Lecture 12b

Derivation of the Pulse Shape:

- For detectors with arbitrary geometry, the Shockly-Ramo theorem in Appendix D is used.
- For the simpler case of parallel plate geometry, separated by a comparatively small spacing (compared with the width and length of the plates, neglecting edge effects) we use a conservation of Energy argument.
- We assume an application of E field sufficient to make electron ion recombination negligible (so negative charges remain free electrons)
- Parallel plate electrodes : $\varepsilon = V/d$, where ε is the electric field intensity, V is voltage across the chamber electrodes and d the distance between the plates and x is the distance of the electron from the positive electrode, thus εx is the electric potential of the ions.
- The energy originally stored across the capacitance C is $\frac{1}{2}CV_0^2$ where V_0 is the initial applied voltage.
- At time t the ions have drifted v^+t toward the cathode, v^+ is the ion drift velocity, similarly v^-t is the distance the electrons have moved toward the anode.
- While moving the ions and electrons undergo multiple collisions, losing energy. The energy lost is $Q\Delta\varphi$, where Q is the total charge and $\Delta\varphi$ the potential change (a function of distance from an electrode). $Q = n_0 e$ where n_0 is the initial number of ion pairs and e the electric charge.
- Conservation of energy:

Original stored energy = energy absorbed by ions + energy absorbed by electrons + remaining stored energy

$$\Rightarrow \frac{1}{2}CV_0^2 = n_0 e\varepsilon v^+ t + n_0 e\varepsilon v^- t + \frac{1}{2}CV_{ch}^2,$$

V_{ch} is the charged voltage

$$\frac{1}{2}C(V_0^2 - V_{ch}^2) = n_0 e\varepsilon(v^+ - v^-)t$$

$$= \frac{1}{2}C(V_0 + V_{ch})(V_0 - V_{ch}) = n_0 e\left(\frac{V_{ch}}{d}\right)(v^+ + v^-)t$$

since $V_0 >> V_R = V_0 - V_{ch}$ we approximate that $V_0 + V_{ch} \approx 2V_0$

and $V_{ch}/d \approx V_0/d$

$$\approx \frac{1}{2}C(2V_0)V_R = n_0 e\left(V_0/d\right)(v^+ + v^-)t$$

$$V_R = \left(n_0 e/dC\right)(v^+ + v^-)t$$

which describes the initial signal pulse.

- We can consider the charge induction on the plate as the amount of charge changed by the ion moving an amount v^+t which gives a potential change of

$n_0 e v^+ t/dC$, as though it induced a charge across a capacitance C by $n_0 e v^+ t/d$, a similar argument is made for the electron.

- After a time $t^- \equiv X / v^-$ the electrons reach the anode, where their total contribution of $n_0 e v^- t^- / dC = n_0 e x / dC$ is given :
$$V_R = (n_0 e / dC)(v^+ t + x)$$
- The ions reach the cathode after a time $t^+ \equiv (d - X)/v^+$ and the signal no longer increases: $V_R = (n_0 e / dC)(d - x + x) = n_0 e / C$
- When RC >> t^+ the amplitude of the signal pulse is given by $V_{\max} = n_0 e / C$ and is independent of where the ion pairs are formed in the chamber
- In electron sensitive operation $t^- << RC << t^+$
$$V_R = (n_0 e / C)(x / d)$$
neglecting ion drift and pulse amplitude depends on the position of the interaction of the radiation.
- Fig 5.16 shows the effect of various time constants on the chamber.

Gridded Ion Chamber:

- To remove the position dependence in the electron sensitive regime a Frisch grid can be employed
- All ion pairs are formed between the grid and cathode (using collimation)
- The pulse shape results from the formation of n_0 ion pairs at a distance y from the grid, where d is the grid/anode spacing.
- No signal voltage is recorded in the movement of electrons or ions in the grid cathode volume.
- Once electrons pass through the grid, a potential develops across the load resistor.
- The voltage is described by the time dependent equation: $V_R = (n_0 e / dC) v^- t$ where d is now the grid anode spacing, leading to a maximum voltage: $V_{\max} = n_0 e / C$, which is identical to the previous response of both ions and electrons, only now is the product of electrons only and is independent of position (since each electron pass through the same potential)

Pulse amplitude:

- The maximum pulse amplitude from either the gridded or standard ion chamber is $V_{\max} = n_0 e / C$
- For a 1MeV particle n_0 can be estimated:
$$n_0 = E / W \cong 10^6 eV / 35 eV / ionpair = 2.86 \times 10^4$$
- C is then on the order of 10-10 farads, then the pulse amplitude is V_{max}= $(2.86 \times 10^4)(1.6 \times 10^{-19} C)/10^{-10} F = 4.58 \times 10^{-5} V$

Statistical limit to Energy Resolution:

Assume an α-particle of E_d = 5.5 MeV, W value = 30 eV/ion pair and Fano factor 0.15

$$n_0 = E_d/W = 5.5\text{x}10^6/30 = 1.83\text{x}10^5 \text{ ion pairs}$$

$$\sigma_{n_0}^2 = F_{n_0} = 0.15\left(1.83\times10^5\right) = 2.75\times10^4$$

$$\sigma_{n_0} = \sqrt{Fn_0} = \sqrt{2.75\times10^4} = 166$$

Assuming a Gaussian Distribution $FWHM\left(n_0\right) = 2.35\sigma_{n_0} = 390$

$$FWHM\left(E\right) = 2.35\sigma_{n_0}W = 390\left(30eV\right) = 11.7keV,$$

this would be the conventional ion chamber energy resolution limit
Or as a percentage of the deposited energy

$$R = \frac{2.35\sigma_{n_0}W}{E_d} = \frac{11.7\times10^3\,eV}{5.5\times10^6\,eV} = 0.213\%$$

Pulsed Particle Spectroscopy:

- Gas pressure content and arbitrary size make ion detectors a good choice for some applications although semiconductors are generally used for most of the spectroscopy applications
- Used in low level α-particle measurement
- Used for large detector construction and can produce resolutions for α particle measurements of 35-45 keV for 5Mev particles
- One can use the shape of Bragg curve to look at the positional deposition of energy, leading to what is called Bragg curve spectroscopy
- Pulse shape analysis can differentiate between particles of different types (atomic number and/or charge) through their Bragg curve differences.