Lecture 4

Interactions with Fast Electrons

- Lose energy at a lower rate than heavy charged particles, leading to longer path lengths.

Specific Energy Loss:

- Losses due to collision (ionization and excitation)

$$-\left(\frac{dE}{dx}\right)_c = \frac{2\pi e^4 Nz}{m_o v^2}\left(\ln\frac{m_o v^2 E}{2I^2(1-\beta^2)} - (\ln 2)\left(2\sqrt{1-\beta^2} - 1 + \beta^2\right) + \left(1-\beta^2\right) + \frac{1}{8}\left(1-\sqrt{1-\beta^2}\right)^2\right)$$

where $\beta = v/c$, and I is the average excitation and ionization potential of the absorber.

- Losses due to radiation (i.e. Bremsstrahlung)

$$-\left(\frac{dE}{dx}\right)_r = \frac{NEz(z+1)e^4}{137 m_o^2 c^4}\left(4\cdot\ln\frac{2E}{m_o c^2} - \frac{4}{3}\right)$$

- Total stopping power is the sum of the two energy loss equations.

$$\frac{dE}{dx} = \left(\frac{dE}{dx}\right)_c + \left(\frac{dE}{dx}\right)_r$$

- The ratio of the specific energy loss is $\frac{(dE/dx)_r}{(dE/dx)_c} = \frac{EZ}{700}$, where E[MeV]

Electron Range and Transmission Curves:

Absorption of monoenergetic e^-:

- Similar to the discussion of α's, one can acquire an extrapolated range by plotting ΔIntensity vs. thickness of an absorber.
- The e^-'s that penetrate the furthest are those with the straightest path (least loss of E).
- Range is not well defined since paths can vary greatly and are tortuous. ****
- One can make a crude estimate for range:
  - ~2mm/MeV for low density materials.
  - ~1mm/MeV for moderate density materials.

- For a reasonable approximation the product of the range and the density of the absorber is constant for differing materials with the same initial velocity e^-.
- Fig. 2.14 shows this relation for 2 materials (Si & NaI) which have very differing properties and atomic #'s.

Absorption of β Particles:

- "Soft" or low energy β particles are readily absorbed even in small thickness of an absorber.
- Empirically (without fundamental derivation) they behave with exponential losses:

$$\frac{I}{I_o} = e^{-nt}$$

  Where I_o – counting rate without an absorber, I – counting rate with absorber, t – thickness of absorber in [g/cm^2], n – an absorption coefficient (similar to μ in γ attenuation).
- Fig. 2.15 shows the relation of β^- with ^{185}W with an endpoint of 0.43 MeV.
- The absorption coefficient correlates well to end point energy, shown in Fig 2.16 for the endpoint (E_m), average (E_{avg}), and mean of E_m & E_{avg} energies.

Backscattering:

- Due to large angle scattering of e^-'s, the impending e^- may re-emerge from the surface it entered. This is backscatter.
- This can be a problem in a detectors "entrance window", where the e^-'s that backscatter are not detected.
- Effect is largest for low incident energy e^- and high atomic number absorbers.

Positron β^+ interactions:

- Still driven by coulombic interactions, so similar to β^-, only interaction is annihilation.

<u>Interaction of γ-rays</u> (photons)

Mechanisms:

- γ's will interact in one of four ways: Compton Scattering; Raleigh (Coherent) Scattering; Photoelectric Absorption; and Pair Production.

Photoelectric Absorption:

- γ is absorbed into the atom exciting an e^- and ejecting it.

- Generally the e^- is from an inner shell (K).
- Energy of the (photo) e^- $E_e = h\nu - E_b$, where $h\nu$ is the energy of the γ-ray and E_b is the binding energy of the e^-.
- This leaves a vacancy and subsequent characteristic x-ray (γ) or Auger e^-.
- This is the primary mode of interaction for low energy γ's or high z absorbers.
- The probability of interaction is approximated by:

$$\tau \cong const \times \frac{z^n}{E_\gamma^{3.5}}$$

Where n varies from 4-5 over the γ ray energy region of interest.

- This shows why high z materials are used for shields (Pb) & detectors.
- Note: When looking at absorption curves, the discontinuities are commiserate with the binding energies of the e^-'s. Generally seen for the K-shell (K-edge) it is energetically favorable to emit a photoelectron just above the K-shell energy. Just below the K-edge there is not enough energy to excite the e^-, thus there is much less absorption below the K-edge.

Compton Scattering:

- This is an energy & momentum conserved "billiard ball" reaction (assume the binding energy is negligible).
- It can be shown that:

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_o c^2}(1 - \cos\theta)}$$

Where $h\nu'$ is the energy of the scattered γ, $h\nu$ is the energy of the incident γ, $m_o c^2$ is the rest mass energy of the recoil e^-, and θ is the scattered γ angle.

- This equation shows direction but not intensity.
- Some energy is always retained by the incident γ, even when $\theta=\pi$.
- The probability of Compton scattering per atom of the absorber depends on the number of electrons available as scattering targets and thus increases with z and gradually falls off with increasing energy.
- The angular scattering of Compton scattering is predicted by the Klein-Nashina formula for differential scattering cross section ($d\sigma/d\Omega$):

$$\frac{d\sigma}{d\Omega} = zr_o^2 \left(\frac{1}{1 + \alpha(1 - \cos\theta)}\right)^2 \left(\frac{1 + \cos^2\theta}{2}\right)\left(1 + \frac{\alpha^2(1 - \cos\theta)^2}{(1 + \cos^2\theta)[1 + \alpha(1 - \cos\theta)]}\right)$$

Where $\alpha \equiv \frac{h\nu}{m_o c^2}$, r_o is the classical electron radius, and $d\sigma/d\Omega$ has azimuthal symmetry. A graph of the distribution is seen in Fig. 2.19.

- Fig. 2.19 is a polar plot of the number of photons Compton scattered into a unit solid angle at the scattering angle θ.
- Note that the distribution moves forward with increasing energy.

Pair Production:

- γ energy must exceed the level to produce 2 electrons (1.02 MeV).
- Probability of pair production is low until E_γ reaches several MeV, and rises sharply with energy.
- Must take place in the coulomb field of the nucleus.
- Fig. 2.20 shows the relative importance of each of the interactions with energy.

Coherent Scattering (Raleigh):

- No excitation or ionization, but involves all of the electrons in the absorber atom.
- Incident and scattered photon directions are different (can contribute to noise in imaging).
- Probability is high for low energy γ's (< a few 100 keV for common materials).
- Most prominent in high z absorbers.
- Gives the sky its overall blue color.
- Average deflection angle increases with decreasing energy.

<u>Gamma Ray Attenuation</u>

- Describe the statistical removal of γ's from the mono-energetic beam.
- Defined by the probabilities of each interaction previously described:
$$\mu = \tau(photoelectric) + \sigma(Compton) + \kappa(pair)$$
μ is the linear attenuation coefficient.
- $\frac{I}{I_o} = e^{-\mu t}$ (loss of relative intensity for thickness (t) of the absorber).
- Can also define a mean free path, λ:
$$\lambda = \frac{\int_0^\infty xe^{-\mu x}dx}{\int_0^\infty e^{-\mu x}dx} = \frac{\left.\frac{e^{-\mu x}}{-\mu}\left(x - \frac{1}{\mu}\right)\right|_0^\infty}{\left.\frac{e^{-\mu x}}{-\mu}\right|_0^\infty} = \frac{0 - \frac{1}{\mu^2}}{0 - \frac{1}{\mu}} = \frac{1}{\mu}$$
- Note that the curve in Fig. 2.21 is under "good" geometry conditions (i.e. point source, large distance between source and detector etc.).
- The mass attenuation coefficient = μ/ρ, where ρ is the density of the medium.
- μ/ρ for a compound (like scaling) is $\left(\frac{\mu}{\rho}\right)_c = \sum_i w_i \left(\frac{\mu}{\rho}\right)_i$, where w_i is the weight fraction of element *i* in *c*.
- Using μ/ρ we can rewrite the attenuation relation:

$$\frac{I}{I_o} = e^{-\left(\frac{\mu}{\rho}\right)\rho t}$$, where ρt is the mass thickness.

- This is useful for discussing the energy loss in fast electrons and charged particles since for absorber materials of similar protons and neutrons, the particle will encounter about the same number of electrons for a given (equal) mass thickness.
- Stopping power and range expressed in ρt will be about the same for absorbers of similar z.

Buildup:

- To account for poor geometry in a source detector set up we can rewrite the attenuation as:

$$\frac{I}{I_o} = B(t, E_\gamma)e^{-\mu t}$$

- This correction for broad beam geometry keeps the exponential term for the major changes and uses a simple multiplicative correction factor.
- $B(t,E_\gamma)$ will vary with γ-energy and detector type and accounts for secondary (scattered) γ's in the absorber (generally the source has little or no geometry).
- $B(t,E_\gamma)$ is greater than one and is determined experimentally.
- Rule of thumb: $B(t,E_\gamma)$ for a thick slab tends to be equal to the thickness of the absorber in units of λ, provided the detector responds to a broad range of γ's.