Pulse Processing and Shaping

Pulse Shaping

- With faster interaction rates (>50 μs) the total charge collection reaching a pre-amp may show varying pulse heights due to pulse pile-up (Fig. 16.8a).
- By pulse shaping the overlap can be reduced or eliminated, shown in Fig. 16.8b.
- The pile up is a result of the pulse tails not decaying fully to 0 before the next pulse occurs.
- The shaping generally takes place in the pre-amp.

CR and RC Shaping:

- RC shaping refers to the use of passive resistor-capacitor networks to carry out the pulse shaping and is differentiated into integrator (RC) or differentiator (CR) networks.
- For both cases the time constant is defined as $\tau=RC$, and τ is in seconds if R is in ohms and C in Farads.
- Both operations can be thought of as filtering in the frequency domain to improve SNR in the time domain by removing unwanted frequencies from the signal.

CR Differentiator (High Pass Filter):

- Fig. 16.9 illustrates the results of a CR filter to a step function, an exponential decay; the circuit configuration is also listed.
- We denote Q as the charge stored across the capacitor which affects the signal input as Q/C producing the result:

$$E_{in} = \frac{Q}{C} + E_{out}$$

Differentiating wrt time:

$$\frac{dE_{in}}{dt} = \frac{1}{C}\frac{dQ}{dt} + \frac{dE_{out}}{dt} = \frac{1}{C}i + \frac{dE_{out}}{dt}$$

The current is: $i = \frac{E_{out}}{R}$

Replacing i:

$$\frac{dE_{in}}{dt} = \frac{E_{out}}{RC} + \frac{dE_{out}}{dt} = \frac{E_{out}}{\tau} + \frac{dE_{out}}{dt}$$

Rearranging:

$$E_{out} + \tau\frac{dE_{out}}{dt} = \tau\frac{dE_{in}}{dt}$$

For small τ the first order "correction", dE_{out}/dt is negligible so:

$$E_{out} \cong \tau \frac{dE_{in}}{dt}$$

For a large τ, τ will dominate the equation and:

$$\tau \frac{dE_{out}}{dt} \cong \tau \frac{dE_{in}}{dt} \quad or \quad E_{out} \cong E_{in}$$

For example, a step voltage input:

$$E_{in} = \begin{cases} E(t \geq 0) \\ 0(t < 0) \end{cases}$$

We use Kirchoff's voltage law:

$$E_{in} - \frac{Q}{C} - iR = 0$$

In terms of Q:

$$\frac{E_{in}}{R} - \frac{Q}{RC} - \frac{dQ}{dt} = 0$$

Rearranging:

$$\frac{Q}{\tau} + \frac{dQ}{dt} = \frac{E_{in}}{R}$$

Overall solution is the sum of the homogeneous and particular solution $Q(t)=Q_h(t)+Q_p(t)$. For homogeneous the forcing term is set to 0:

$$\frac{dQ_h}{dt} + \frac{Q_h}{\tau} = 0$$

First order solution is:

$$Q_h(t) = Ae^{-\frac{t}{\tau}}$$

And for t>0 the particular solution is given by:

$$\frac{dQ_p}{dt} + \frac{Q_p}{\tau} = \frac{E}{R}$$

Since E is constant in this problem (no time dependence), Q_p must also be constant, so plug in and solve for the constant:

$$Q_p(t) = B \Rightarrow 0 + \frac{B}{\tau} = \frac{E}{R}, \qquad B = CE$$

- Total solution is then:

$$Q(t) = Ae^{-\frac{t}{\tau}} + CE$$

- Now for the boundary conditions that when $t = 0$, $Q = 0$:

$$Q(0) = A + CE = 0 \Rightarrow A = -CE$$

So:

$$Q(t) = CE\left(1 - e^{-\frac{t}{\tau}}\right)$$

- We are solving for $E_{out}=i(t)R$:

$$E_{out} = \frac{dQ(t)}{dt} R = \frac{RCE}{\tau} e^{-\frac{t}{\tau}} = Ee^{-\frac{t}{\tau}}$$

Now let's try with a sinusoidal E_{in}:

$$E_{in} = E_i \sin(2\pi ft)$$

Where it can be shown that:

$$\frac{E_{out}}{E_{in}} = |A|\sin(2\pi ft + \theta)$$

Where:

$$|A| = \frac{1}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}, \quad \theta = \tan^{-1}\left(\frac{f_1}{f}\right), \quad and \quad f_1 = \frac{1}{2\pi\tau}$$

- For high frequency inputs ($f>>f_1$) the amplitude approaches 1, but for low frequencies ($f<<f_1$) the amplitude goes to 0, thus a high pass filter.

RC Integrator (Low Pass Filter):

- Fig. 16.10 illustrates the results of an RC filter to a step function, a quasi-logarithmic growth, the circuit configuration is also shown.
- Now $E_{in}=iR + E_{out}$, where:

$$i = \frac{dQ}{dt} = \frac{dV_C}{dt}$$

- In this case the voltage across the capacitor is also the output voltage:

$$i = C\frac{dV_c}{dt}$$

Using $\tau=RC$ and combining equations:

$$E_{in} = \tau\frac{dE_{out}}{dt} + E_{out}$$

Rearranging:

$$\frac{1}{\tau}E_{in} = \frac{dE_{out}}{dt} + \frac{1}{\tau}E_{out}$$

Again for large τ the $(1/\tau)*E_{out}$ term is insignificant:

$$\frac{1}{\tau}E_{in} \cong \frac{dE_{out}}{dt} \quad or \quad E_{out} \cong \frac{1}{\tau}\int E_{in}dt$$

When τ is small the $1/\tau$ dominates:

$$\frac{1}{\tau}E_{in} \cong \frac{1}{\tau}E_{out}, \quad or \quad E_{out} \cong E_{in}$$

Example: Sinusoidal input:

For $E_{in}=E_i sin(2\pi ft)$ it can again be shown that:

$$\frac{E_{out}}{E_{in}} = |A|\sin(2\pi ft + \theta)$$

Where:

$$|A| = \frac{1}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}}, \quad \theta = \tan^{-1}\left(\frac{f}{f_2}\right), \quad and \quad f_2 = \frac{1}{2\pi\tau}$$

- Note that for high frequencies ($f >> f_2$), $|A|$ goes to 0 and for low pass frequencies ($f << f_2$), $|A|$ now approaches 1 and behaves as a low pass filter.

Example: Step voltage input: $E_{in} = \begin{cases} E(t \geq 0) \\ 0(t < 0) \end{cases}$

The output is now: $E_{out} = E\left(1 - e^{-\frac{t}{\tau}}\right)$

CR-RC Shaping:

- Fig.16.11 illustrate a shaping network which is composed of sequential differentiating and integrating stages, denoted as CR-RC networks.
- The general solution of the response of the combined network to a step voltage of amplitude E at $t = 0$ is:

$$E_{out} = \frac{E\tau_1}{\tau_1 - \tau_2}\left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}}\right)$$

For unequal time constants (τ_1 & τ_2) of stages in the network.

- For equal time constants ($\tau_1 = \tau_2 = \tau$) the solution is:

$$E_{out} = E\left(\frac{t}{\tau}\right)e^{-\frac{t}{\tau}}$$

- Fig. 16.12 shows the response of a CR-RC network to a step voltage with four pairs of differentiator (+) integrator time constants.
- The ballistic deficit is an affect caused by applying time constants on the order of the rise time (in an attempt to get the voltage to the baseline quickly). At this point the step voltage no longer appears as such, and some amplitude is lost.

Gaussian or CR-(RC)^n Shaping:

- If a single CR differentiator is followed by n stages of RC integration, the solution is:

$$E_{out} = \frac{E}{n!}\left(\frac{1}{\tau}\right)^n e^{-\frac{t}{\tau}}$$

The output approaches a Gaussian line shape for n≈4.

- For a given τ, $n = 4$ will take four times as long to return to the baseline, but if τ is adjusted to the same time frame as $n = 1$ then the shape is Gaussian and returns to the baseline more quickly than just for a single CR-RC shaping circuit.

Other Shaping Strategies:

- Active filter shaping provides for similar shaping using active components (circuits) incorporating diodes and transistors.
- Triangular shaping will generally use active elements to produce a triangular output, which has benefits over a Gaussian. Trapezoidal shaping puts a "flat top" on the pulse shape which can remove problems with the ballistic deficit, where pulse heights would differ for shorter τ selections.
- All pulses from trapezoidal shaping would have the same amplitude.

Pole-Zero Cancellation:

- For input pulses having a finite decay, CR-RC differentiator-integrator create a slight 0 cross over or undershoot in the output, which recovers back to 0 with a time characteristic of preamplifier decay time.
- The undershoot will affect the signal amplitude of another signal that arrives at the time.
- Fig 16.15 shows an input, CR-RC shaping and undershoot, for an input step with a finite decay time. An added resistance R_{pz} in the differentiator stage can correct for this problem.

Laplace Transform Aside:

- The Laplace transform is defined as:

$$\overline{F}(s) = \int_0^\infty f(t)e^{-st}\,dt$$

Where t is the real or time domain variable and s is the complex variable.

LT example 1: $f(t) = e^{-at}$

$$\overline{F}(s) = \int_0^\infty e^{-at}e^{-st}\,dt = \int_0^\infty e^{-(a+s)t}\,dt = \frac{1}{a+s}$$

LT example 2: $f(t) = \begin{cases} E & t > 0 \\ 0 & t \le 0 \end{cases}$

$$\overline{F}(s) = \int_0^\infty Ee^{-st}\,dt = \frac{E}{s}$$

LT example 3: Differentiator Circuit

$$E_{in} - \frac{Q}{C} - E_{out} = 0$$

$$E_{in} - \frac{Q}{C} - iR = 0$$

$$E_{in} - \frac{1}{C}\int^{t} idt' - iR = 0$$

Apply the LT:

$$\int_0^\infty E_{in} e^{-st} dt - \frac{1}{C}\left[\int_0^\infty \left(\int^{t} idt'\right) e^{-st} dt\right] - R\int_0^\infty ie^{-st} dt = 0$$

Using the LT property:

$$\int^{t} \bar{f}(t)dt' = \frac{\bar{f}(t)}{s} = \frac{\bar{F}(s)}{s} \quad for \quad \int^{t} idt'$$

Gives:

$$\bar{E}_{in}(s) = \frac{1}{C}\frac{\bar{I}(s)}{s} - R\bar{I}(s) = 0$$

We now solve for the transform of the current (*I(s)*):

$$\bar{I}(s) = \frac{\bar{E}_{in}(s)}{\frac{1}{Cs} + R}$$

The output of the signal transformation:

$$\bar{I}(s)R = \frac{\bar{E}_{in}(s)R}{\frac{1}{Cs} + R} = \frac{s\tau}{1 + s\tau}\bar{E}_{in}(s) = \bar{E}_{out}(s)$$

The factor $s\tau/(1+s\tau)$ is called the "transfer function" (*T(s)*) or what is multiplied by the input voltage to get the response (output voltage).

We can write:

$$\bar{T}(s) = \frac{s\tau}{1 + s\tau} = \frac{s}{s + \frac{1}{\tau}}$$

Which has a pole (infinite value) an $s=-1/\tau$.

- Consider using a CR differentiator to turn a step function into a simple exponential. Then use this to represent signal from a preamplifier. The signal is fed into a CR-RC pulse shaping circuit.
- If we use the LT notation for the output of the CR differentiator we again use $\bar{T}(s) = \frac{s\tau}{1 + s\tau}$ where τ is the time constant and s is the Laplace variable.

- In LT space the transfer functions multiply, so if we want an overall transfer function from the CR different from the pre-amp and the CR portion of the CR-RC pulse shape circuit we get:

$$\left(\frac{\tau_1 s}{1+\tau_1 s}\right)\left(\frac{\tau_2 s}{1+\tau_2 s}\right)=\frac{\tau_1\tau_2 s^2}{(1+\tau_1 s)(1+\tau_2 s)}$$

Just the product of each CR.

- If we now add our undershoot correction, R_{pz} in parallel with the capacitor in the CR part of the CR-RC circuit, the overall transfer function is:

$$\frac{s\tau_1\tau_2(1+sR_{pz}C_1)}{(1+s\tau_2)(sR_{pz}C_1\tau_1+R_{pz}C_1+\tau_1)}$$

- We now choose $R_{pz}=\tau_2/C_1$ and let $k=(\tau_1+\tau_2)/\tau_1\tau_2$ so that $T(s)$ reduces to:

$$\bar{T}(s)=\frac{s}{s+k}$$

With a single pole at $s=-k$.

Which ensures that the network produces a simple exponential decay for a step input.

- What exactly does R_{pz} do? R_{pz} allows an attenuated replica of the input pulse to pass through to the output. If the input is a positive step with a finite decay, then the slight undershoot from a normal CR differentiator can be cancelled by the slight admixture of the input pulse.
- Generally this value is selected by inspection and adjustment of an oscilloscope.

Baseline shift:

- The origin of the problem, baseline shift, stems from the problem that we generally deal with a train of pulses, not individual isolated pulses.
- Looking at Fig. 16.11 note that for a DC capacitive coupled circuit everything to the right of the capacitor must average to 0. This means if we were left with some voltage, a current must flow through the resistor and capacitor to ground (Impossible for a DC circuit).
- Thus we expect the signal to average to 0, or have equal amounts above and below the 0 line to give us the 0 average.
- The problem is exacerbated by a variable baseline shift depending on the pulse height (again from pulse pile-up), due to the randomness of pulse events.
- Fig. 16.16 shows the averaging for a regular pulse train, and one for random pulse train events.
- Fig. 16.17 shows non-polar and bi-polar pulses and their correction (bi-polar) to baseline shift.
- A bi-polar shift can correct this problem (in theory) by producing two lobes of equal area above and below the 0, negating a baseline correction requirement (since this pulse can now pass the capacitor).

Baseline Restoration:

- Corrections can be implemented by shaping circuits (to produce the bi-polar signals).
- Using an active circuit can restore the baseline. Such a circuit is illustrated in Fig. 16.18.
- The baseline-restorer circuit makes use of a switch to restore the signal quickly to zero, before the next pulse.
- Such a circuit should be placed near the end of the pulse shaping since another capacitor would introduce a new baseline shift.

Other Pulse Shaping Methods

Double Differentiation or CR-RC-CR Shaping:

- Fig. 16.19 illustrates the effect of CR-RC-CR shaping, which shows a bi-polar output for an input step function. Plot shows result for all τ's being equal.

Single Delay Line (SDL) Shaping:

- Implemented by using a shorter cable at the receiving end.
- Produces a reflection when the signal reaches the end.
- Shown in Fig. 16.20 an SDL network produces a rectangular output pulse from a step voltage input. As noted the delay transmission line is shorted at the end and the travel time T is the time to travel through the delay transmission line.
- Fig. 16.21 shows how the technique can be applied to various signal shapes with various results. For a pulse with a decay time on the order of the delay time a pulse with undershoot is formed (a). With the application of sending an attenuated reflection the undershoot is eliminated (b). Finally (c) illustrates the effect of SDL shaping on a rectangular shaped input pulse whose length exceeds the delay line down and back time.

Double Delay Line (DDL) Shaping:

- DDL can produce a better formed bipolar shape.
- Fig. 16.22 illustrates the effect of a DDL shaping network (with equal delay time) applied to a step input waveform. The result is a bipolar rectangular pulse.