



# Radiation Detection and Measurement

Lecture 10

Chapter 4: General properties of  
radiation detectors

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# Detection efficiency

- In general, all radiation detectors will give rise to pulse for each quantum of radiation that interacts with its active volume.
- For  $\alpha$  and  $\beta$  particles, which leave a large ionization trail, it is easy to arrange these detectors to have 100% efficiency.
- However, uncharged particles tend to travel large distances between interactions where some may not trigger inside the active volume (photons & neutrons), and have efficiencies less than 100%

# Detection efficiency

- Absolute efficiency:
  - $\varepsilon_{\text{abs}} = \# \text{ of pulse recorded} / \# \text{ of quanta emitted.}$
- Intrinsic efficiency:
  - $\varepsilon_{\text{int}} = \# \text{ of pulses recorded} / \# \text{ of quanta incident on detector.}$  And the # of quanta incident on the detector is the fraction of the solid angle subtended by the detector:

$$\# \text{ of incident quanta} = \frac{\Omega}{4\pi} \times \# \text{ of radiation quanta emitted}$$

# Detection efficiency

- By substitution  $\varepsilon_{abs} = \frac{\Omega}{4\pi} \cdot \varepsilon_{int}$ , where  $\frac{\Omega}{4\pi}$  is also called the geometric efficiency.
- The peak efficiency is then the quanta that deliver their full energy to the detector. This will generally be at the high end of the spectrum in a peak, where lower energy deposition contributes to the spectrum at lower energies.

# Detection efficiency

- Total efficiency is the total count over all energies (the area under the entire curve on a differential pulse height spectrum),  
 $\epsilon_{\text{total}}$ .

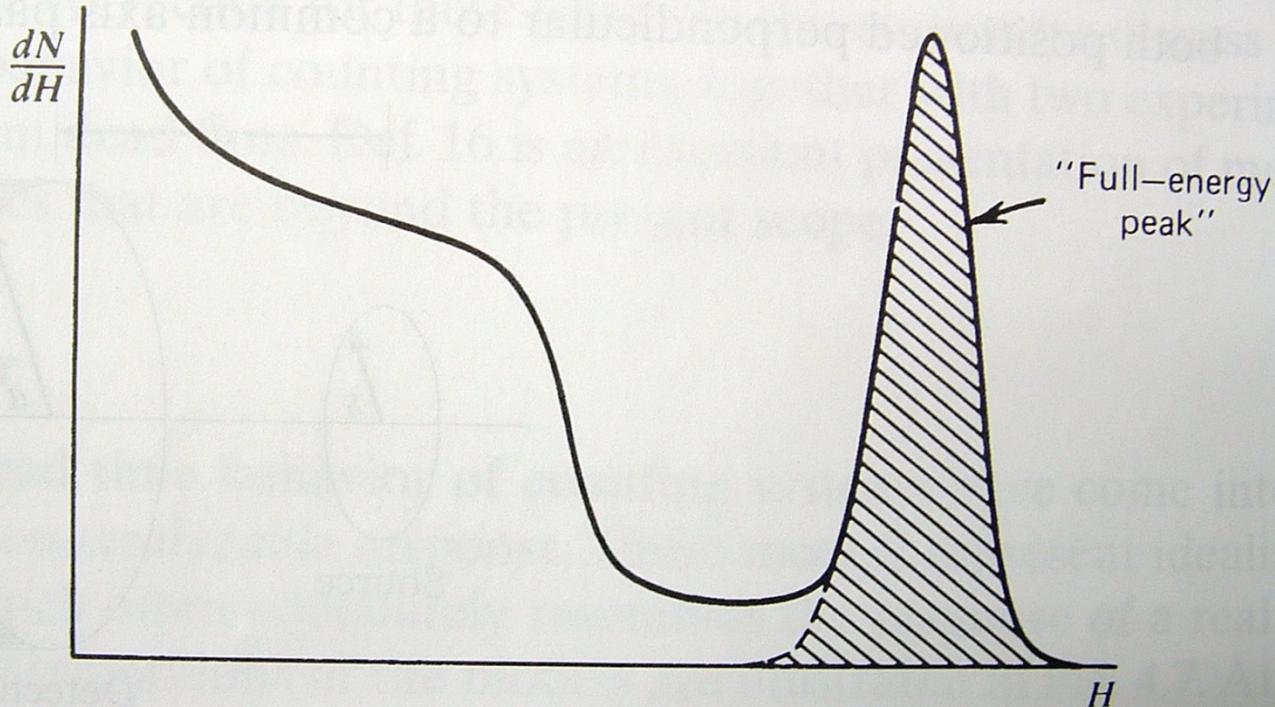
# Detection efficiency

- Total efficiency is the total count over all energies (the area under the entire curve on a differential pulse height spectrum),  $\varepsilon_{total}$ .
- The peak efficiency is then the area under the curve at the peak,  $\varepsilon_{peak}$ .
- Then the peak-to-total ratio,  $r$ , is:
- The intrinsic peak efficiency is then:

$$r = \frac{\varepsilon_{peak}}{\varepsilon_{total}}$$

$$\varepsilon_{IP} = \frac{\# \text{ of recorded pulses in full energy peak}}{\# \text{ of quanta incident on detector}}$$

# Full energy peak example



**Figure 4.6** Example of the full-energy peak in a differential pulse height spectrum

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# Detection efficiency

- Inversely, the number of pulses recorded in the full energy peak, given S number of quanta from the source:

$$N = \varepsilon_{IP} \left( \frac{\Omega}{4\pi} \right) S \quad \text{for a point source}$$

# Detection efficiency

- The subtended angle in steradians is:

$$\Omega = \int_A \frac{\cos(\alpha)}{r^2} dA$$

- Where  $r$  is the distance from the source to the detector element  $dA$ , and  $\alpha$  is angle between the normal to the surface element  $dA$  and the source direction.
- For a large volume source, an integral over the volume of the source must also be included.

# Detection efficiency

- For a point source and a right circular cylindrical detector of radius  $a$ :

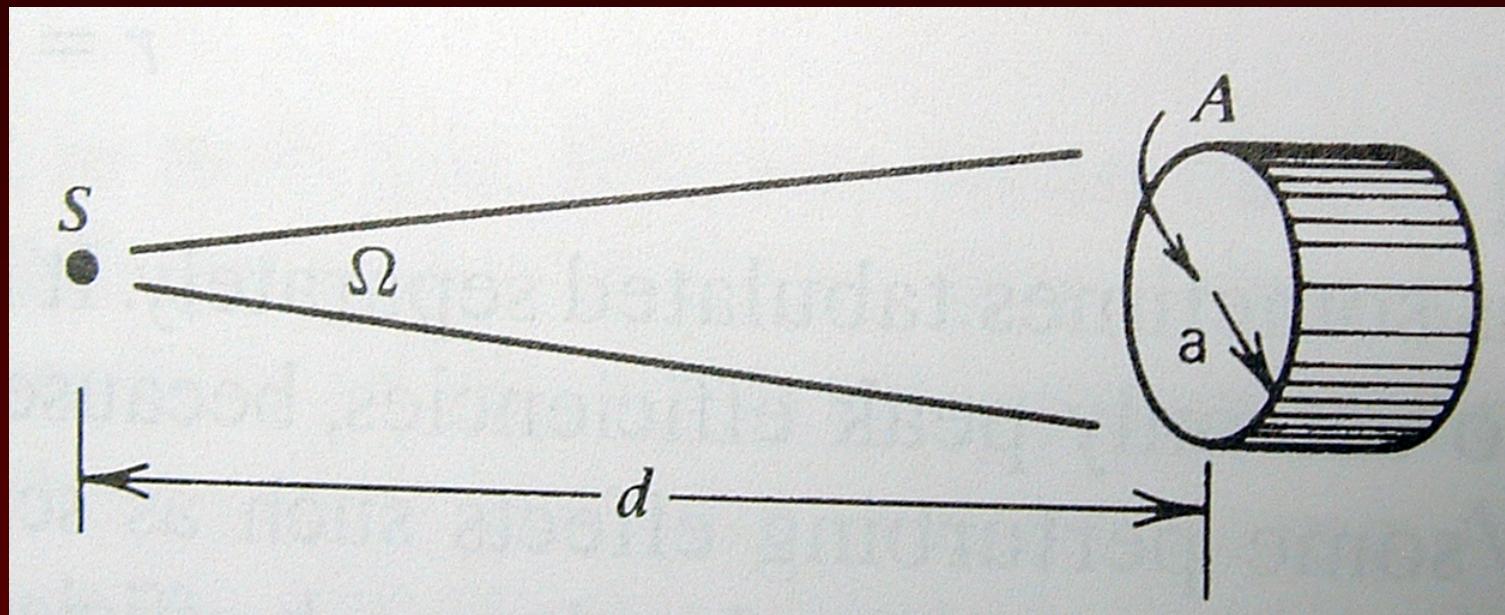
$$\Omega = 2\pi \left( 1 - \frac{d}{\sqrt{d^2 + a^2}} \right)$$

- $\Omega$  reduces for  $d \gg a$  to:

$$\Omega \simeq \frac{A}{d^2} = \frac{\pi a^2}{d^2}$$

– Where  $d$  is the distance from the detector.

# Right cylinder geometry



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# Detection efficiency

- For another common configuration of a circular disk ( $r = s$ ) source emitting isotropic radiation aligned with a circular disk ( $r = a$ ) detector both positioned perpendicular to a common axis through their centers a distance  $d$  apart:

$$\Omega = \frac{4\pi a}{s} \int_0^{\infty} \frac{\exp(-dk) J_1(sk) J_1(ak)}{k} dk$$

– Where  $J_1(x)$  are the Bessel functions of  $x$

# Detection efficiency

- Through a numerical solution method one can approximate the answer to:

$$\Omega \cong 2\pi \left[ 1 - \frac{1}{(1+\beta)^{\frac{1}{2}}} - \frac{3}{8} \cdot \frac{\alpha\beta}{(1+\beta)^{\frac{5}{2}}} + \alpha^2[F1] - \alpha^3[F2] \right]$$

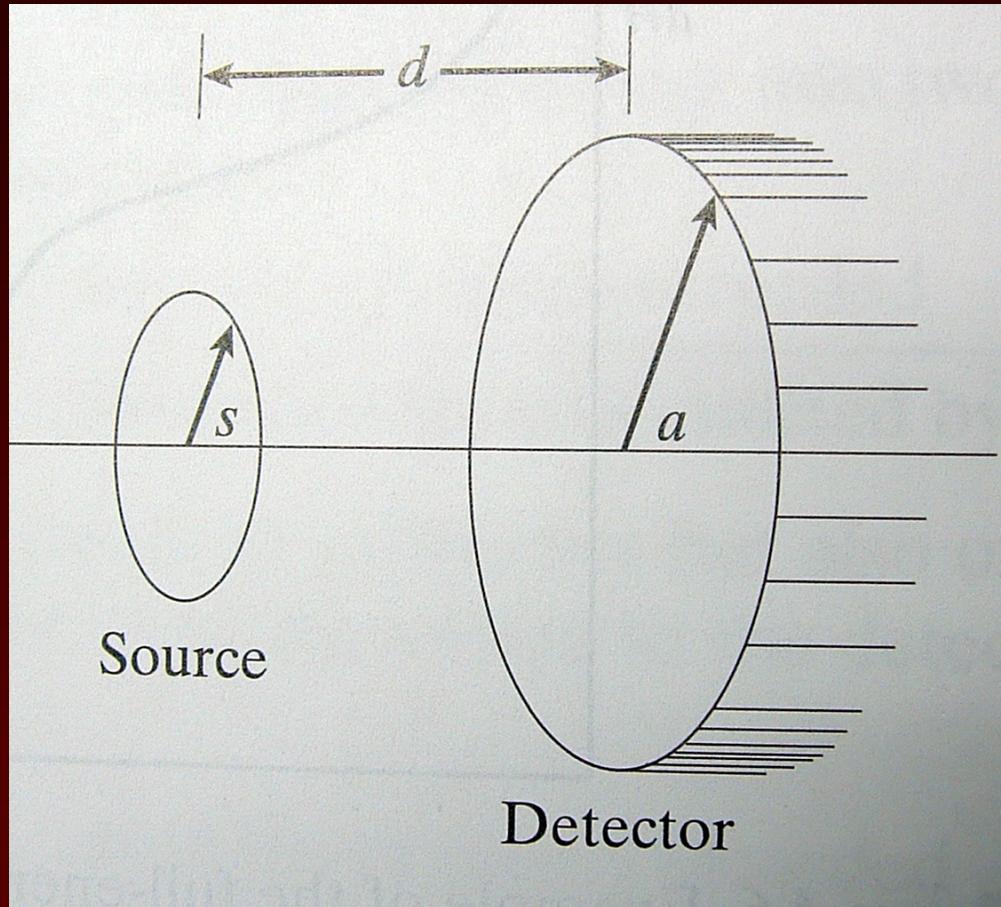
- Where:

$$F1 = \frac{5}{16} \cdot \frac{\beta}{(1+\beta)^{\frac{7}{2}}} - \frac{35}{16} \cdot \frac{\beta^2}{(1+\beta)^{\frac{9}{2}}}$$

$$F2 = \frac{35}{128} \cdot \frac{\beta}{(1+\beta)^{\frac{9}{2}}} - \frac{315}{256} \cdot \frac{\beta^2}{(1+\beta)^{\frac{11}{2}}} + \frac{1155}{1024} \cdot \frac{\beta^3}{(1+\beta)^{\frac{13}{2}}}$$

- And  $\alpha \equiv (a/d)^2$ ,  $\beta \equiv (s/d)^2$ , which becomes inaccurate when source or detector diameters get too large compared with  $d$ .

# Uniform cylindrical disk geometry

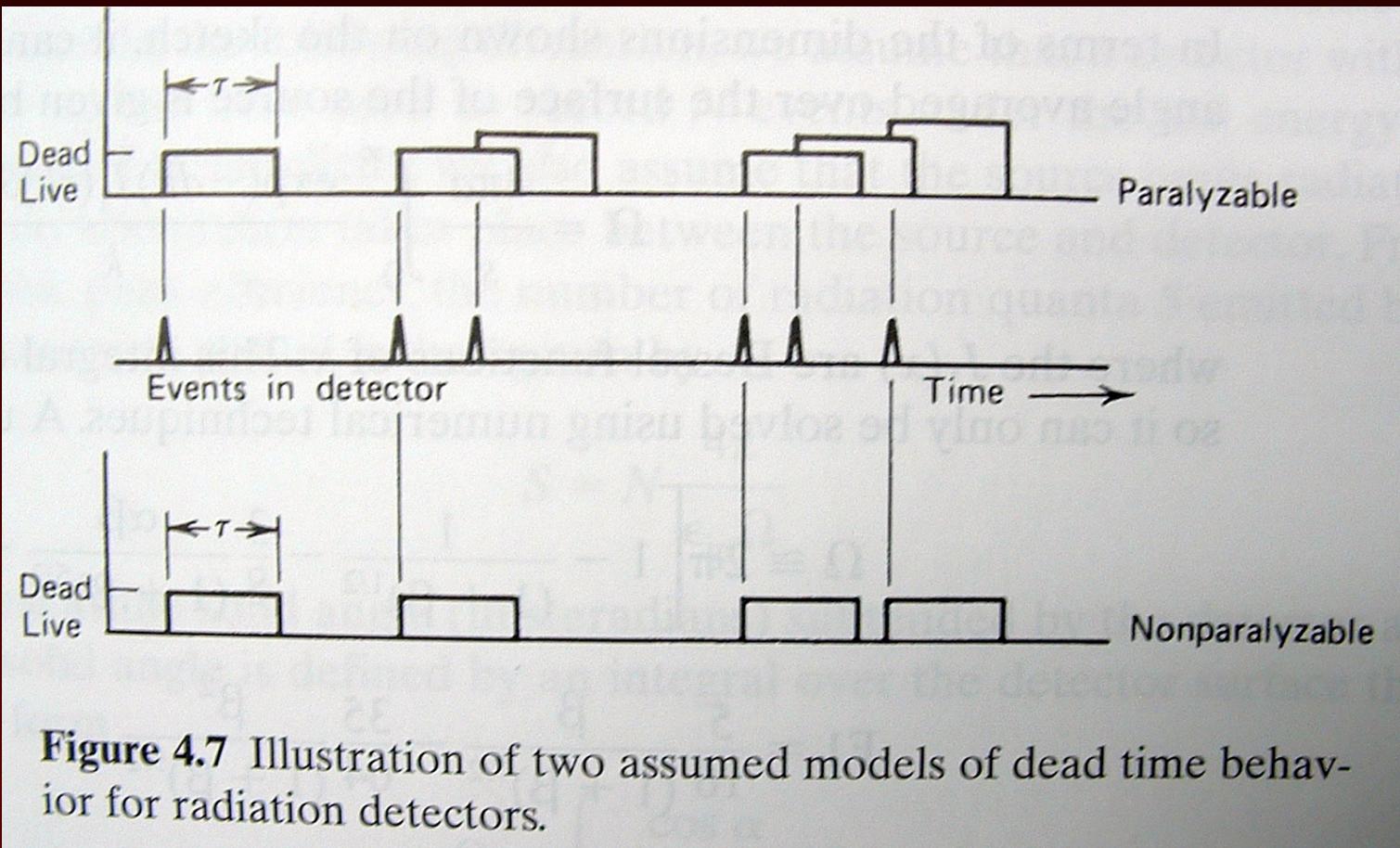


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# Dead time

- Dead time is the time required to separate two events so that they are recorded as two pulses.
- There are two models of dead time behavior:
  - Paralyzable: true events that occur during the dead period, although not recorded, are assumed to extend the dead time by another period following the lost event.
  - Non-paralyzable: true events that occur during the dead time are lost and assumed to have no effect on the behavior of the detector.
- We define the following variables:  $n$ -true interaction rate;  $m$ -recorded count rate;  $\tau$ -system dead time.

# Dead time models



**Figure 4.7** Illustration of two assumed models of dead time behavior for radiation detectors.

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# Non-paralyzable System

- The total time dead is  $m\tau$ , and the loss rate is  $nm\tau$  and  $n-m$ , therefore
  - $n-m = nm\tau$ .
- For the true count rate:

$$n = \frac{m}{1 - m\tau}$$

# Paralyzable System:

- Dead periods are not a fixed length, so we need a different analysis, but we note  $m$  is also the rate of occurrences of time intervals between true events which exceed  $\tau$ .
- We invoke the distribution of intervals between random events occurring at an average rate  $n$  to be:

$$P_1(T)dT = ne^{-nT}dT$$

- Where  $P_1(T)$  is the probability of observing an interval whose length lies within  $dT$  about  $T$ . (Recall

$$P_1(T) = P(0) \times ndt \text{ and } P(0) = \frac{(nt)^0 e^{-nt}}{0!}$$

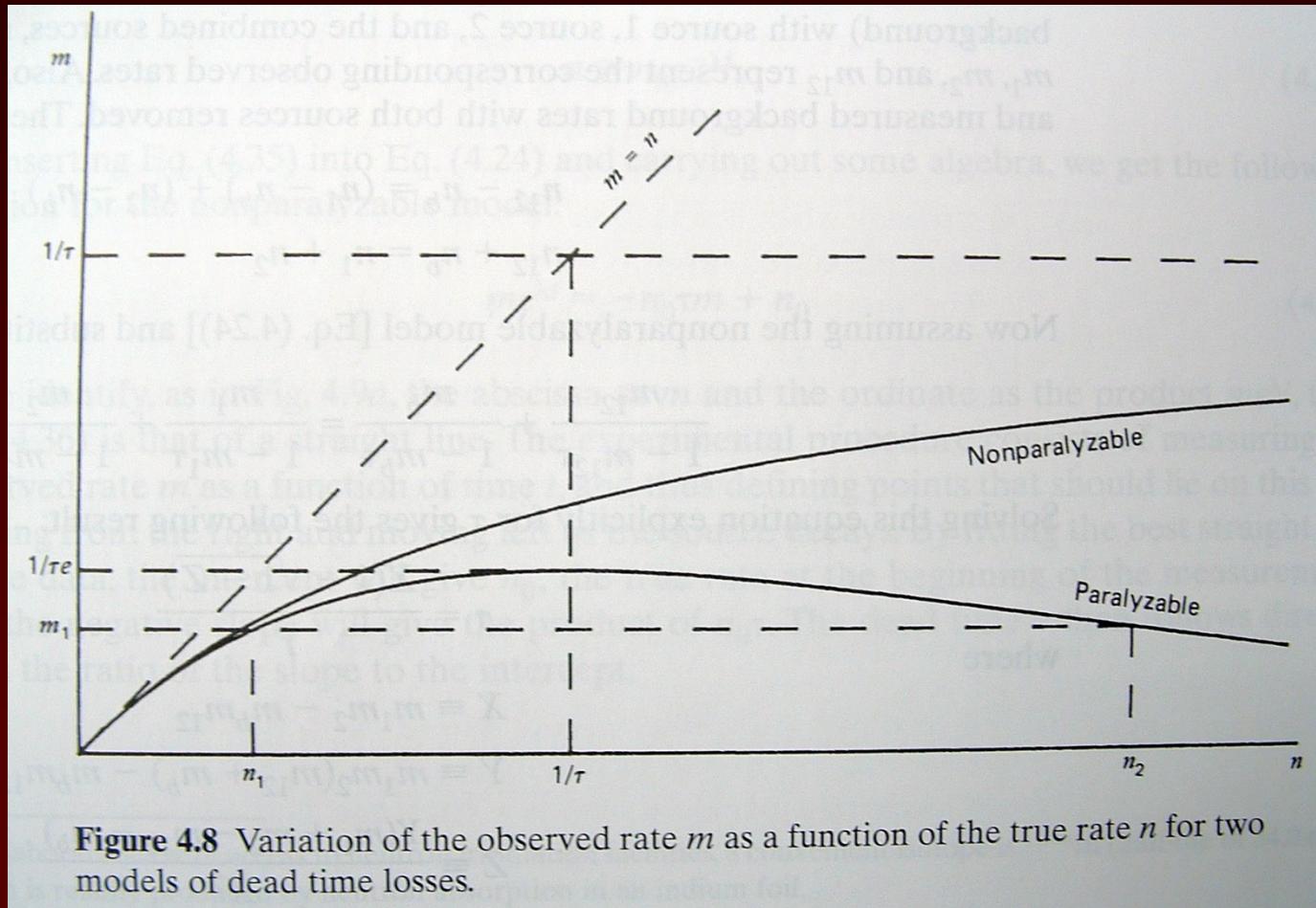
# Paralyzable System:

- The probability of having intervals larger than  $\tau$  is:

$$P_2(T) = \int_{\tau}^{\infty} P_1(T) dT = e^{-nt}$$

- which when multiplied by the true rate  $n$ , gives us the rate of occurrence:  
$$m = ne^{-nt}$$
- One can then compare the two models of how  $m$  varies as a function of  $n$  (Fig. 4.8).

# Paralyzable System:



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# Paralyzable System:

- Note that a non-paralyzable system approaches an asymptote value for the observed rate of  $1/\tau$  (where the counter is just recovering from one dead time before beginning another).
- Note that the paralyzable system goes through a maximum.
- For low rates ( $n \ll 1/\tau$ ) both systems produce count rates of:  
$$m \cong n(1 - n\tau)$$

# Methods of dead time measurement: 2 sources

- $n_i$  is the counting rate of source  $i$  ( $i=1$  or  $2$ ) including background
- $n_{12}$  is the counting rate of sources 1 & 2 including background
- $n_b$  is the background rate.
  - $n_{12} - n_b = (n_1 - n_b) + (n_2 - n_b) \Rightarrow n_{12} + n_b = n_1 + n_2$

# Methods of dead time measurement: 2 sources

- Assuming a non-paralyzable model we substitute true count rates for observed count rates:

$$\frac{m_{12}}{1 - m_{12}\tau} + \frac{m_b}{1 - m_b\tau} = \frac{m_1}{1 - m_1\tau} + \frac{m_2}{1 - m_2\tau}$$

# Methods of dead time measurement: 2 sources

- Solving for  $\tau$ :

$$\tau = \frac{x(1 - \sqrt{1 - z})}{y}$$

– Where

$$x = m_1 m_2 - m_b m_{12}$$

$$y = m_1 m_2 (m_{12} + m_b) - m_b m_{12} (m_1 + m_2)$$

$$z = \frac{y(m_1 + m_2 - m_{12} - m_b)}{x^2}$$

# Methods of dead time measurement: 2 sources

- For  $m_b = 0$ :

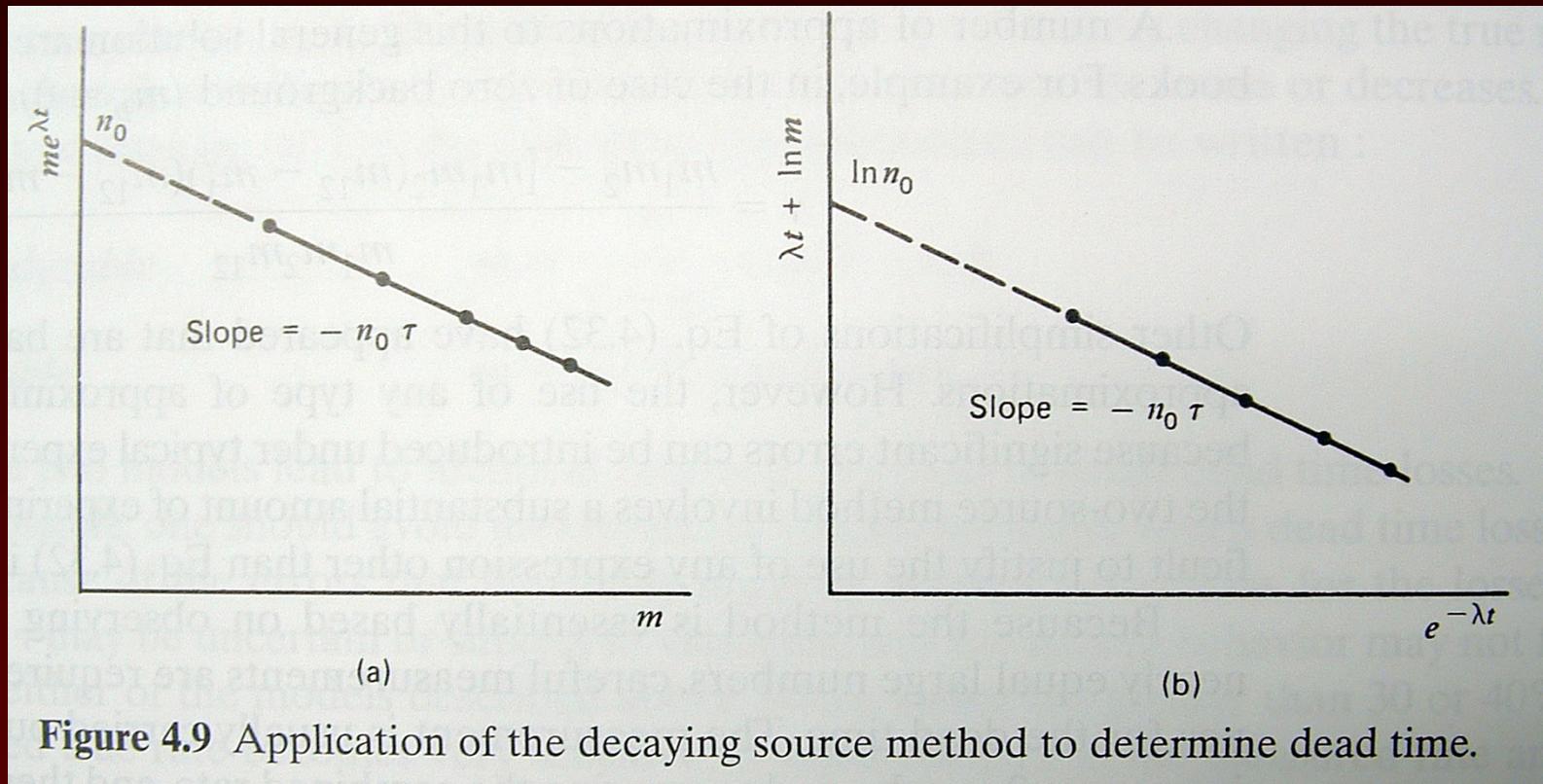
$$\tau = \frac{m_1 m_2 - [m_1 m_2 (m_{12} - m_1)(m_{12} - m_2)]^{\frac{1}{2}}}{m_1 m_2 m_{12}}$$

- This method requires using sources with two equally large numbers and best results are obtained by using sources active enough to result in fractional dead time  $m_{12}\tau$  of at least 20%.

# Methods of dead time measurement: decaying source

- Based on the known behavior of the true rate of  $n$ :  $n=n_o e^{-\lambda t} + n_b$
- Where  $n_o$  is the true rate at the beginning, and  $\lambda$  is the decay constant.

# Decaying source method plot



**Figure 4.9** Application of the decaying source method to determine dead time.

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# Methods of dead time measurement: decaying source

- For negligible background  $n \simeq n_o e^{-\lambda t}$  and assuming a non-paralyzable model

$n = m/(1-m\tau)$ :

$$n_o e^{-\lambda t} = \frac{m}{1 - m\tau} \Rightarrow m e^{\lambda t} = -n_o \tau m + n_o$$

- Where if we set  $y = m e^{\lambda t}$  and plot  $y$  vs.  $m$ , we get a straight line of slope  $-n_o \tau$  with a  $y$ -intercept of  $n_o$  (which gives us  $\tau$  by the ratio of the slope to the  $y$  intercept)

# Methods of dead time measurement: decaying source

- For the paralyzable model  $m = n e^{-n\tau}$ .
- Taking a ln:  $\ln(m) = -n\tau + \ln(n)$  and substituting we get:

$$\begin{aligned}\ln(m) &= -n_o e^{-\lambda t} \tau + \ln(n_o e^{-\lambda t}) \Rightarrow \\ \lambda t + \ln(m) &= -n_o e^{-\lambda t} \tau + \ln(n_o)\end{aligned}$$

- Again we make the LHS = y = and  $e^{-\lambda t} = x$ , we get a line function of “x” and can determine the dead time from the intercept and slope.

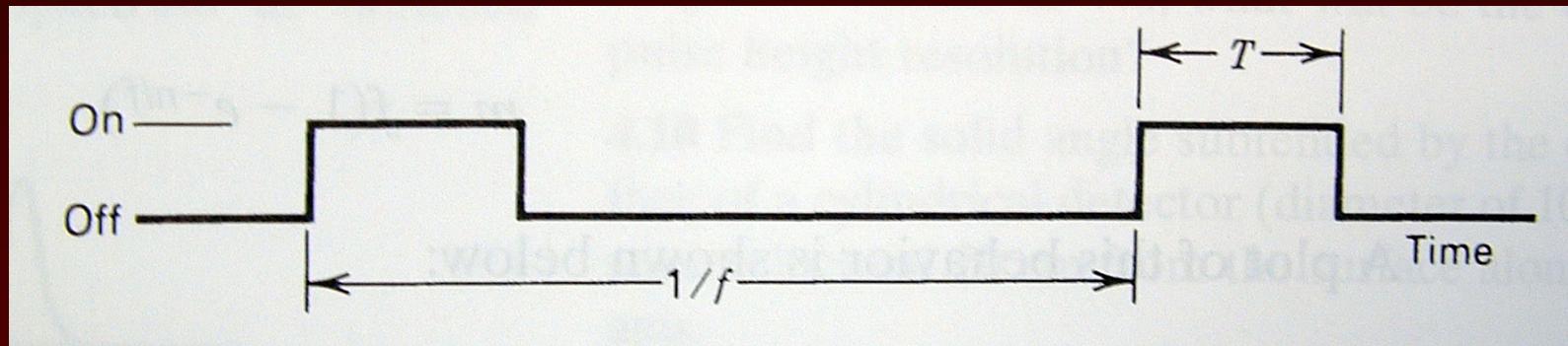
# Statistics of dead time losses

- Distorts the statistics away from Poisson behavior if large enough (~10-20%).

# Dead time losses from pulsed sources

- Previous analyses have assumed a constant source. Now we look at a pulsed source (like a Linac which can produce pulsed x-rays).
- We assume a constant relation over a time  $T$ , where the pulses occur with period of  $1/f$  which depends on the dead time of the detector  $\tau$ .

# Dead time loss from a pulsed source schematic



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# Dead time losses from pulsed sources

1. For  $\tau \ll T$ : our previous analysis holds (the pulses have little effect).
2. For  $\tau < T$ : only a small number of counts may be registered by the detector during a single pulse. This results in an analysis beyond the scope of this class or text.
3. For  $\tau > T$  but less than the “off time” between pulses ( $1/f - T$ ): There is a registration of a 0 or 1 count per pulse (This analysis is also applicable to sources not constant over  $\tau$ ).

# Dead time losses from pulsed sources

- We define  $f$  to be the source pulse frequency.
- The probability of observed count from a pulse =  $m/f$ .
- The average number of true events per source pulse =  $n/f$ .
- The probability of at least one true event occurs per source pulse is:

$$P(> 0) = 1 - P(0) = 1 - e^{-\frac{n}{f}} = 1 - e^{-\frac{n}{f}}$$

# Dead time losses from pulsed sources

- This equals the probability of at least one observed count from a pulse:

$$1 - e^{-\frac{n}{f}} = \frac{m}{f} \Rightarrow m = f \left( 1 - e^{-\frac{n}{f}} \right)$$

- Note in the plot of observed vs. true count rate, there is no dead time influence ( $m$  vs.  $n$ ).

# Dead time losses from pulsed sources

- Solving for a true count rate ( $n$ ):

$$n = f \cdot \ln\left(\frac{f}{f - m}\right)$$

- Which is a correction to adjust from the observed to the true count rate.
- This is valid only under  $T < \tau < (1/f - T)$ .

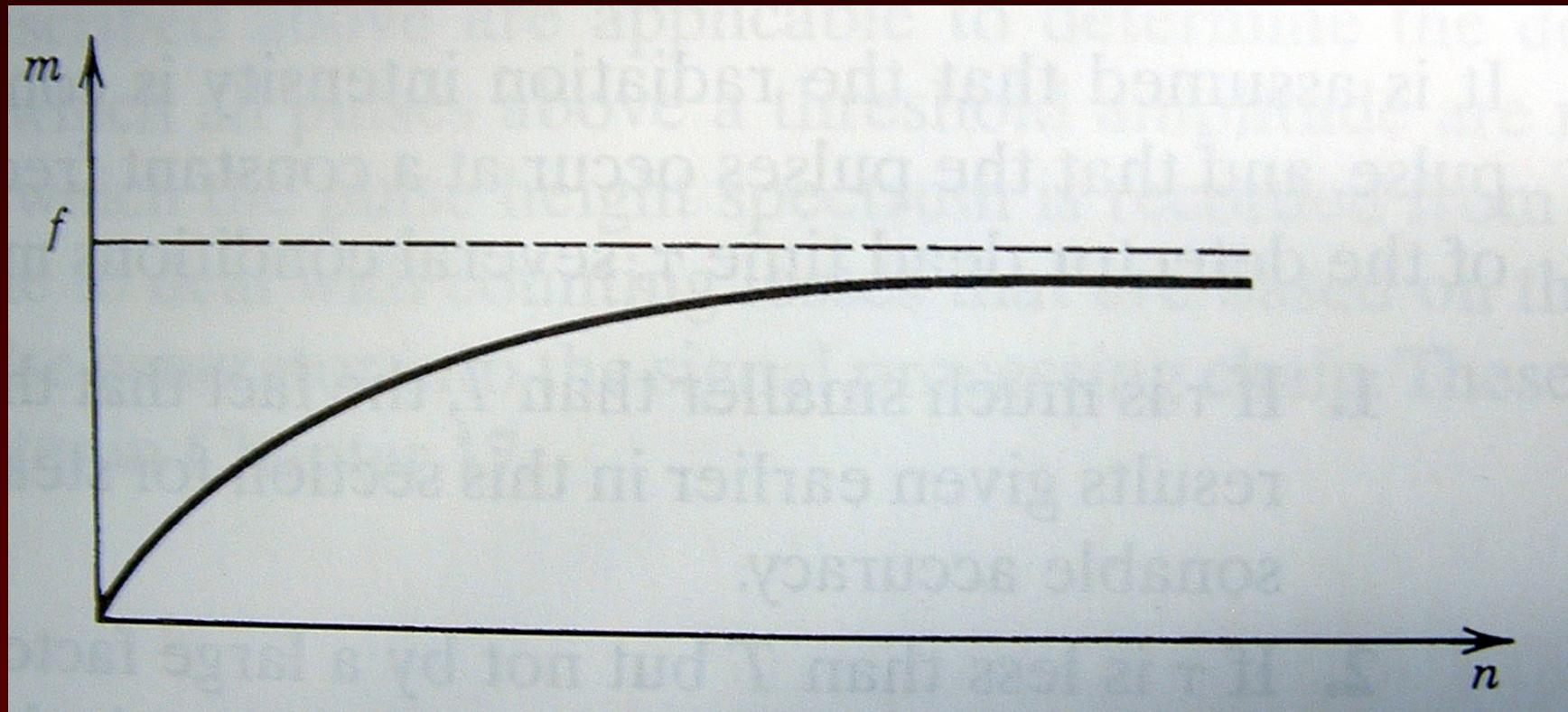
# Dead time losses from pulsed sources

- We can approximate this expression for small dead time losses, (small  $n$  and  $m$  relative to  $f$ )  $m \ll f$  we get:

$$n \cong \frac{m}{1 - \frac{m}{2f}}$$

- If we compare this to the non-paralyzable model, where  $n = m/(1-m\tau)$ , we get an effective dead time of  $1/2f$  in the low loss limit.
- Since this value is  $\frac{1}{2}$  the source pulsing period it can be many times longer than the actual physical dead time of the detector system

# Plot of observed counting rate vs. true counting rate for pulsed source



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