2m+1)!}{m!m!}\tilde{x}^{3m+2}(1-\tilde{x})^{2m+1}(3\tilde{x}^2+2\tilde{x}+1)^m(4-3\tilde{x})^m}$$

HW p.254 #48,49,50 p.257 #88

4 p. 257 #88

Find the probability that in a random sample of size $n = 4$ from the continous uniform population of Exercise 8.46, the smallest value will be at least 0.20.

As seen from Exercise 8.48, the density of the population is given by

$$f(x) = 1,\ 0 < x < 1$$

By Theorem 8.16 the density of the smallest value of the population is

$$g_1(y_1) = 4(1 - y_1)^3,\ 0 < y_1 < 1$$

We want to find $P(y_1 > 0.20)$.

$$P(y_1 > 0.20) = \int_{0.20}^{1} 4(1 - y_1)^3 dy_1$$

$\boxed{P(y_1 > 0.20) = 0.4096 \text{ or about } 40.96\%}$