Detection Efficiency

- In general, all radiation detectors will give rise to pulse for each quantum of radiation that interacts with its active volume.
- For α and β particles, which leave a large ionization trail, it is easy to arrange these detectors to have 100% efficiency.
- However, uncharged particles tend to travel large distances between interactions where some may not trigger inside the active volume (photons & neutrons), and have efficiencies less than 100%.
- Absolute efficiency: ε_{abs}=# of pulse recorded/# of quanta emitted.
- Intrinsic efficiency: ε_{int}=# of pulses recorded/# of quanta incident on detector. And the # of quanta incident on the detector is the fraction of the solid angle subtended by the detector:

$$\#\ \ of\ \ incident\ \ quanta = \frac{\Omega}{4\pi} \times\ \ \#\ \ of\ \ radiation\ \ quanta\ \ emitted$$

- By substitution $\varepsilon_{abs} = \frac{\Omega}{4\pi} \cdot \varepsilon_{\text{int}}$, where $\frac{\Omega}{4\pi}$ is also called the geometric efficiency.
- The peak efficiency is then the quanta that deliver their full energy to the detector. This will generally be at the high end of the spectrum in a peak, where lower energy deposition contributes to the spectrum at lower energies.
- Total efficiency is the total count over all energies (the area under the entire curve on a differential pulse height spectrum), ε_{total}.
- The peak efficiency is then the area under the curve at the peak, ε_{peak}.
- Then the peak-to-total ratio, *r*, is:

$$r = \frac{\varepsilon_{peak}}{\varepsilon_{total}}$$

- The intrinsic peak efficiency is then:

$$\varepsilon_{IP} = \frac{\#\ \ of\ \ recorded\ \ pulses\ \ in\ \ full\ \ energy\ \ peak}{\#\ \ of\ \ quanta\ \ incident\ \ on\ \ \det ector}$$

- Inversely, the number of pulses recorded in the full energy peak, given S number of quanta from the source:

$$N = \varepsilon_{IP}\left(\frac{\Omega}{4\pi}\right)S \text{, for a point source}$$

  The subtended angle in steradians is:

$$\Omega = \int_A \frac{\cos(\alpha)}{r^2}\,dA$$

  Where *r* is the distance from the source to the detector element *dA*, and α is angle between the normal to the surface element *dA* and the source direction.
- For a large volume source, an integral over the volume of the source must also be included.

– For a point source and a right circular cylindrical detector of radius *a*:

$$\Omega = 2\pi\left(1 - \frac{d}{\sqrt{d^2 + a^2}}\right)$$

– Ω reduces for *d* >> *a* to:

$$\Omega \cong \frac{A}{d^2} = \frac{\pi a^2}{d^2},$$

Where *d* is the distance from the detector.

– For another common configuration of a circular disk ($r = s$) source emitting isotropic radiation aligned with a circular disk ($r = a$) detector both positioned perpendicular to a common axis through their centers a distance *d* apart:

$$\Omega = \frac{4\pi a}{s}\int_0^\infty \frac{\exp(-dk)J_1(sk)J_1(ak)}{k}\,dk$$

Where $J_1(x)$ are the Bessel functions of *x*.

– Through a numerical solution method one can approximate the answer to:

$$\Omega \cong 2\pi\left[1 - \frac{1}{(1+\beta)^{\frac{1}{2}}} - \frac{3}{8}\cdot\frac{\alpha\beta}{(1+\beta)^{\frac{5}{2}}} + \alpha^2[F1] - \alpha^3[F2]\right]$$

Where:

$$F1 = \frac{5}{16}\cdot\frac{\beta}{(1+\beta)^{\frac{7}{2}}} - \frac{35}{16}\cdot\frac{\beta^2}{(1+\beta)^{\frac{9}{2}}}$$

$$F2 = \frac{35}{128}\cdot\frac{\beta}{(1+\beta)^{\frac{9}{2}}} - \frac{315}{256}\cdot\frac{\beta^2}{(1+\beta)^{\frac{11}{2}}} + \frac{1155}{1024}\cdot\frac{\beta^3}{(1+\beta)^{\frac{13}{2}}}$$

And $\alpha \equiv (a/d)^2$, $\beta \equiv (s/d)^2$, which becomes inaccurate when source or detector diameters get too large compared with *d*.

Dead Time

– Dead time is the time required to separate two events so that they are recorded as two pulses.
– There are two models of dead time behavior:
  - Paralyzable: true events that occur during the dead period, although not recorded, are assumed to extend the dead time by another period following the lost event.
  - Non-paralyzable: true events that occur during the dead time are lost and assumed to have no effect on the behavior of the detector.
– We define the following variables: *n*-true interaction rate; *m*-recorded count rate; τ-system dead time.

Non-paralyzable System:

- The total time dead is $m\tau$, and the loss rate is $nm\tau$ and n-m, therefore
  - $n-m = nm\tau$.
- For the true count rate: $n = \dfrac{m}{1-m\tau}$

Paralyzable System:

- Dead periods are not a fixed length, so we need a different analysis, but we note m is also the rate of occurrences of time intervals between true events which exceed τ.
- We invoke the distribution of intervals between random events occurring at an average rate n to be:

  $$P_1(T)dT = ne^{-nT}dT \ ,$$

  Where $P_1(T)$ is the probability of observing an interval whose length lies within dT about T. (Recall $P_1(T)=P(0)$ x ndt and $P(0) = \dfrac{(nt)^0 e^{-nt}}{0!}$)
- The probability of have intervals larger than τ is:

  $$P_2(T) = \int_{\tau}^{\infty} P_1(T)dT = e^{-nt}$$

  which when multiplied by the true rate n, gives us the rate of occurrence:

  $$m = ne^{-nt}$$
- One can then compare the two models of how m varies as a function of n (Fig. 4.8).
- Note that a non-paralyzable system approaches an asymptote value for the observed rate of $1/\tau$ (where the counter is just recovering from one dead time before beginning another).
- Note that the paralyzable system goes through a maximum.
- For low rates ($n<<1/\tau$) both systems produce count rates of:

  $$m \cong n(1-n\tau)$$

<u>Methods of Dead Time Measurement:</u>

Two Source Method:

- n_i is the counting rate of source i (i=1 or 2) including background
- n_{12} is the counting rate of sources 1 & 2 including background
- n_b is the background rate.
- $n_{12}-n_b=(n_1-n_b)+(n_2-n_b) \Rightarrow n_{12}+n_b=n_1+n_2$.
- Assuming a non-paralyzable model we substitute true count rates for observed count rates:

$$\frac{m_{12}}{1-m_{12}\tau}+\frac{m_b}{1-m_b\tau}=\frac{m_1}{1-m_1\tau}+\frac{m_2}{1-m_2\tau}$$

- Solving for τ:

$$\tau=\frac{x\left(1-\sqrt{1-z}\right)}{y}$$

Where: $x=m_1m_2-m_bm_{12}$, $y=m_1m_2\left(m_{12}+m_b\right)-m_bm_{12}\left(m_1+m_2\right)$, and $z={y\left(m_1+m_2-m_{12}-m_b\right)}\big/{x^2}$.

- For $m_b = 0$:

$$\tau=\frac{m_1m_2-\left[m_1m_2\left(m_{12}-m_1\right)\left(m_{12}-m_2\right)\right]^{\frac{1}{2}}}{m_1m_2m_{12}}$$

- This method requires using sources with two equally large numbers and best results are obtained by using sources active enough to result in fractional dead time $m_{12}\tau$ of at least 20%.

Decaying Source Method:

- Based on the known behavior of the true rate of n:

$$n=n_oe^{-\lambda t}+n_b$$

Where n_o is the true rate at the beginning, and λ is the decay constant.

- For negligible background $n \cong n_oe^{-\lambda t}$ and assuming a nonparalyzable model $n=m/(1-m\tau)$:

$$n_oe^{-\lambda t}=\frac{m}{1-m\tau}\Rightarrow me^{\lambda t}=-n_o\tau m+n_o$$

Where if we set $y = me^{\lambda t}$ and plot y vs. m, we get a straight line of slope $-n_o\tau$ with a y-intercept of n_o (which gives us τ by the ratio of the slope to the y intercept).

- For the paralyzable model $m = ne^{-n\tau}$.
- Taking a ln: $ln(m)=-n\tau + ln(n)$ and substituting we get:

$$\ln(m)=-n_oe^{-\lambda t}\tau+\ln\left(n_oe^{-\lambda t}\right)\Rightarrow \lambda t+\ln(m)=-n_oe^{-\lambda t}\tau+\ln\left(n_o\right)$$

- Again we make the LHS = y = $\lambda t+\ln(m)$ and $e^{-\lambda t} = x$, we get a line function of "x" and can determine the dead time from the intercept and slope.

Statistics of Dead Time Losses:

- Distorts the statistics away fro Poisson behavior if large enough (~10-20%).

Dead Time Losses from Pulsed Sources:

- Previous analyses have assumed a constant source. Now we look at a pulsed source (like a Linac which can produce pulsed x-rays).

- We assume a constant relation over a time T, where the pulses occur with period of $1/f$ which depends on the dead time of the detector τ.
  1. For $\tau << T$: our previous analysis holds (the pulses have little effect).
  2. For $\tau < T$: only a small number of counts may be registered by the detector during a single pulse. This results in an analysis beyond the scope of this class or text.
  3. For $\tau > T$ but less than the "off time" between pulses ($1/f$ - T): There is a registration of a 0 or 1 count per pulse (This analysis is also applicable to sources not constant over τ).
- We define f to be the source pulse frequency.
- The probability of observed count from a pulse = m/f.
- The average number of true events per source pulse = n/f.
- The probability of at least one true event occurs per source pulse is:

$$P(>0) = 1 - P(0) = 1 - e^{-\bar{x}} = 1 - e^{-\frac{n}{f}}$$

- This equals the probability of at least one observed count from a pulse:

$$1 - e^{-\frac{n}{f}} = \frac{m}{f} \Rightarrow m = f\left(1 - e^{-\frac{n}{f}}\right)$$

- Note in the plot of observed vs. true count rate, there is no dead time influence (m vs. n).
- Solving for a true count rate (n):

$$n = f \cdot \ln\left(\frac{f}{f - m}\right)$$

  Which is a correction to adjust from the observed to the true count rate.
- This is valid only under $T < \tau < (1/f - T)$.
- We can approximate this expression for small dead time losses, (small n and m relative to f) $m << f$ we get:

$$n \cong \frac{m}{1 - \frac{m}{2f}}$$

- If we compare this to the non-paralyzable model, where $n = m/(1-m\tau)$, we get an effective dead time of $1/2f$ in the low loss limit.
- Since this value is ½ the source pulsing period it can be many times longer than the actual physical dead time of the detector system.