2

a) The Risk of 0-1 loss is defined as

$$R = P(Y=1) \cdot P(g(x) \neq 1 \mid Y=1) +$$
$$P(Y=-1) \cdot P(g(x) \neq -1 \mid Y=-1)$$

$$= \theta \cdot P(g(x) \neq 1 \mid Y=1) + (1-\theta) \cdot P(g(x) \neq -1 \mid Y=-1)$$

$$= \theta \cdot P(g(x) = -1 \mid Y=1) + (1-\theta) \cdot P(g(x) = 1 \mid Y=-1)$$

$$= \theta \cdot \underset{x \sim p_+}{P}(g(x) = -1) + (1-\theta) \cdot \underset{x \sim p_-}{P}(g(x) = 1)$$

This can be written as

$$R = P(Y=1) \cdot \underset{x \sim p_+(x)}{E}[\mathbb{1}(g(x) \neq +1] +$$

$$P(Y=-1) \cdot \underset{x \sim p_-(x)}{E}[\mathbb{1}(g(x) \neq -1]$$

Suppose $P(Y=1)=1$, then our risk is

$$R(g) = \mathop{E}_{x \sim p_+(x)} \left[\mathbb{1}(g(x) \neq +1) \right],$$ this

gives us that $p^{tr} = p_+(x)$

we may then estimate R as

$$\hat{R}(g) = \frac{1}{|p^{tr}|} \sum_{i=1}^{|p^{tr}|} \mathbb{1}(g(x_i) \neq 1)$$

A similar argument holds when $P(Y=1)=0$. These are the only scenarios where we may estimate R, when we know p^{tr} is only a single probability distribution, not a mix of many.