Could twisted bilayer graphene be a nodal superconductor?

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We discuss a simple physical manifestation of the possibility of nodal quasiparticles in twisted tri/bilayer graphene, and their manifestations in transport experiments. Please do not share!

In MATBG the superconductivity is strongest at filling $\nu = -2 - \epsilon$ with a BKT transition temperature in the range $T_{BKT} \sim 1 - 3K$. A recent STM measurement finds a tunneling gap $\Delta \approx 2.5meV$ with the ratio $\frac{2\Delta}{k_B T_{BKT}} \approx 20$. We thus have a heirarchy of energy scales $k_B T_{BKT} \ll \Delta \ll E_F$.

Further the shape of the tunneling spectrum suggests the presence of nodal quasiparticles with a linear density of states. Here we explore some simple consequences of the presence of such nodal quasiparticles which may be visible in transport experiments.

We focus on the effects of the nodal quasiparticles on the phase stiffness K. This is defined as the free energy cost for a long wavelength distortion of the phase ϕ of the superconductor:

$$F = \int d^2x \frac{K}{2} \left(\nabla\phi\right)^2 \tag{1}$$

Note that in two dimensions K has dimensions of energy. It is common to also write $K = \frac{\hbar^2 n_s}{m^*}$ to define a superfluid density n_s (m^* is the normal state quasiparticle mass). For $\nu = -2 - \epsilon$, we expect that $K(T = 0) \equiv K_0 \sim \frac{\hbar^2 \epsilon}{a_M^2 m^*}$ where a_M is the moire lattice constant. Thus the ratio $\frac{K_0}{E_F}$ is a dimensionless number which we may imagine is much smaller than 1, but which we do not really know much about.

Now consider the temperature dependence of K, first within a BCS-like theory that ignores renormalization by vortex fluctuations (i.e the physics of the BKT transition). Generally thermal excitation of quasiparticles will reduce the phase stiffness. The crucial point is that as $T_{BKT} \ll \Delta$, this reduction of the phase stiffness can be obtained by considering the low-T limit where only a few quasiparticles are thermally excited. The T-dependence of K can then provide meaningful information about the gap structure of superconductor. In a gapped superconductor $K^g_{BCS}(T) = K_0 - O(e^{-\beta\Delta})$, and hence the effect of thermally induced quasiparticles on K is hardly visible for

$k_BT_{BKT} \ll \Delta$. In contrast in a nodal superconductor

$$K^n_{BCS}(T) = K_0 - Ak_BT \tag{2}$$

with the coefficient

$$A = \left(\frac{\ln 2}{\pi}\right)\left(\frac{\alpha^2 v_F p}{v_\Delta}\right) \tag{3}$$

where $v_F =$ Fermi velocity, $v_\Delta = \frac{1}{k_F}\frac{d\Delta}{d\theta}$ at the node, p is the number of distinct Dirac nodes. α is a Fermi liquid parameter—specifically, the current carried by a nodal quasiparticle is $\mathbf{j} = \alpha \mathbf{v}_F$. In standard BCS theory $\alpha = 1$ and we will assume $\alpha \sim O(1)$ here as well.

This linear-T dependence is a classic signature of a nodal superconductor and is seen in the cuprates where K is obtained by measuring the penetration depth. Crucially $K^n_{BCS}(T)$ goes to zero at a temperature T_0 with

$$k_BT_0 \sim \frac{K_0 v_\Delta}{v_F p} \tag{4}$$

As we have thus far ignored vortex fluctuations this gives an *upper bound* on T_c in the assumed nodal superconductor. In practice the BKT transition will occur before $K(T)$ goes to zero. The vortex unbinding happens when

$$\frac{K(T)}{k_BT} = \frac{2}{\pi} \tag{5}$$

Approximating $K(T)$ by $K^n_{BCS}(T)$ we get an estimate of T_{BKT}:

$$K_0 - Ak_BT_{BKT} \approx \frac{2}{\pi}k_BT_{BKT} \tag{6}$$

so that

$$k_BT_{BKT} \approx \frac{K_0}{A + \frac{2}{\pi}} \tag{7}$$

The expected temperature dependence of $K(T)$ is as shown in Fig. 1.

We do not have confident estimates of v_F/v_Δ or of p but we may guess that $v_F/v_\Delta \sim E_F/\Delta \approx 5$, and that p is 2 or 4 based on normal state landau level degeneracy. This gives a big value for A implying that $T_{BKT} \approx T_0$. Note that even if $K_0 \sim E_F$, the nodal quasiparticle mechanism significantly limits the value of T_0 and hence of T_{BKT}.

In a $2d$ superconductor, it is hard to measure $K(T)$ by the standard methods used for bulk superconductors but we can instead exploit the BKT physics itself. As is well known for $T > T_{BKT}$ the I-V curve is linear but it becomes non-linear for $T < T_{BKT}$. For $T \to T^-_{BKT}$, $V \sim I^3$ which is often used to identify the transition. For $T < T_{BKT}$, we have

$$V \sim I^{a(T)} \tag{8}$$

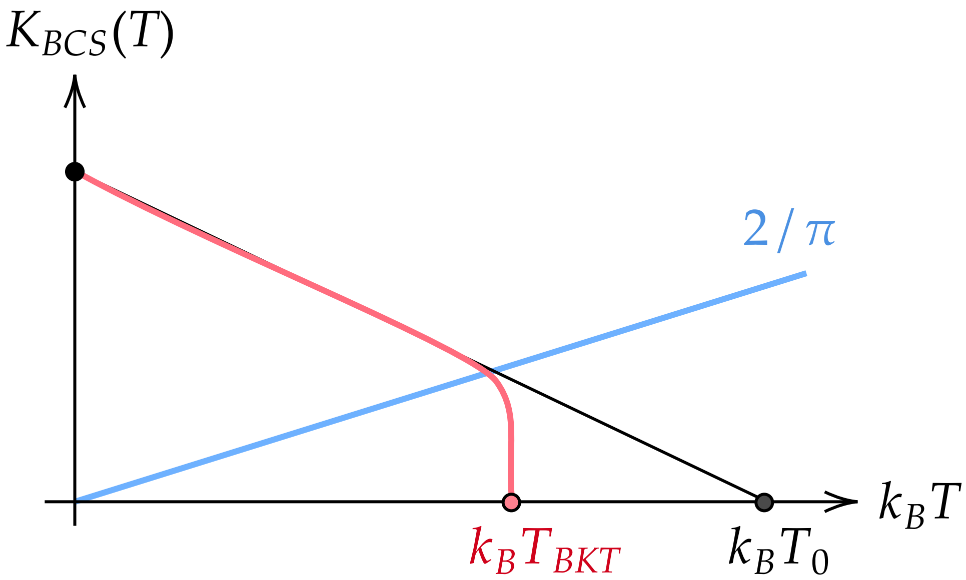

FIG. 1: Expected behavior of phase stiffness versus temperature. Black line is the BCS expectation for a nodal superconductor at $T \ll \Delta$. The SC is unstable to vortex unbinding once $K(T)$ goes below the blue line. The red curve is the true behavior of $K(T)$ including both quasiparticle and vortex effects.

with

$$a(T) = 1 + \frac{\pi K(T)}{k_B T} \tag{9}$$

(at least for low current densities). It follows that by measuring $a(T)$ we can extract $K(T)$.