Math 332 A - Mathematical Statistics

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HW p.380 #80,#82 p.378 #42, p.448 #18

1 p. 380 #80

Four coins were tossed 160 times and 0,1,2,3, or 4 heads showed, respectively, 19, 54, 58, 23, and 6 times. Use the 0.05 level of significance te suppose that the coins are balanced and randomly tossed.

H_0 : The population is binomial with $\theta = 0.5, n = 4$.
H_1 : The population is not binomial with $\theta = 0.5, n = 4$.
$\alpha = 0.05$

Num. heads	**Totals**
0	19
1	54
2	58
3	23
4	6

The expected values for a binomial population with $\theta = 0.5, n = 4$ and a sample size of 160 is:

Num. heads	**Totals**
0	10
1	40
2	60
3	40
4	10

$\sum_{i=0}^{4} \frac{(f_i - e_i)^2}{e_i} = 18.89167$
However, the value of $\chi^2_{\alpha=0.05,4} = 9.488$, which is less than our sum.
Thus, we must reject the null hypothesis.

$\boxed{\text{The coins are not equally weighted.}}$

2 p. 380 #82

Each day, Monday through Saturday, a baker bakes three large chocolate cakes, and those not sold on the same day are given away to a food bank. Use the data shown in the following table to test at the 0.05 level of significance whether they may be looked upon as values of a binomial random variable:

Number of cakes sold	**Number of days**
0	1
1	16
2	55
3	228

The total number of days is 300. The toal number of cakes sold is 810.
Thus, the expected number of cakes sold per day is 2.7 cakes.
Thus, we test the population against a binomial population with $\theta = 2.7/3 = 0.9$.
The expected values for a binomial population with $\theta = 0.0, n = 3$ and a sample size of 300 is:

Number of cakes sold	**Number of days**
0	0.3
1	8.1
2	72.9
3	218.7

$\sum_{i=0}^{3} \frac{(f_i - e_i)^2}{e_i} = 14.12894019$
However, the value of $\chi^2_{\alpha=0.05,3} = 7.815$, which is less than our sum.
Thus, we must reject the null hypothesis.

$\boxed{\text{The cakes sold are not binomially distributed.}}$

3 p. 378 #42 (part 1)

To compare two kinds of front-end designs, six of each kind were installed on a certain make of compact car. Then each car was run into a concrete wall at 5 miles per hour, and the following are the costs of repairs (in dollars):

Design 1: 127 168 143 165 122 139
Design 2: 154 135 132 171 153 149

Use the four steps of page 354 to test at the 0.01 level of significance whether the difference between means of these two samples is significant.

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$
$\alpha = 0.01$

By some simple calculations, $\overline{x}_1 = 144$, $\overline{x}_2 = 149$, $s_1^2 = 302.67$, $s_2^2 = 168.33$, $s_p^2 = 235.5$
We will perform a two tailed test with t as our test statistic.

$$t = \frac{\overline{x}_2 - \overline{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{149 - 144}{15.346\sqrt{\frac{1}{6} + \frac{1}{6}}} = 0.564332$$

$$t_{0.005,10} = 3.169$$

Since $0.564332 < 3.169$, we accept the null hypothesis.

4 p. 378 #42 (part 2)

Design 1: 127 168 143 165 122 139
Design 2: 154 135 132 171 153 149

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$
$\alpha = 0.01$

By some simple calculations, $\overline{x}_1 = 144,\ \overline{x}_2 = 149,\ SS(Tr) = 75, SSE = 2826$
$MS(Tr) = 75$. $MSE = 282.6$
We will perform a one tailed test with f as our test statistic.

$$f = \frac{MS(Tr)}{MSE} = 0.26539$$

$$f_{0.01,1,5} = 16.3$$

Since $0.26539 < 16.3$, we accept the null hypothesis.

5 p. 448 #18

Three groups of siz guinea pigs each were injected, respectively, with 0.5 milligram, 1.0 milligram, and 1.5 milligrams of a new tranquilizer, and the following are the number of minutes it took them to fall asleep:

0.5 mg: 21, 23, 19, 24, 25, 23
1.0 mg: 19, 21, 20, 18, 22, 20
1.5 mg: 15, 10, 13, 14, 11, 15

Test at the 0.05 level of significance whether the null hypothesis that differences in dosage have no effect can be rejected. Also, estimate the parameters μ, α_1, α_2, and α_3 of the model used in the analysis.

I finally decided to use an online ANOVA calculator.
The result from the calculator gives a test-statistic value of $f_{0.05,2,15} = 39.32432$, which has an extremely small p-value (less than 0.0001).
The null hypothesis can certainly be rejected since the p-value is smaller than 0.05.

Also, by some simple calculations -

$$\mu = \overline{\overline{x}} = 18.5$$

$$\alpha_1 = \overline{x}_1 - \mu = 4$$

$$\alpha_2 = \overline{x}_2 - \mu = 1.5$$

$$\alpha_3 = \overline{x}_3 - \mu = -5.5$$