Consider a vector field X with Lagrangian

$$\mathcal{L} = \frac{1}{2} X \wedge \star(\alpha \Pi_T + \beta \Pi_L + \gamma \star d) X, \tag{1}$$

with $\Pi_T = d^\dagger d/\Box, \Pi_L = dd^\dagger/\Box$. We invert this by writing the propagator as

$$D_X = A\Pi_T + B\Pi_L + C \star d. \tag{2}$$

We then use the orthogonality of $\Pi_{T/L}$ as well as (as usual, $\Box = d^\dagger d + dd^\dagger$ is negative-definite)

$$(\star d)^2 = -\Box \Pi_T, \qquad \star d \Pi_T = \Pi_T \star d = \star d, \tag{3}$$

where $\star d$ is viewed as a matrix with vector indices. The sign on this first equation is important, and follows from the fact that when acting on p-forms in D-dimensional Euclidean space, the adjoint of d is

$$d^\dagger = (-1)^{Dp+D+1} \star d \star . \tag{4}$$

For us $D = 3$ and $p = 1$, so that $d^\dagger = - \star d \star$ (alternatively one can just write out $\star d \star d$ explicitly).

This gives the conditions

$$\begin{aligned} \alpha A - \gamma C \Box &= 1 \\ \beta B &= 1 \\ \gamma A + \alpha C &= 0 \end{aligned} \tag{5}$$

so that

$$D_X = \frac{1}{\Box + \alpha^2/\gamma^2} \left(\frac{\alpha}{\gamma^2} \Pi_T - \frac{1}{\gamma} \star d \right) + \frac{1}{\beta} \Pi_L, \tag{6}$$

or in momentum space,

$$D_X^{\mu\nu} = \frac{1}{q^2 - \alpha^2/\gamma^2} \left[-\frac{\alpha}{\gamma^2} \left(\delta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + \frac{i}{\gamma} \varepsilon^{\mu\nu\lambda} q_\lambda \right] + \frac{1}{\beta} \frac{q^\mu q^\nu}{q^2}. \tag{7}$$

This is such that D_X is the inverse of the kernel in (1). If we just want to e.g. invert the kernel on coexact forms (viz. those with $\Pi_L X = 0$), we simply need drop the last $1/\beta$ term in the above expression.

As a check, note that this gives the correct topologically massive propagator when we take $\alpha = -\Box/e^2, \gamma = ik/2\pi, \beta = 0$.