

## Energy Resolution:

Statistical considerations:

- We define  $n_0$  to be the number of avalanches formed from  $n_0$  original ion pairs and  $A$  to be the multiplication factor for an individual avalanche:

$$M = \frac{1}{n_0} \sum_{i=1}^{n_0} A_i = \bar{A}$$

and from the charge contribution due to each avalanche,  $eA_i$  we get  $M=Q/en_0$  or  $Q=n_0eM$ .

- By error propagation we get for  $Q$ :

$$\left(\frac{\sigma_Q}{Q}\right)^2 = \left(\frac{\sigma_{n_0}}{n_0}\right)^2 + \left(\frac{\sigma_M}{M}\right)^2$$

from “ $M$ ” equation we get:

$$\sigma_M^2 = \left(\frac{1}{n_0}\right)^2 \sum_{i=1}^{n_0} \sigma_A^2 = \left(\frac{1}{n_0}\right)^2 n_0 \sigma_A^2 = \frac{\sigma_A^2}{n_0}$$

- Substituting into the  $Q$  error equation:

$$\left(\frac{\sigma_Q}{Q}\right)^2 = \left(\frac{\sigma_{n_0}}{n_0}\right)^2 + \frac{1}{n_0} \left(\frac{\sigma_A}{\bar{A}}\right)^2,$$

where the relation is between the fractional variance of the pulse amplitude

$\left(\frac{\sigma_Q}{Q}\right)^2$ , ion pair fluctuation  $\left(\frac{\sigma_{n_0}}{n_0}\right)^2$ , and single-electron multiplication variation  $\left(\frac{\sigma_A}{\bar{A}}\right)^2$ .

- 1) Variations in the number of ion pairs: In terms of the Fano factor

$$\sigma_{n_0}^2 = Fn_0 \quad \text{or} \quad \left(\frac{\sigma_{n_0}}{n_0}\right)^2 = \frac{F}{n_0}.$$

- 2) Variations in single-electron avalanches: We assume that the probability of ionization is dependent only on electric field strength ( $\epsilon$ ) and independent of previous history. It can then be shown the expected distribution in the number of electrons in a given gas from an avalanche should be predicted by a Furry distribution.

- The Furry Distribution:

$$P(A) = \frac{\left(1 - \frac{1}{\bar{A}}\right)^{A-1}}{\bar{A}}$$

where  $A$  is the avalanche multiplication and  $\bar{A}$  is the mean value of  $A(=M)$ .

- If  $A$  is reasonably large then:

$$P(A) \cong \frac{e^{-\frac{A}{\bar{A}}}}{\bar{A}} \quad \text{with} \quad \left( \frac{\sigma_A}{\bar{A}} \right)^2 = 1$$

- In strong electric fields, the probability of ionization by an electron can no longer be considered independent of its past history, and the experimental description is no longer valid. The exponential description is replaced by a Polya distribution:

$$P(A) = \left( \frac{A(1+\theta)}{\bar{A}} \right)^\theta \exp\left( -\frac{A(1+\theta)}{\bar{A}} \right),$$

where  $\theta$  is a parameter related to the number of electrons energies whose energy exceeded a threshold energy for ionization ( $0 < \theta < 1$ ).

- The relative variance:

$$\left( \frac{\sigma_A}{\bar{A}} \right)^2 = \frac{1}{\bar{A}} + b, \quad b = (1+\theta)^{-1} \quad \text{with observed values of } b \sim 0.5.$$

$$\text{For large } \bar{A} : \left( \frac{\sigma_A}{\bar{A}} \right)^2 \cong b.$$

3) Overall statistical limit: Recall  $\left( \frac{\sigma_Q}{Q} \right)^2 = \left( \frac{\sigma_{n_0}}{n_0} \right)^2 + \frac{1}{n_0} \left( \frac{\sigma_A}{\bar{A}} \right)^2$

With the replacements above:

$$\left( \frac{\sigma_Q}{Q} \right)^2 = \frac{F}{n_0} + \frac{b}{n_0} = \frac{1}{n_0} (F + b)$$

where  $F \sim 0.05-0.20$  and  $b \sim 0.4-0.7$ .

- The relative square root:  $\frac{\sigma_Q}{Q} = \left( \frac{F + b}{n_0} \right)^{\frac{1}{2}}$
- Using  $W = E/n_0$ , where  $E$  is the energy deposited and  $W$  the energy required per ion pair:

$$\frac{\sigma_Q}{Q} = \left( \frac{W(F + b)}{E} \right)^{\frac{1}{2}} \quad \text{and} \quad W(F + b) = C$$

A constant for a given fill gas ( $C$ ):

$$\frac{\sigma_Q}{Q} = \left( \frac{C}{E} \right)^{\frac{1}{2}}$$

or the relative energy resolution is inversely proportional to the square root of the energy deposited.

## Other Factors Affecting Resolution:

- Data at gas pressures  $\sim 1$  atm confirm the  $1/E$  dependence, but resolution tends to worsen at higher pressures.
- The relation is seen in Fig. 6.12 showing the confirmation of the  $1/E$  dependence.
- Other adverse factors include: electronic noise; geometric nonconformities in the chamber; variation of operating parameters (e.g. unstable applied voltage, gas impurity, aging physical condition of the chamber); and the count-rate effect where space charge build up at different rates; table 6.3 shows various changes in  $M$  with changing detector parameters.

## Time Characteristics of the Signal Pulse:

- A similar analysis for pulse type ion chambers can be performed for proportional chambers with 2 differences:
  - 1) The output of the pulse is divided into two regions. One during the drift time required for the free electrons created by the radiation to travel from their original formation to the avalanche region; the second time is from the avalanche formation to completion.
  - 2) The bulk of the output pulse is attributable to the positive ion drift from the multiplication region near the anode to the cathode. The latter part of the pulse rises slowly, but in practice is not observed due to the finite shaping times of the subsequent electronic circuit.
- Start with the energy absorbed ( $dE$ ) by the motion of a positive charge  $Q$  through a difference in electric potential  $d\phi$ :

$$dE = -Qd\phi$$

- For a cylindrical field:  $d\phi = -\varepsilon(r)dr$
- Substituting in for  $d\phi$ :

$$\frac{dE}{dr} = Q\varepsilon(r) = Q \frac{v_o}{r \cdot \ln\left(\frac{b}{a}\right)}$$

- Energy absorbed at a fixed distance  $\rho$  from the surface of the anode, given  $Q = n_0 c$ , then the positive ions give:

$$E^+ = \int_{a+\rho}^b \frac{dE}{dr} dr = \frac{Qv_o}{\ln(b/a)} \int_{a+\rho}^b \frac{dr}{r} = \frac{Qv_o}{\ln(b/a)} \ln\left(\frac{b}{a+\rho}\right)$$

- Energy absorbed by the negatively charged electrons inward to the anode:

$$E^- = -\frac{Qv_o}{\ln(b/a)} \int_{a+\rho}^a \frac{dr}{r} = \frac{Qv_o}{\ln(b/a)} \ln\left(\frac{a+\rho}{a}\right)$$

- The total energy absorbed  $\Delta E = E^+ + E^-$ :

$$\Delta E = \frac{Qv_o}{\ln(b/a)} \left( \ln\left(\frac{b}{a+\rho}\right) + \ln\left(\frac{a+\rho}{a}\right) \right) = \frac{Qv_o}{\ln(b/a)} \ln\left(\frac{b}{a+\rho} \cdot \frac{a+\rho}{a}\right) = Qv_o$$

- Fig. 6.13 shows the geometry for the cylindrical proportional counter for the previous analysis and the diagram shows  $V_R$  the voltage measured across the lead resistance  $R$  and  $V_{Ch}$  the voltage across the chamber or detector.
- The energy absorbed comes from the energy stored in the capacitor:

$$\frac{1}{2} CV_{Ch}^2 = \frac{1}{2} CV_0^2 - \Delta E \rightarrow \frac{1}{2} C(V_{Ch} + V_0)(V_{Ch} - V_0) = -\Delta E$$

Assuming  $V_{Ch} + V_0 \approx 2V_0$  and  $V_R = V_{Ch} - V_0$ :

$$V_R = \frac{\Delta E}{CV_0} = \frac{QV_0}{CV_0} = \frac{Q}{C}$$

which is the same result as parallel geometry.

- Most of the absorbed energy comes from the larger, further moving positive ions, we can test this by looking at the ratio of energy absorption:

$$\frac{E^-}{E^+} = \frac{\ln\left(\frac{a+\rho}{a}\right)}{\ln\left(\frac{b}{a+\rho}\right)}$$

for  $a=25 \mu\text{m}$ ,  $b=1 \text{ cm}$  (tube dimensions);  $\rho=3 \mu\text{m}$ ;  $E^-/E^+=0.019$  ( $<2\%$ ).

- We make an assumption that the signal pulse stems from the positive ions, we use a drift velocity and mobility argument to obtain the pulse shape.

$$v^+(r) = \frac{\mu \mathcal{E}(r)}{p} = \left(\frac{\mu}{p}\right) \left(\frac{V_0}{\ln(b/a)}\right) \left(\frac{1}{r}\right)$$

- Using a motion integration ( $\int dx = \int v dt$ ):

$$\int_a^{r(t)} \frac{dr}{v^+(r)} = \int_0^t dt \Rightarrow r(t) = \left(\frac{2\mu}{p} \frac{V_0}{\ln(b/a)} t + a^2\right)^{\frac{1}{2}}$$

- The time required to collect the ions can be found by setting  $r(t) = b$

$$t^+ = \frac{(b^2 - a^2)p \ln(b/a)}{2\mu V_0}$$

- So the energy absorbed by the motion of the ions as a function of time:

$$E^+(t) = \frac{QV_0}{\ln(b/a)} \int_a^{r(t)} \frac{dr}{r} = \frac{QV_0}{\ln(b/a)} \ln\left(\frac{r(t)}{a}\right)$$

- Replacing  $r(t)$  from the equation above and setting  $V_R = (E^+(t)/CV_0)$ :

$$V_R(t) = \frac{Q}{C} \frac{1}{\ln(b/a)} \ln\left(\frac{2\mu V_0}{a^2 p \cdot \ln(b/a)} t + 1\right)^{\frac{1}{2}}$$

which is the pulse shape with time.

## Spurious Pulses:

- Secondary pulses generated from various causes (some unknown) include: Optical photons from excited atoms produced in avalanche; the photons reach the cathode and may produce a photo-electron and another avalanche; interaction of the positive ion upon collection at the cathode; field emission of electrons in the cathode wall; meta stable states of the counter gas; and electrons delayed by space charge effects of previous avalanches.
- Such pulses can be reduced or eliminated by timing choice, pulse height discrimination, or lowering the applied excitation voltage.

## Detection Efficiency and Counting Curves:

### Selection of Operating Voltage:

- The pulse originates from a finite number of ion pairs in order to have amplitude large enough to exceed the discrimination level of the counting system. Since gas multiplication factors vary with the applied voltage, a “counting curve” is used to select an appropriate operating voltage, seen in plots on pages 184 & 185.
- Note the plateaus where counting rates do not increase appreciably for a range of voltages.

### Alpha Counting:

- $\alpha$ -particles from radioactive sources are generally mono-energetic and have a range less than the dimensions of the chamber. This situation would constitute pulses of a single peak in a histogram of pulse height, generally efficiency is 100%.

### Beta Counting:

- $\beta$ -particles have a range that exceeds the dimensions of a proportionality chamber.
- $\beta$  pulses have pulse heights smaller than  $\alpha$ 's, also note the wider distribution of pulse heights for  $\beta$ -particles (page 185).

### Mixed Sources:

- Many applications will want to determine the contribution from mixed  $\alpha$  and  $\beta$  sources. One can achieve this simply by setting up 2 channels set with windows at each of the energies by windowing different pulse heights.