Radiological Sciences @ OUHSC

Radiation Detection and Measurement

Lecture 10

Chapter 4: General properties of radiation detectors

Detection efficiency

- In general, all radiation detectors will give rise to pulse for each quantum of radiation that interacts with its active volume.
- For α and β particles, which leave a large ionization trail, it is easy to arrange these detectors to have 100% efficiency.
- However, uncharged particles tend to travel large distances between interactions where some may not trigger inside the active volume (photons & neutrons), and have efficiencies less than 100%

Detection efficiency

- Absolute efficiency:
  - ε_{abs} = # of pulse recorded/# of quanta emitted.
- Intrinsic efficiency:
  - ε_{int} = # of pulses recorded/# of quanta incident on detector. And the # of quanta incident on the detector is the fraction of the solid angle subtended by the detector:

$$\#\ \ of\ \ incident\ \ quanta = \frac{\Omega}{4\pi} \times \ \#\ \ of\ \ radiation\ \ quanta\ \ emitted$$

Detection efficiency

- By substitution $\varepsilon_{abs} = \frac{\Omega}{4\pi} \cdot \varepsilon_{\text{int}}$, where $\frac{\Omega}{4\pi}$ is also called the geometric efficiency.
- The peak efficiency is then the quanta that deliver their full energy to the detector. This will generally be at the high end of the spectrum in a peak, where lower energy deposition contributes to the spectrum at lower energies.

Detection efficiency

- The peak efficiency is then the quanta that deliver their full energy to the detector. This will generally be at the high end of the spectrum in a peak, where lower energy deposition contributes to the spectrum at lower energies.
- Total efficiency is the total count over all energies (the area under the entire curve on a differential pulse height spectrum), ε_{total}.

Detection efficiency

- The peak efficiency is then the area under the curve at the peak, εpeak.
- Then the peak-to-total ratio, *r*, is:

$$r = \frac{\varepsilon_{peak}}{\varepsilon_{total}}$$

- The intrinsic peak efficiency is then:

$$\varepsilon_{IP} = \frac{\#\ \ of\ \ recorded\ \ pulses\ \ in\ \ full\ \ energy\ \ peak}{\#\ \ of\ \ quanta\ \ incident\ \ on\ \ detector}$$

Full energy peak example

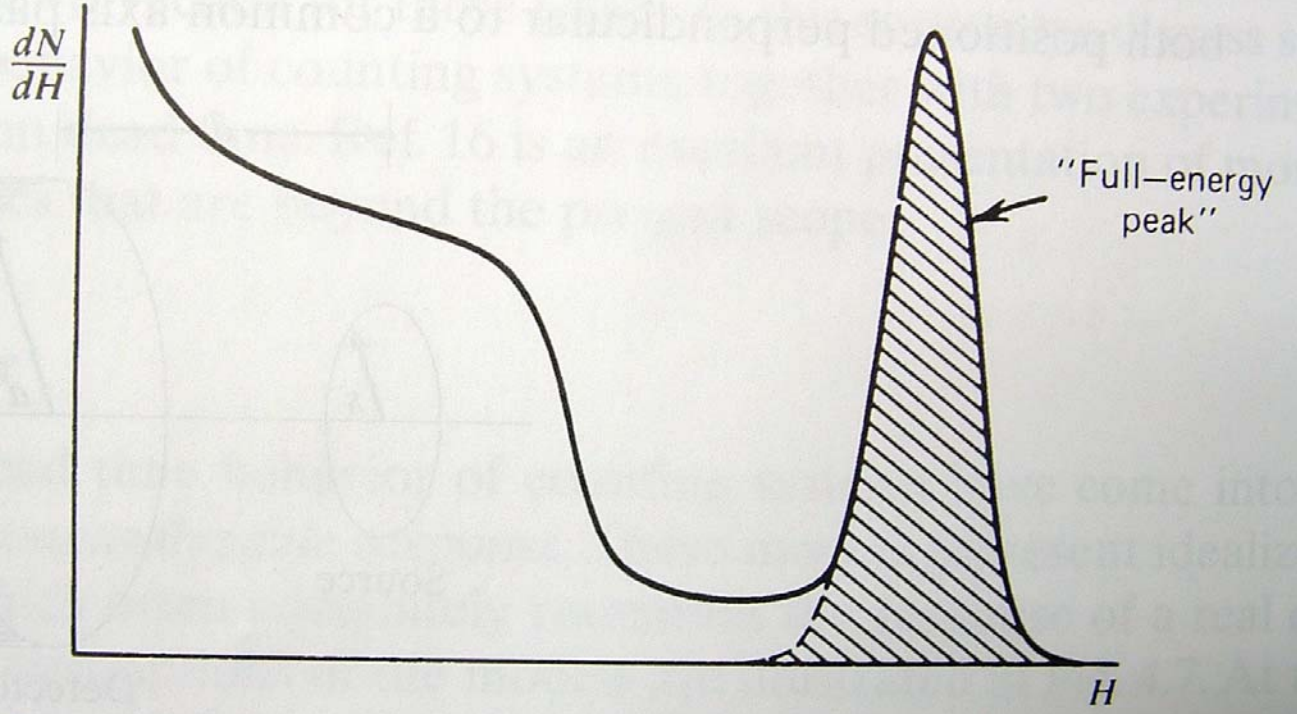


Figure 4.6 Example of the full-energy peak in a differential pulse height sp

Detection efficiency

- Inversely, the number of pulses recorded in the full energy peak, given S number of quanta from the source:

$$N = \varepsilon_{IP}\left(\frac{\Omega}{4\pi}\right)S$$ for a point source

Detection efficiency

- The subtended angle in steradians is:

$$\Omega = \int_A \frac{\cos(\alpha)}{r^2} dA$$

- Where *r* is the distance from the source to the detector element *dA*, and α is angle between the normal to the surface element *dA* and the source direction.
- For a large volume source, an integral over the volume of the source must also be included.

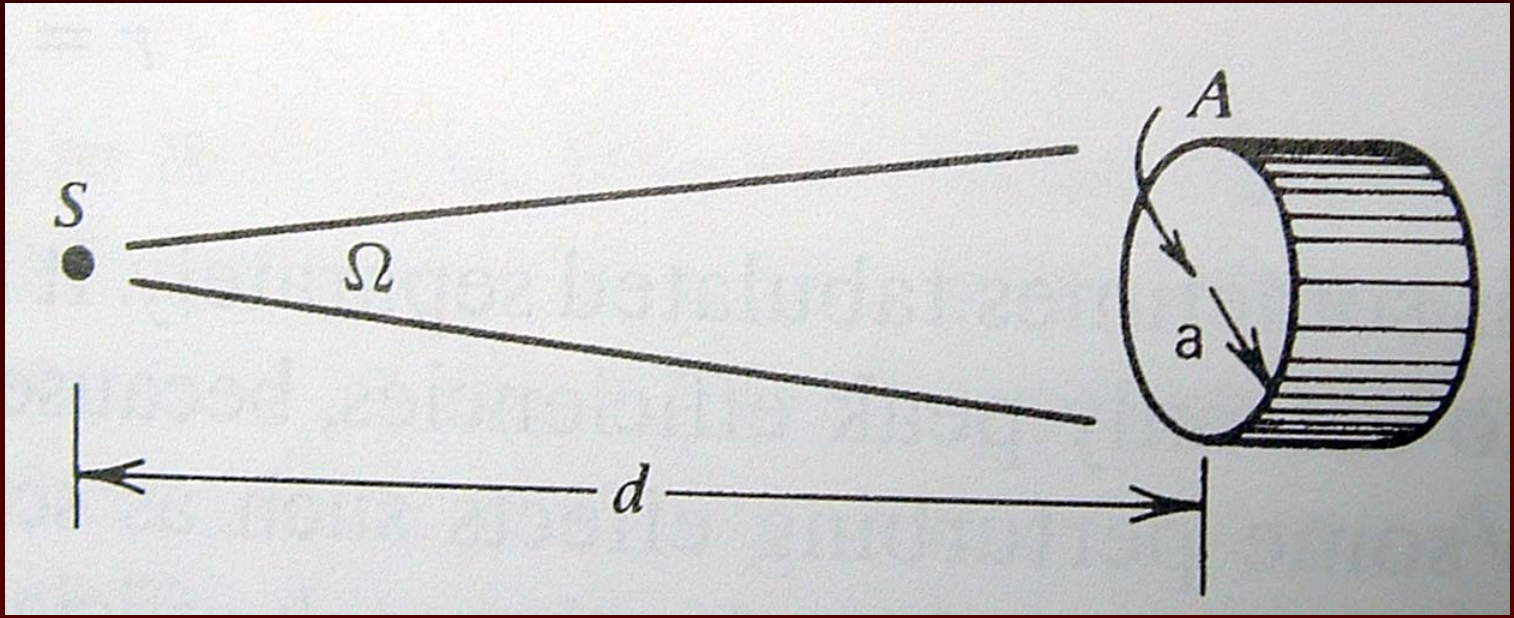
Detection efficiency

- For a point source and a right circular cylindrical detector of radius *a*:

$$\Omega = 2\pi\left(1 - \frac{d}{\sqrt{d^2 + a^2}}\right)$$

- Ω reduces for *d* >> *a* to: $$\Omega \cong \frac{A}{d^2} = \frac{\pi a^2}{d^2}$$
  - Where *d* is the distance from the detector.

Right cylinder geometry



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Detection efficiency

- For another common configuration of a circular disk (*r* = *s*) source emitting isotropic radiation aligned with a circular disk (*r* = *a*) detector both positioned perpendicular to a common axis through their centers a distance *d* apart:

$$\Omega = \frac{4\pi a}{s}\int_0^\infty \frac{\exp(-dk)J_1(sk)J_1(ak)}{k}\,dk$$

  – Where $J_1(x)$ are the Bessel functions of *x*

Detection efficiency

- Through a numerical solution method one can approximate the answer to:

$$\Omega \cong 2\pi\left[1-\frac{1}{(1+\beta)^{\frac{1}{2}}}-\frac{3}{8}\cdot\frac{\alpha\beta}{(1+\beta)^{\frac{5}{2}}}+\alpha^{2}[F1]-\alpha^{3}[F2]\right]$$

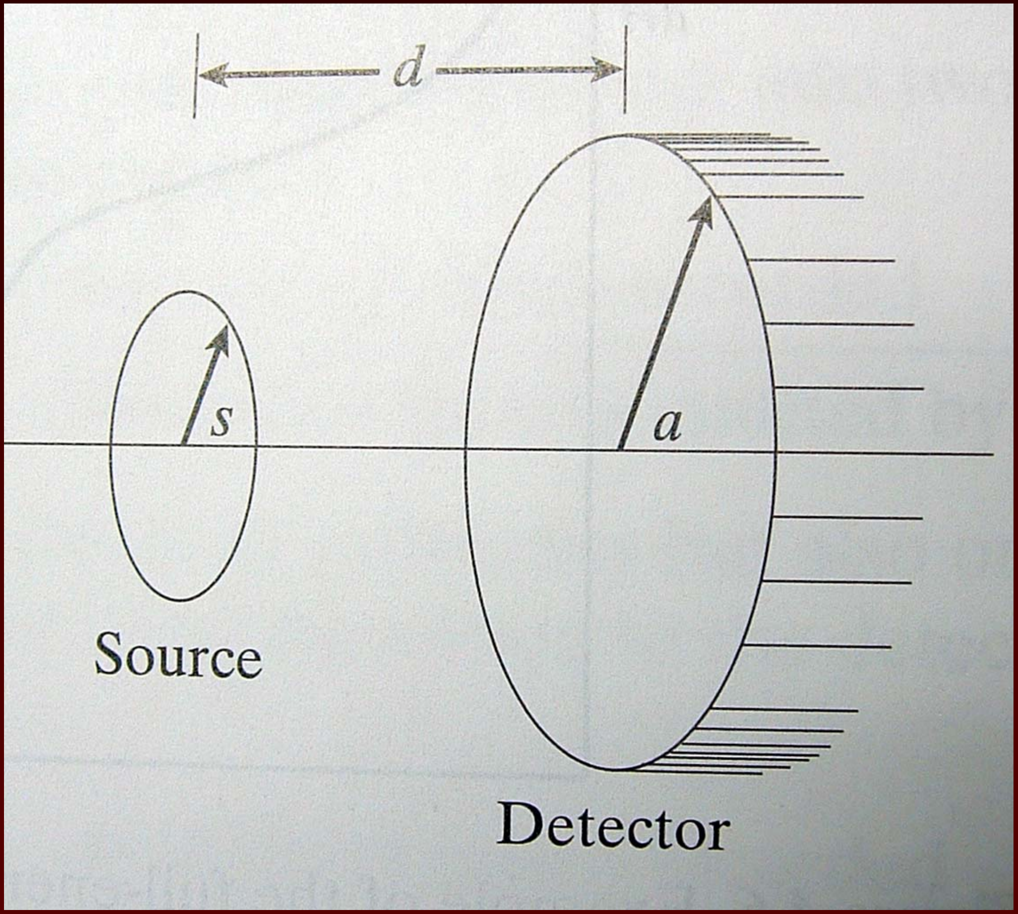
  – Where:

$$F1=\frac{5}{16}\cdot\frac{\beta}{(1+\beta)^{\frac{7}{2}}}-\frac{35}{16}\cdot\frac{\beta^{2}}{(1+\beta)^{\frac{9}{2}}}$$

$$F2=\frac{35}{128}\cdot\frac{\beta}{(1+\beta)^{\frac{9}{2}}}-\frac{315}{256}\cdot\frac{\beta^{2}}{(1+\beta)^{\frac{11}{2}}}+\frac{1155}{1024}\cdot\frac{\beta^{3}}{(1+\beta)^{\frac{13}{2}}}$$

  – And $\alpha \equiv (a/d)^2$, $\beta \equiv (s/d)^2$, which becomes inaccurate when source or detector diameters get too large compared with *d*.

Uniform cylindrical disk geometry



Dead time

- Dead time is the time required to separate two events so that they are recorded as two pulses.
- There are two models of dead time behavior:
  - Paralyzable: true events that occur during the dead period, although not recorded, are assumed to extend the dead time by another period following the lost event.
  - Non-paralyzable: true events that occur during the dead time are lost and assumed to have no effect on the behavior of the detector.
- We define the following variables: *n*-true interaction rate; *m*-recorded count rate; τ-system dead time.

Dead time models

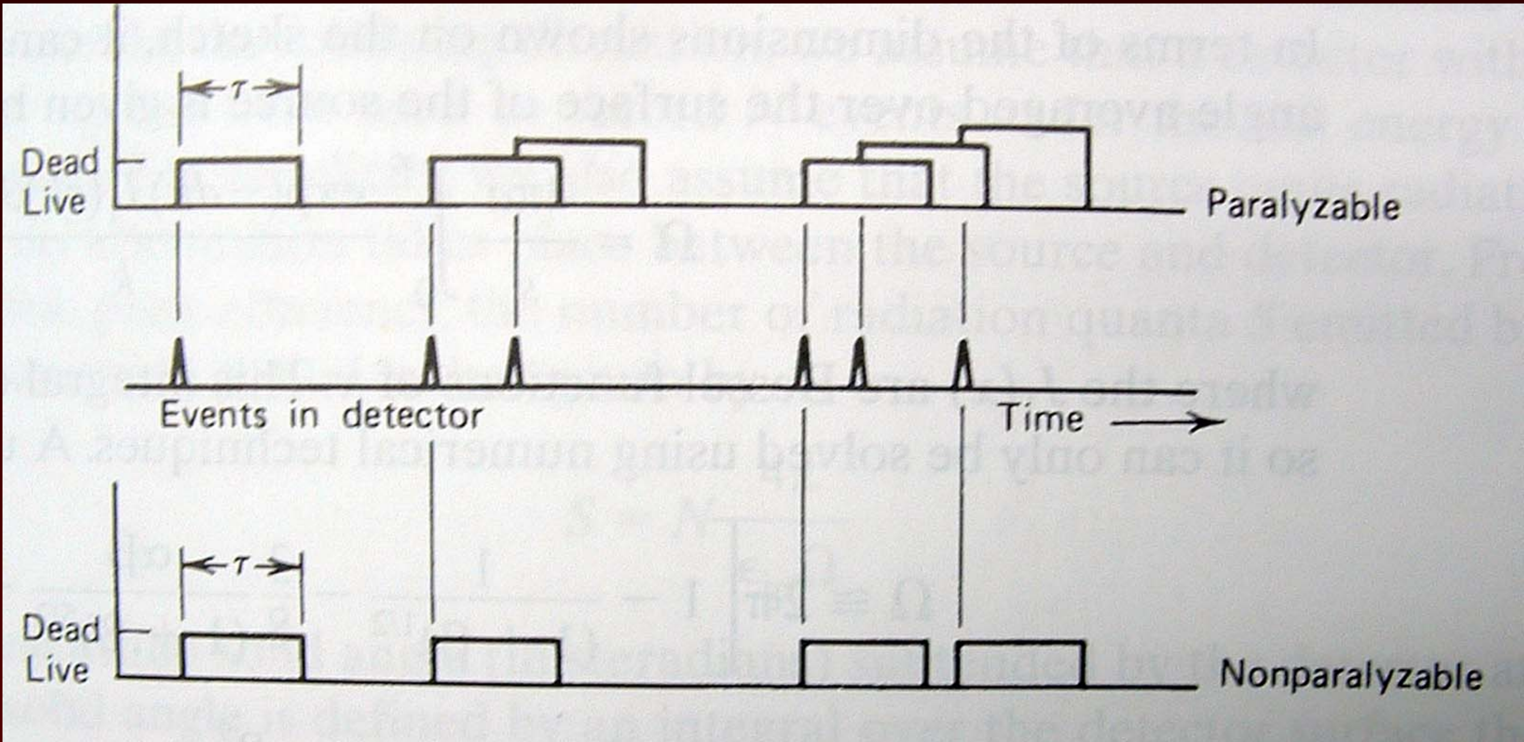


Figure 4.7 Illustration of two assumed models of dead time behavior for radiation detectors.

Non-paralyzable System

- The total time dead is $m\tau$, and the loss rate is $nm\tau$ and n-m, therefore
  - $n - m = nm\tau$.
- For the true count rate: $$n = \frac{m}{1 - nm\tau}$$

Paralyzable System:

- Dead periods are not a fixed length, so we need a different analysis, but we note m is also the rate of occurrences of time intervals between true events which exceed τ.
- We invoke the distribution of intervals between random events occurring at an average rate *n* to be:

$$P_1(T)dT = ne^{-nT}dT$$

  - Where $P_1(T)$ is the probability of observing an interval whose length lies within dT about T. (Recall $P_1(T)=P(0) \times ndt$ and $P(0)=\frac{(nt)^0 e^{-nt}}{0!}$)

Paralyzable System:

- The probability of have intervals larger than τ is:

$$P_2(T) = \int_{\tau}^{\infty} P_1(T)dT = e^{-nt}$$

- which when multiplied by the true rate *n*, gives us the rate of occurrence:

$$m = ne^{-nt}$$

- One can then compare the two models of how *m* varies as a function of *n* (Fig. 4.8).

Paralyzable System:

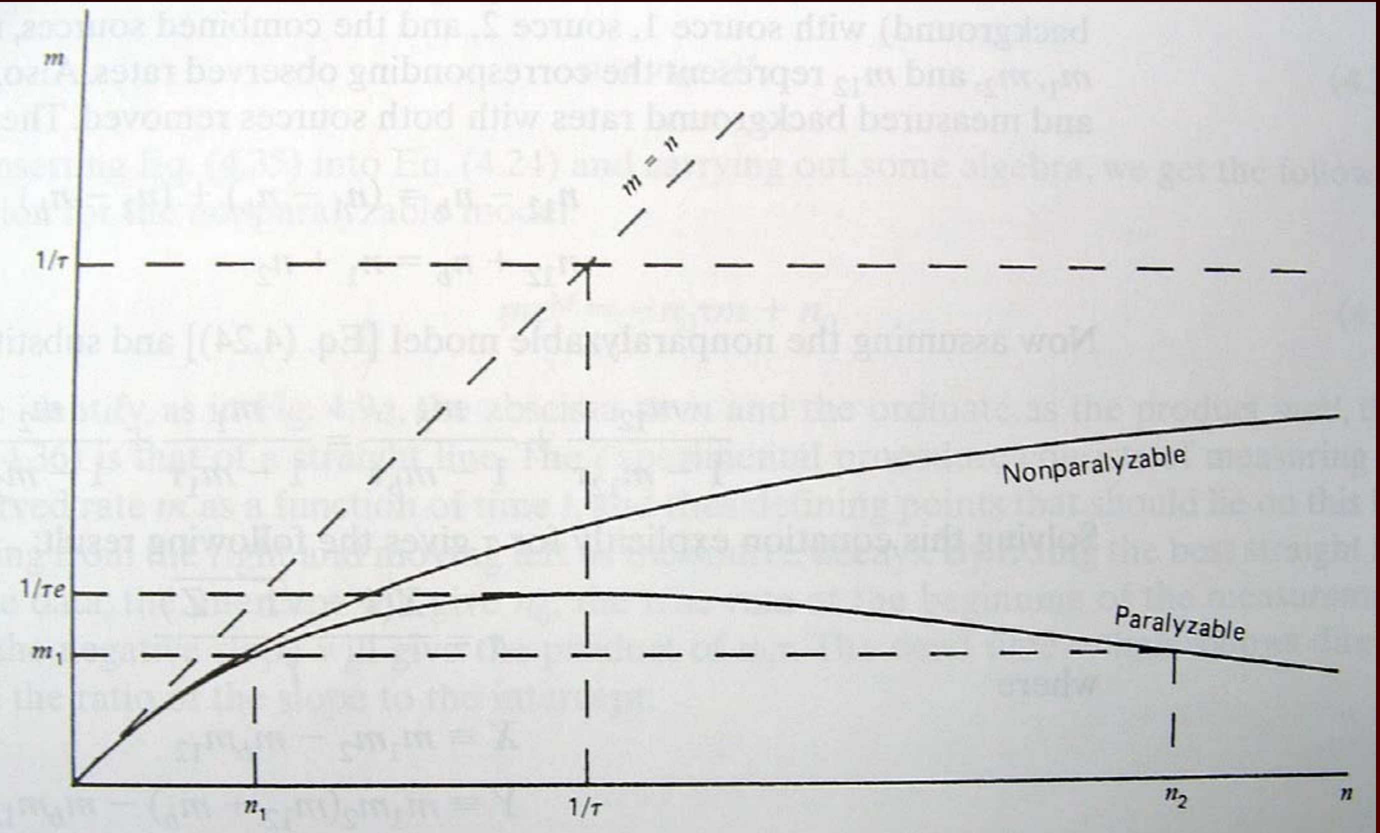


Figure 4.8 Variation of the observed rate m as a function of the true rate n for two models of dead time losses.

Paralyzable System:

- Note that a non-paralyzable system approaches an asymptote value for the observed rate of $1/\tau$ (where the counter is just recovering from one dead time before beginning another).
- Note that the paralyzable system goes through a maximum.
- For low rates ($n \ll 1/\tau$) both systems produce count rates of: $m \cong n(1 - nt)$

Methods of dead time measurement: 2 sources

- n_i is the counting rate of source i (i=1 or 2) including background
- n_{12} is the counting rate of sources 1 & 2 including background
- n_b is the background rate.
  - $n_{12}-n_b=(n_1-n_b)+(n_2-n_b) \Rightarrow n_{12}+n_b=n_1+n_2$

Methods of dead time measurement: 2 sources

- Assuming a non-paralyzable model we substitute true count rates for observed count rates:

$$\frac{m_{12}}{1-m_{12}\tau}+\frac{m_b}{1-m_b\tau}=\frac{m_1}{1-m_1\tau}+\frac{m_2}{1-m_2\tau}$$

Methods of dead time measurement: 2 sources

- Solving for τ:

$$\tau = \frac{x\left(1-\sqrt{1-z}\right)}{y}$$

  – Where

$$x = m_1 m_2 - m_b m_{12}$$

$$y = m_1 m_2 \left(m_{12} + m_b\right) - m_b m_{12}\left(m_1 + m_2\right)$$

$$z = y\left(m_1 + m_2 - m_{12} - m_b\right) \Big/ x^2$$

Methods of dead time measurement: 2 sources

- For $m_b = 0$:

$$\tau = m_1 m_2 - \frac{\left[m_1 m_2 \left(m_{12} - m_1\right)\left(m_{12} - m_2\right)\right]^{\frac{1}{2}}}{m_1 m_2 m_{12}}$$

- This method requires using sources with two equally large numbers and best results are obtained by using sources active enough to result in fractional dead time $m_{12}\tau$ of at least 20%.

Methods of dead time measurement: decaying source

- Based on the known behavior of the true rate of n: $n=n_o e^{-\lambda t} + n_b$
- Where n_o is the true rate at the beginning, and λ is the decay constant.

Decaying source method plot

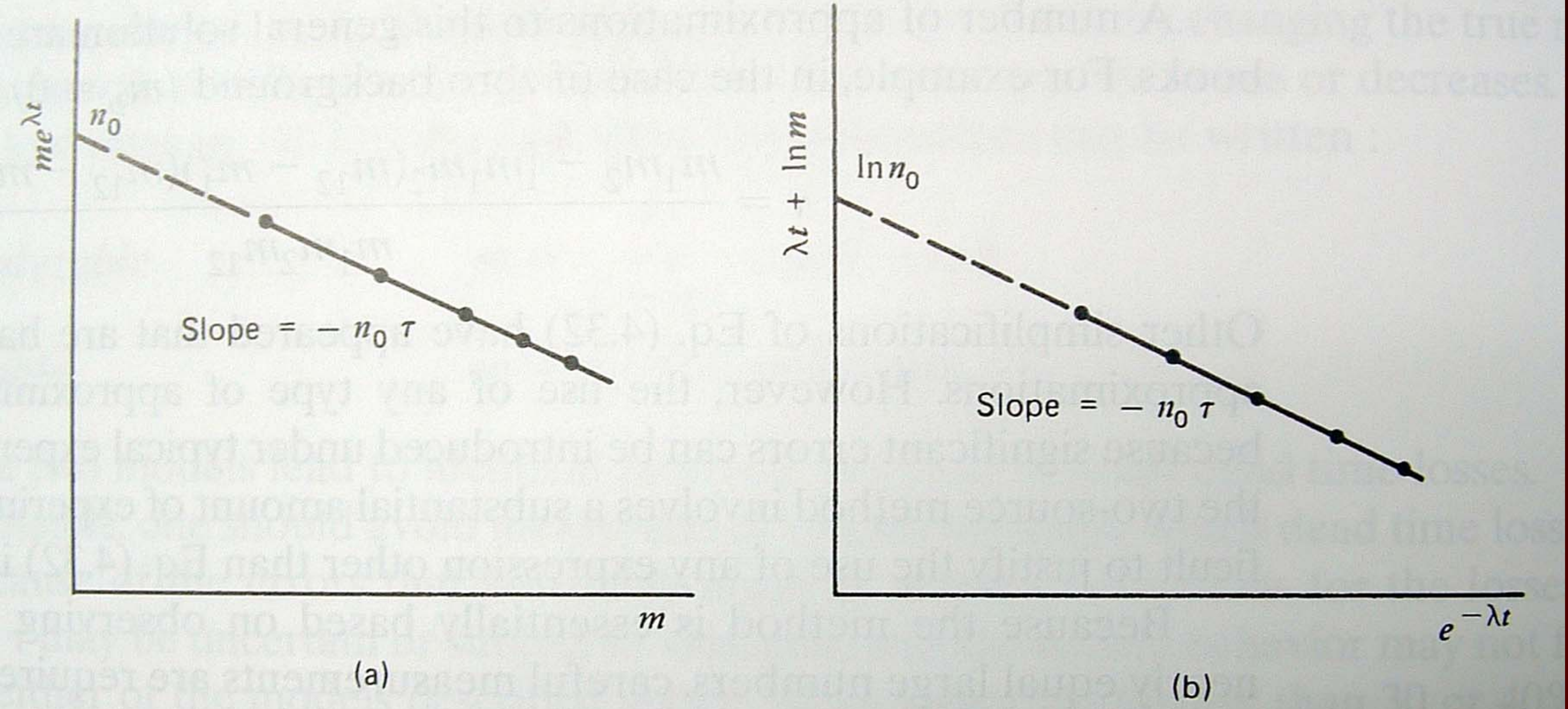


Figure 4.9 Application of the decaying source method to determine dead time.

Methods of dead time measurement: decaying source

- For negligible background $n \cong n_o e^{-\lambda t}$ and assuming a non-paralyzable model

$n = m/(1-m\tau)$:

$$n_o e^{-\lambda t} = \frac{m}{1 - m\tau} \Rightarrow m e^{\lambda t} = -n_o \tau m + n_o$$

  – Where if we set $y = me\text{-}\lambda t$ and plot y vs. m, we get a straight line of slope $-no\tau$ with a y-intercept of no (which gives us τ by the ratio of the slope to the y intercept)

Methods of dead time measurement: decaying source

- For the paralyzable model $m = ne^{-n\tau}$.
- Taking a ln: $ln(m)=-n\tau + ln(n)$ and substituting we get:

$$\ln(m) = -n_o e^{-\lambda t}\tau + \ln\left(n_o e^{-\lambda t}\right) \Rightarrow$$
$$\lambda t + \ln(m) = -n_o e^{-\lambda t}\tau + \ln(n_o)$$

- Again we make the LHS = y = and $e^{-\lambda t} = x$, we get a line function of "x" and can determine the dead time from the intercept and slope.

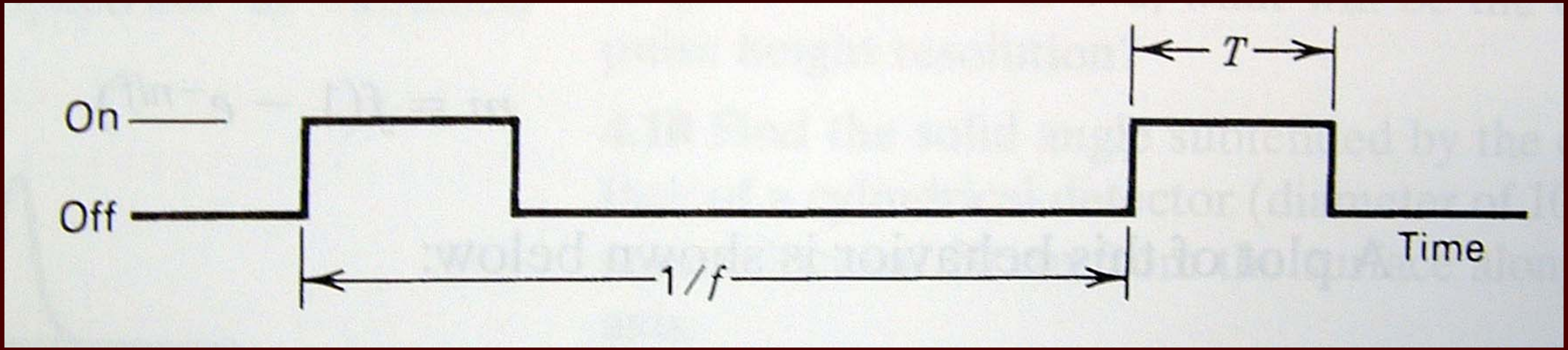
Statistics of dead time losses

- Distorts the statistics away fro Poisson behavior if large enough (~10-20%).

Dead time losses from pulsed sources

- Previous analyses have assumed a constant source. Now we look at a pulsed source (like a Linac which can produce pulsed x-rays).
- We assume a constant relation over a time *T*, where the pulses occur with period of 1/*f* which depends on the dean time of the detector τ.

Dead time loss from a pulsed source schematic



Dead time losses from pulsed sources

1. For $\tau << T$: our previous analysis holds (the pulses have little effect).
2. For $\tau < T$: only a small number of counts may be registered by the detector during a single pulse. This results in an analysis beyond the scope of this class or text.
3. For $\tau > T$ but less than the “off time” between pulses ($1/f - T$): There is a registration of a 0 or 1 count per pulse (This analysis is also applicable to sources not constant over τ).

Dead time losses from pulsed sources

- We define f to be the source pulse frequency.
- The probability of observed count from a pulse = *m/f*.
- The average number of true events per source pulse = *n/f*.
- The probability of at least one true event occurs per source pulse is:

$$P(>0) = 1 - P(0) = 1 - e^{-\bar{x}} = 1 - e^{-\frac{n}{f}}$$

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Dead time losses from pulsed sources

- This equals the probability of at least one observed count from a pulse:

$$1-e^{-\frac{n}{f}}=\frac{m}{f}\Rightarrow m=f\left(1-e^{-\frac{n}{f}}\right)$$

  - Note in the plot of observed vs. true count rate, there is no dead time influence (*m* vs. *n*).

Dead time losses from pulsed sources

- Solving for a true count rate (*n*):

$$n = f \cdot \ln\left(\frac{f}{f-m}\right)$$

  - Which is a correction to adjust from the observed to the true count rate.
  - This is valid only under $T < \tau < (1/f - T)$.

Dead time losses from pulsed sources

- We can approximate this expression for small dead time losses, (small *n* and *m* relative to *f*) $m << f$ we get:

$$n \cong \frac{m}{1 - \frac{m}{2f}}$$

- If we compare this to the non-paralyzable model, where $n = m/(1-m\tau)$, we get an effective dead time of $1/2f$ in the low loss limit.
- Since this value is ½ the source pulsing period it can be many times longer than the actual physical dead time of the detector system

Plot of observed counting rate vs. true counting rate for pulsed source