uc3m | Universidad Carlos III de Madrid

University Degree in Economics
2022-2023

Bachelor Thesis

Econometrics meets AI: Predicting Airbnb prices using Machine Learning

---

José Jaén Delgado

1st Andrés Modesto Alonso Fernández
2nd Ricardo Aler Mur

Getafe, 2023

Abstract

Predictive models for Airbnb rental prices are built using Data Mining techniques and Machine Learning algorithms. The goal is to provide an accurate and scalable statistical toolkit for economic agents to make optimal decisions. It is found that Artificial Intelligence models outperform traditional econometric methods at the expense of lower interpretability, for which a solution is proposed.

Keywords: Algorithms, Machine Learning, Bayesian Statistics, Explainable AI

Mediante técnicas de minería de datos y algoritmos de Machine Learning se estiman diferentes modelos predictivos para precios de Airbnb. El objetivo es proveer a los agentes económicos de una herramienta estadística precisa y escalable para tomar decisiones óptimas. Se encuentra que los modelos de Inteligencia Artificial superan en rendimiento predictivo a los métodos econométricos clásicos al precio de una menor interpretabilidad, para lo cual se propone una solución.

Palabras clave: Algoritmos, Machine Learning, Estadística Bayesiana, IA Explicable

Contents

List of Figures

List of Tables

1 Introduction

"*Econometrics is the original Data Science*" Joshua Angrist (2020)

Economic agents face various utility optimization problems on a daily basis. Firms seek to maximize revenue subject to a cost function and input constraints. Households, on the other hand, choose optimal consumption bundles given a specific budget and individual preferences. Likewise, central banks conduct monetary policy by setting interest rates to attain price stability, maximizing social welfare.

Nowadays, society is immersed in the Big Data era, where numerous data sources are available for agents to make informed decisions. Pernagallo and Torrisi (2019) argue that real time communication networks and continuous technological breakthroughs result in information overload, ultimately overwhelming economic agents. What used to be scarce data has become massively accessible to the public.

Hardly any market is resilient to this issue and even experienced users may struggle to effectively process data to make profitable transactions. Agents might benefit from applying Machine Learning (ML) to large datasets.

Consequently, Artificial Intelligence (AI) could be used to overcome agents' limited computational capacity in the form of ML and Deep Learning (DL) models. However, outputs that stem from sophisticated algorithms suffer from the so-called 'black box' problem. Said issue is a longstanding concern related with the lack of interpretability of AI models. Rudin (2019) reviewed the tradeoffs of favoring such models over interpretable ones and concluded that the former are not always necessary to successfully fulfill predictive tasks.

In this project predictive accuracy and model interpretability commutation is studied for the real estate market. An AI framework for solving economic agents' utility optimization problem is proposed using online Airbnb listings data. Several ML and DL models will be built to accurately predict rental pricing in Los Angeles for June 2022, providing eXplainable Artificial Intelligence (XAI) methods to understand contributing factors to final predictions.

The rest of the document is structured as follows: Section 2 briefly introduces concepts and definitions related to AI. Section 3 reviews AI applications in Econometrics. Section 4 presents the utility maximization problem that is solved for real estate economic agents, describes the relevant dataset and presents Statistical Inference results. Section 5 explains ML notation and methodological aspects. Section 6 provides insights into modeling and XAI. Finally, Section 7 concludes and identifies some limitations of the AI framework.

2 Basic Concepts of AI

Grewal (2014) defines AI as "*the mechanical simulation system*" of human intelligence that collects knowledge and information for "*processing intelligence of universe and disseminating it to the eligible in the form of actionable intelligence*". Indeed, what has made AI stand out is the ability to build intelligent systems that reproduce human cognition.

Machine Learning, as expounded by Mitchell (1997), is a subfield of AI "*concerned with the question of how to construct computer programs that automatically improve with experience*". When the family of learning algorithms used for predictive tasks are Neural Networks, ML is called Deep Learning. DL specifically focuses on developing various model architectures that mimic biological neural networks.

Despite being used almost interchangeably, AI models and algorithms are not identical concepts. An algorithm is "*a procedure that is run on data to create a model*" (Brownlee, 2020). Algorithms *learn* by being fit to datasets. Contrastingly, a model is the output of AI algorithms, as well as the set of assumptions and restrictions on the joint distribution of the predictors and target variable. Thus, an AI model encompasses several algorithms and defines theoretical and empirical rules followed by the latter.

Even though ML and DL are usually linked with Data Science rather than Statistics, a considerable number of AI algorithms are built upon the latter (if not the vast majority). Not coincidentally, as it will be rapidly noticed by Data Analytics professionals such as econometricians and statisticians when reading Section 5 and Section 6, AI algorithms extend or slightly change 'traditional' quantitative methods.

3 Econometrics and AI

In fact, econometricians have been using ML and DL algorithms for some time. For example, Angrist and Frandsen (2019) resorted to ML for automating the selection of control variables, decreasing bias in the reduced form equation of the Two-Stage Least Squares (TSLS) estimator. With respect to policy evaluation, Dube et al. (2021) estimated the impact of minimum wages on employment and income with Classification And Regression Tree (CART) algorithms, greedy function approximations and regularized logistic regression. Hansen et al. (2018) studied how external communication of internal deliberation impacted monetary policy decisions of the Federal Open Market Committee (FOMC) using Natural Language Processing. These are just few examples of how econometricians benefitted from adopting ML.

With respect to the specific study of rental prices, econometricians have largely focused on hedonic models, a class of linear regression models that estimates the monetary value contribution of different variables such as location, air pollution and number of rooms to prices. Apart from deriving causal effects, sheer prediction was also in the interest of certain research.

Recently, ML and DL algorithms have been been widely adopted by econometricians, showing considerable improvement in predictive performance over OLS estimation method. Limsombunchai et al. (2004) compared Artificial Neural Networks with classical hedonic price models and concluded that DL algorithms outperformed the latter in predictive terms. Embaye et al. (2021) resorted to algorithms such as Random Forest and Gradient Boosting Machine, procuring better results than OLS estimation. It was noted, however, that the explanatory capacity of ML algorithms was scarce.

Even though AI algorithms clearly surpassed OLS, this is not to say that it is possible to adopt them for every purpose. Econometrics is the "*the application of mathematics and statistical methods to the analysis of economic data*" (Kayode and Tang, 2013), mainly aiming to establish causality between economic variables. ML and DL are mostly about finding functions that best map covariates with the target variable. Optimal data fitting is not enough to explain relationships between variables, let alone derive causal effects.

Moreover, lack of interpretability is hindering human capacity to oversight and understand the predictions of AI models. While econometricians have developed numerous statistical tests and draw much of their attention to correctly interpreting model outputs, the AI community's focalpoint has been refining performance of algorithms. Nevertheless, there is growing interest in enhancing model interpretability. As a demonstration, XAI methods will be applied to the best performing algorithm in Section 6.

4 Utility Maximization Problem & Airbnb Data

4.1 Utility function

The statistical toolkit to be built with AI algorithms serves the purpose of assisting economic agents in making optimal decisions. Naturally, an utility function has to be defined to understand how predictive models can contribute to such goal.

Let the i^{th} agent's problem be expressed as:

$$\begin{aligned} \arg\max_{\hat{p}_i} \; u_i(\hat{p}_i, s_{ij}) = & \sum_{j=1}^{n} \beta_{ij} u_i(s_{ij}) - |p_i - \hat{p}_i| \\ \text{s.t. } \hat{p}_i = & f(p_i|\mathbf{x}_i; \mathbf{w}_i) \end{aligned} \tag{1}$$

where $\beta_{ij} \in (0, 1)$ is the importance given by the agent to $u_i(s_{i,j})$, a function dependent on unobservable subjective factors $s_{i,j}$. p_i is the actual listing price and $\hat{p}_i$ is the predicted price. $f(p_i|\mathbf{x}_i; \mathbf{w}_i)$ is a target function mapping data $\mathbf{x}_i$ and parameters $\mathbf{w}_i$ to p_i. Note that if either data or prior beliefs were available for $u_i(s_{ij})$, it could be incorporated into AI algorithms and thus estimated.

Since the first term of the utility maximization problem cannot be tackled by AI, predictive models optimize agents' utility by providing accurate predictions in the sense that $|p_i - \hat{p}_i|$ is minimized.

In economic terms, Airbnb guests benefit from identifying fair priced listings, while real estate firms and Airbnb hosts obtain price margins for their marketing strategy.

4.2 Listings Data

The datasets used in this thesis were created by Inside Airbnb, a mission driven project that seeks to debate the role of renting residential homes to tourists with data. Inside Aibnb collects quarterly information about Airbnb listings across different countries via web scrapping and makes it available for the public.

Concretely, the city of Los Angeles was chosen to be analyzed during June 2022. A total of 42,041 houses are included with 74 different characteristics. After some data cleaning and feature engineering steps to be expounded in the next section 41,853 listings and 75 predictors were used for modeling. Although the reader may check the appendix for further details, the variables of the listings dataset can be generally classified into four groups: geographical data, personal information about hosts (host name, host profile picture, total active time in Airbnb, etc), residential houses characteristics (number of bathrooms, amenities, availability, etc) and rating metrics of listings (cleanliness, communication, etc).

Additionally, a specific dataset of Airbnb guests' reviews was incorporated into the analysis, amounting to 1,511,891 opinions about 32,413 distinct listings. Only comments written in English were considered, so the final dataset ended up with 1,460,047 records. The dimensions of the listings and reviews datasets considerably vary, making it imperative to perform Data Mining operations before combining both into a joint database. For such purpose, different data preprocessing techniques had to be applied to the reviews dataset in order to turn originally unstructured data into an acceptable format for predictive modeling.

4.3 Feature Engineering

Feature Engineering is the "*process of using domain knowledge to select and transform the most relevant variables from raw data*" (Ng, 2019). For instance, the target variable price was transformed by taking logarithms to address skewed data. Also, since some information originally available in the datasets was of no predictive use, several predictors were ruled out. For an explanation of the final data matrix please refer to the appendix.

4.3.1 Data Imputation Algorithms

One of the drawbacks of web scrapping is that it may not be possible to retrieve all the data about specific individuals. Additionally, sometimes hosts simply do not post complete information about listings.

Although generally perceived as an analytical obstacle, missing data can be indicative of certain patterns. Indeed, creating features by one-hot encoding the presence of null values has proven to be highly effective in Kaggle competitions (Pandey, 2020).

Once missing data is accounted for with one-hot encoding, data imputation comes into play. Imputing missing values is much preferable over eliminating observations that lack some variables since considerable information loss would be caused. Nonetheless, it is sometimes justified to directly delete some data rows if their number is negligible. Actually, in this project hosts that did not complete the registration process were discarded.

A custom k-nearest neighbor algorithm (k-NN) and MICE algorithm (Multiple Imputation by Chained Equations) were applied to impute data.

For the custom k-NN algorithm, three statistical distance matrices were computed on non-missing data: Mahalanobis distance for quantitative predictors, Jaccard distance for binary variables and Hamming distance for the rest of categorical features. Mahalanobis distance was preferred over commonly used Euclidean distance since the former accounts for correlation between covariates and remains unchanged after scale changes. As for Jaccard and Hamming distance matrices, they were suitable metrics for qualitative data. Based on a combination of these distances (called Gower distance matrix), the six nearest neighbors were identified for each listing in the dataset. Leveraging such information, missing data was imputed by taking the mean value of the neighbors (and then adjusted for binary and categorical variables).

As for MICE algorithm, it was implemented as described by van Buuren and Groothuis-Oudshoorn (2011). MICE combines predicted values for missing data from m copies of the original incomplete dataset into a pooled estimate. Prediction is flexible since different estimators are allowed (Bayesian Ridge Regression and Random Forest were chosen). Algorithm 2 in the appendix summarizes how MICE works.

4.3.2 Natural Language Processing

Natural Language Processing (NLP) techniques were used to extract information from the unstructured reviews dataset. Emojis, symbols, punctuation and internet links were removed to clean the data. Then text was forced to lowercase and non-English comments were eliminated as to maximize homogeneity among users' comments. Most importantly, the opinion expressed by Airbnb guests was extracted by perfoming Sentiment Analysis.

Unfortunately, no score rating metric was available so training a NLP model was unfeasible since there were no labels. However, by defining regular expressions (regex) and tweaking VADER (Valence Aware Dictionary and sEntiment Reasoner), a lexicon and rule-based sentiment analysis tool attuned to social media created by developers Hutto and Gilbert (2014), sentiment was estimated in an unsupervised manner.

Gender was guessed based on a database containing 40,000 first names collected by Michael (2006), since hosts' names were available.

4.3.3 Computer Vision

The original dataset provided personal information about hosts, among which an internet link to their profile picture was accessible. For those hosts that did upload a photograph, a Computer Vision (CV) model was passed to predict their gender.

Resorting to transfer learning, a pre-trained model built by Serengil (2020) was used to guess hosts' gender. Firms and couples were assigned a third category within the gender feature. Sample images displaying the CV model performance are shown:

Figure 1: Face Recognition with CV

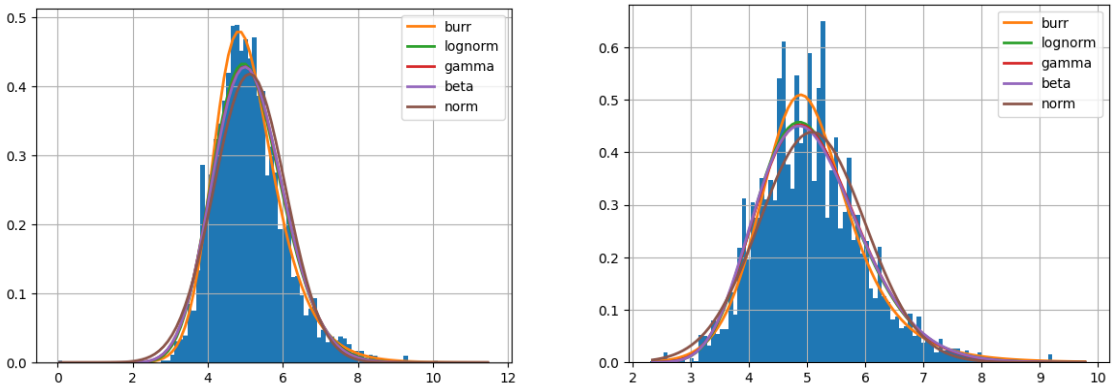
4.4 Tackling online myths: Statistical Inference

According to Davidson and Gleim (2022) the wage gap persists even in the sharing economy. Statistical Inference can shed light on the truth of such statement by verifying whether the median price set by women is truly lower than the median price of men.

Let y_m denote the transformed median price, formally it is sought to test:

$$H_0 : y_m^{\text{♀}} = y_m^{\text{♂}}$$
$$H_1 : y_m^{\text{♀}} < y_m^{\text{♂}}$$

A Bayesian hypothesis testing approach is used as suggested by Gelman et al. (1995). Probability distributions for describing the likelihood function and prior beliefs of y^i for $i \in \{\text{♀}, \text{♂}\}$ are needed. Five continuous density functions were fitted to transformed prices.



(a) Female prices

(b) Male prices

Figure 2: Probability distributions

According to goodness of fit metrics such as sum of squared error, Akaike's Information Criteria (AIC) and Bayesian Information Criteria (BIC), the Burr Type III distribution was the preferred density function.

Now a prior belief has to be defined for a parameter of the Burr distribution. The conditional distribution is $y^i|\theta \sim \text{Burr}(\omega, \theta)$, and a conjugate prior is $\theta \sim \text{Gamma}(\alpha, \beta)$.

It is shown in the appendix that the marginal distribution of y^i after obtaining the posterior distribution $p(\theta|y_1^i, \ldots, y_n^i)$ takes the form $y^i \sim \text{Pareto}\left(\alpha + n, \beta + \sum_{i=1}^{n} \ln(1 + y_i^{-\omega})\right)$.

Thus, we can obtain the probability that the median female price is less than the median male price by sampling from the predictive distribution for each gender.

After 300 Monte Carlo simulations, it was obtained that $\Pr(H_1|y) = 0$. In fact, comparing the generated samples it could not be found $|y_m^{\female} - y_m^{\male}| \geq 0.01$ across any Monte Carlo simulation. Therefore it is concluded that no evidence of a *wage* gap in Airbnb exists for Los Angeles in 2022. Moreover, a frequentist sign test yielded the same outcome as the bayesian test, indicating that prior beliefs did not influence the final conclusion.

5 Methodology

5.1 Machine Learning notation

Despite mutually sharing various concepts, ML has adopted new terminology for well-established labels in Econometrics.

- Regression in ML is only applied in cases where the '*target*' (dependent variable or regressand) is continuous. For discrete targets '*classification*' is performed (even though logistic regression might be used for such tasks).
- The sample data to estimate parameters is renamed to '*training set*'.
- Regressors, predictors or covariates become '*features*' and parameters are often referred to as '*weights*'.
- Optimizing a cost function to get parameter estimates is called '*learning*'. It can either be '*supervised*', in case there are known labels for the target function or '*unsupervised*' otherwise.

For the sake of consistency, ML nomenclature will be used, although when necessary the econometric analogues will be mentioned.

Thus, the listings dataset $\mathcal{D} = \{\mathbf{x}_i, y_i\}_{i=1}^n$ is composed of feature vectors $\mathbf{x}_i \in \mathfrak{R}^p$ and a target $\mathbf{y} \in \mathfrak{R}$. Each $\mathbf{x}_i$ contains data for the i^{th} listing, concretely p variables or features. If stacked together, feature vectors lead to feature matrix $\mathbf{X} \in \mathfrak{R}^{n \times p}$.

Since there exists a target vector $\mathbf{y} = \{y_i\}_{i=1}^n$ with $y_i \in \Re \ \forall i$, predicting Airbnb prices is a *supervised learning* task (specifically a *regression* problem). Such problem will be solved by estimating the target function $f(\mathbf{y}|\mathbf{X}; \mathbf{w})$, which maps feature vectors $\mathbf{x}_i$ to target vector $\mathbf{y}$ through weights $\mathbf{w} \in \Re^p$ (non-parametric models are not considered).

Dataset $\mathcal{D}$ will be randomly partitioned into three disjoint sets: training set $\mathcal{T}$, validation set $\mathcal{V}$ and test set $\mathcal{F}$ so that $\mathcal{T} \cup \mathcal{V} \cup \mathcal{F} = \mathcal{D}$. Specifically, these sets amount to 77%, 14%, 9% of total data (training, validation and test, respectively).

Algorithms learn from the training set $\mathcal{T}$ optimizing a cost function (sum of individual loss functions) $J(\mathbf{w}, \boldsymbol{\lambda}) = \frac{1}{n} \sum_i L_i(\mathbf{w}, \boldsymbol{\lambda})$ and $\hat{w}_i$ are obtained by performing model validation. This entails that hyperparameters $\boldsymbol{\lambda}$ (user-specified values to control the learning process) are also tuned on the validation set.

Root Mean Squared Error (RMSE) is the chosen model performance metric to select the 'best' or 'champion' model. Chai (2014) lists some desirable properties such as sharing in the same measurement units as the target and satisfying the triangle inequality requirement for a distance metric.

After having learned from the training set and been refined with the validation set, the AI algorithm can be assessed in the test set as to obtain an unbiased estimate of model performance.

Figure 12 in the appendix provides a graphical illustration of the AI framework.

5.2 Hyperparameter selection procedure

During the refining process, not only weights are optimized to attain accurate results but hyperparameters $\boldsymbol{\lambda}$ are also included in the cost function. These hyperparameters serve the purpose of regularizing AI algorithms, acting as "*penalty terms in the loss function*" (Bishop, 1995). Shrinking $\mathbf{w}$ during the learning process through $\boldsymbol{\lambda}$ leads to better generalization on test data. Otherwise AI algorithms tend to *overfit* the training data, procuring noticeable worse results when exposed to non-training instances.

Finding optimal hyperparameters is no trivial task since $\boldsymbol{\lambda}$ is manually selected by the AI practitioner. Some popular methods are Grid Search and Random Search, where a

vector of user-defined hyperparameter values are tried during training. Although for some tasks such approaches are sufficient, they are highly inefficient. Bergstra et al. (2011) developed a series of greedy algorithms that leverage past evaluations of hyperparameter values to select the most promising $\boldsymbol{\lambda}$ to evaluate in $J(\mathbf{w}, \boldsymbol{\lambda})$. For that purpose, Bayesian Inference is performed on a probabilistic model (surrogate model).

Two probability distributions are defined based on the following rule: values of $\boldsymbol{\lambda}$ that increase model accuracy are part of $l(\boldsymbol{\lambda})$, whereas poor perfoming values of hyperparameters belong to $g(\boldsymbol{\lambda})$. Performance is measured in terms of a threshold y^*. Tree Parzen Estimator (TPE) is a greedy algorithm that maximizes the expected improvement in model performance based on hyperparameter values drawn from these probability distributions. Clearly, the ratio $\frac{g(\boldsymbol{\lambda})}{l(\boldsymbol{\lambda})}$ is desired to be minimized, since optimal hyperparameter values $\boldsymbol{\lambda}^*$ should be drawn from probability distribution $l(\boldsymbol{\lambda})$ rather than $g(\boldsymbol{\lambda})$. Prior beliefs $p(\boldsymbol{\lambda})$ are updated at each evaluation, yielding posterior distribution $p(\boldsymbol{\lambda}^*|\mathbf{y})$. The following pseudo-code gives additional details on how TPE works to find the best hyperparameters.

Algorithm 1: Tree Parzen Estimator

Input: Prior probability distribution $p(\boldsymbol{\lambda})$

Output: Optimal hyperparameters $\boldsymbol{\lambda}^*$

Set probabilistic domain of hyperparameters: $\boldsymbol{\lambda} \sim p(\boldsymbol{\lambda})$

Define $J(\mathbf{w}, \boldsymbol{\lambda}) = \sum_{i=1}^{n} L_i(\mathbf{w}_i, \boldsymbol{\lambda}_i)$

Construct probability distributions $p(\boldsymbol{\lambda}|\mathbf{y}) = \begin{cases} l(\boldsymbol{\lambda}) & \text{if } y < y^* \\ g(\boldsymbol{\lambda}) & \text{if } y \geq y^* \end{cases}$

Build surrogate model $p(\mathbf{y}|\boldsymbol{\lambda}) = \frac{p(\boldsymbol{\lambda}|\mathbf{y})p(\mathbf{y})}{p(\boldsymbol{\lambda})}$

for $t = 1, \ldots, n$ **do**

`/* Update` $\boldsymbol{\lambda}^{(t)}$ `*/`

$$\boldsymbol{\lambda}^{(t+1)} \leftarrow \arg\max_{y^*} \int_{-\infty}^{y^*} (y^* - y) \frac{p(\boldsymbol{\lambda}^{(t)}|\mathbf{y})p(\mathbf{y})}{p(\boldsymbol{\lambda}^{(t)})} \, dy$$

Obtain optimal weights $\hat{\mathbf{w}} \leftarrow \arg\min_{\mathbf{w}} \frac{1}{n} \sum_{i=1}^{n} L_i(\mathbf{w}, \boldsymbol{\lambda}^*)$

A graphical representation is available in the appendix in Figure 12.

6 Machine Learning Modeling

6.1 Bayesian Ridge Regression

An empirical Bayesian learning framework is proposed as the baseline model. Given prior knowledge on weights $\mathbf{w}$, Bayes' Theorem will be used to update such distribution of beliefs over the model space. Bayesian modeling offers some advantages (Bishop, 2006):

- **Measurability of prediction uncertainty:** Bayesian Inference procures a probability distribution for weights, allowing to quantify uncertainty around model outputs by estimating conditional distribution $p(\mathbf{y}|\mathbf{x})$.
- **Good generalization performance:** Facilitated by imposing a complexity penalty term in the form of a conditional prior probability distribution $p(\mathbf{w}|\boldsymbol{\lambda})$.
- **Sparsity of inferred predictors:** Many of the weights exhibit posterior distributions $p(\mathbf{w}, \boldsymbol{\lambda}, \alpha|\mathbf{y})$ that are sharply peaked around zero, so the learned algorithm makes predictions based on the most relevant features. Thus, feature selection is carried out in a similar fashion to Least Absolute Shrinkage and Selection Operator (LASSO) Regression.

The Bayesian Ridge Regression Model to be estimated is expressed as:

$$\mathbf{y} = f(\mathbf{y}|\mathbf{X}; \mathbf{w}) + \boldsymbol{\varepsilon}. \tag{2}$$

For which a set of restrictions on the joint distribution of $\{\mathbf{x}_i, y_i\}$ is imposed:

- Stochastic process $\{\mathbf{x}_i, y_i\}$ originates from random sampling.
- The parameterized conditional function is linear $f(\mathbf{y}|\mathbf{X}; \mathbf{w}) = \mathbf{X}\mathbf{w}$.
- The prior distribution of target vector $\mathbf{y}$ is $p(\mathbf{y}|\mathbf{X}, \mathbf{w}, \boldsymbol{\alpha}) = \mathcal{N}(\mathbf{X}\mathbf{w}, \sigma^2\mathbf{I}_p)$.
- The prior distribution of coefficient vector $\mathbf{w}$ is $p(\mathbf{w}|\boldsymbol{\lambda}) = \mathcal{N}(\mathbf{0}, \boldsymbol{\lambda}^{-1}\mathbf{I}_p)$.
- ε_i are samples drawn from a noise process such that $\boldsymbol{\varepsilon} \overset{\text{i.i.d}}{\sim} \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I}_p)$.

Even if an economic agent has no real estate market expertise knowledge, Bayesian Inference allows the parameters of a prior distribution to be estimated from data by building hierarchical models. In order to do that, marginal probability functions have to be defined for hyperpriors $\boldsymbol{\lambda}$ and σ^2, introducing an additional level of beliefs. Specifically:

$$p(\boldsymbol{\lambda}) = \prod_{i=0}^{n} \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda_i^{\alpha-1} e^{-\lambda_i/\beta}, \quad p(\sigma^2) = \frac{\theta^{\delta}}{\Gamma(\delta)} \sigma^{2(\delta-1)} e^{-\sigma^2\theta}. \tag{3}$$

Note that hyperpriors follow a Gamma distribution, a selection justified by the fact that when operating with the previously assumed distributions $p(\mathbf{y}|\mathbf{X}, \mathbf{w}, \boldsymbol{\alpha})$ and $p(\mathbf{w}|\boldsymbol{\lambda})$, an analytical result can be derived thanks to conjugate priors. Consequently, there is no need to resort to Markov Chain Monte Carlo (MCMC) algorithms for sampling posterior distributions. As an example, the first three weight posterior distribution plots for the intercept, `description` and `host_since` are presented:

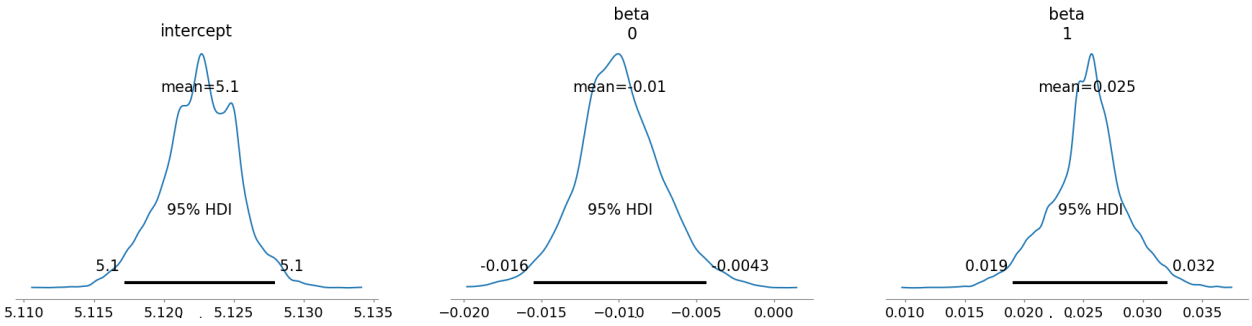


Figure 3: Posterior distribution of selected w_i. Solid line is a 95% credible interval

Note how a credible interval is given for each w_i, allowing to analyze predictive uncertainty rather than obtaining unequivocal point estimates. Furthermore, the interpretation of credible intervals is far clearer than confidence intervals. The zero value is not contained within any of the credible intervals above, so the null hypothesis of no individual statistical significance is rejected with probability 95% for these weights.

It is possible to test whether all weights really contribute to enhance predictive performance. A second specification was estimated using only the most relevant features according to the champion model (to be uncovered in the following subsections). Consequently, model comparison is carried out between a complete model (using all 75 features)

and a restricted one (top 20 relevant features). For such purpose, Deviance Information Criterion (DIC) is used. Let DIC be expressed as $D(\theta) = -2\ln(p(y|\theta)) + C$, where y is the data, θ represents unkwown parameters, $C \in \mathfrak{R}$ and $p(y|\theta)$ is the likelihood function. Just like AIC and BIC, a lower DIC is associated with higher predictive performance. After estimating the expected log pointwise predictive density using Pareto-smoothed importance sampling leave-one-out cross-validation, it was found that the full model is the preferred specification (50,195.4 vs 54,910.2 DIC). Evidence was found in favor of the statistical significance of features different from the most relevant ones.

6.2 Elastic Net Regression

For comparison purposes, a Linear Regression Model is estimated in a frequentist fashion. The weight vector $\mathbf{w}$ is obtained by solving the following optimization problem:

$$\hat{\mathbf{w}} = \arg\min_{\tilde{\mathbf{w}}} (\mathbf{y} - \mathbf{X}\tilde{\mathbf{w}})'(\mathbf{y} - \mathbf{X}\tilde{\mathbf{w}}) + \lambda_1\|\tilde{\mathbf{w}}\|_1 + \lambda_2\|\tilde{\mathbf{w}}\|_2^2 \quad , \tag{4}$$

where $\tilde{\mathbf{w}}$ is a parameter vector, $\|\mathbf{w}\|_1$ is the ℓ_1-norm of the weight vector and λ_1 is the associated regularization term. Likewise, $\|\mathbf{w}\|_2^2$ is the ℓ_2-norm of $\mathbf{w}$ and λ_2 is its regularization term.

Zou and Hastie (2005) combined ℓ_1 and ℓ_2-norm regularization to overcome some drawbacks of LASSO, obtaining a new regularization and feature selection method. The result is that few non-zero weights are left while maintaining properties of Ridge Regression. LASSO tends to vanish many $\mathbf{w}_i$, sometimes resulting in poorer performance due to information loss.

Note that some restrictions on the joint distribution of $\{\mathbf{x}_i, y_i\}$ can be relaxed for this model: heteroskedasticity is allowed, there is no need to define prior distributions and random sampling can be reduced to $\{\mathbf{x}_i, y_i\}$ being jointly ergodic stationary.

Despite being a notably less restrictive model in terms of assumptions, Elastic Net Regression performs worse than Bayesian Ridge Regression (0.5232 vs 0.5104 RMSE on test data, respectively). Thus, as of now the best perfoming model is Bayesian Regression.

6.3 Random Forest

Random Forest algorithm is an example of an "*Ensemble Method*", which combines multiple models. Ho (1998) argued that by aggregating weak learners predictive performance could significantly improve. It is imperative that such learners are not identical. Since Random Forest only consists of Decision Trees, diversity stems from training said algorithm on random subsets of data selected by "*bootstrap aggregating*" or "*bagging*" with a multinomial probability distribution. As Decision Trees are extremely sensitive to the data they are exposed to, training them with dissimilar datasets leads to different predictions $\hat{\mathbf{y}}_i$. Finally, the output of Random Forest is the mean of prediction vector $\hat{\mathbf{y}}$.

Decision Trees are trained with the CART algorithm. Firstly, data is split into two subsets using the j^{th} feature and establishing a threshold t_j (i.e, # amenities ≤ 3). The pair (j, t_j) is selected by minimizing a cost function $J(j, t_j)$. CART recursively performs this operation until $J(j, t_j)$ cannot be reduced anymore or the maximum depth of Decision Trees has been reached (a hyperparameter). Random Forest further increases stochasticity by considering only a random subset of features, trading a higher bias for a lower variance.

Taddy et al. (2015) derived ensembles of Decision Trees through a Bayesian model, allowing Random Forest to be interpreted as "*a sample from a posterior* [distribution] *over trees*". Leveraging that the Dirichlet distribution is a conjugate prior of the multinomial distribution, the bootstrap posterior distribution is also Dirichlet. In practice, an exponential distribution is used to define the prior since the `ensemble` module of `scikit-learn` normalizes the draws from the posterior distribution. This Bayesian adaption of Random Forest has been implemented along with the Frequentist version.

Unlike classical methods, Random Forest and the rest of ML algorithms require little to no restrictions on $\{\mathbf{x}_i, y_i\}$. In fact, there is no need to impose linearity or any probability distribution on the data (only the prior distribution for Bayesian Random Forest), predictors can far exceed the number of observations and redundancy is not a problem as feature selection is carried out by CART algorithm. Random Forest is also insensitive to data scaling due to threshold t_j, and random sampling is not necessary to hold as long as bootstrap procures sufficiently representative samples of data (Géron, 2019).

It is possible to make Random Forest even more stochastic by selecting random thresholds t_j rather than the ones that minimize $J(j, t_j)$. Geurts etl al. (2006) coined this algorithm as Extremely Randomized Trees and found that time complexity is lowered as well as variance.

The following table displays RMSE on test data of the different RF algorithms.

Table 1: Random Forest algorithm results

Frequentist RF	Bayesian RF	Frequentist ERF	Bayesian ERF
0.4551	0.4205	0.4507	0.4572

Notice how all Random Forest algorithms outperform Linear Regression. Bayesian Random Forest is the preferred model as it achieves the smallest RMSE.

6.4 Neural Networks

Artificial Neural Networks (ANN) are Deep Learning algorithms that mimic biological neural circuits by transmitting information through neurons from different layers. Below, a graphical representation of the ANN architecture used in this thesis is shown.

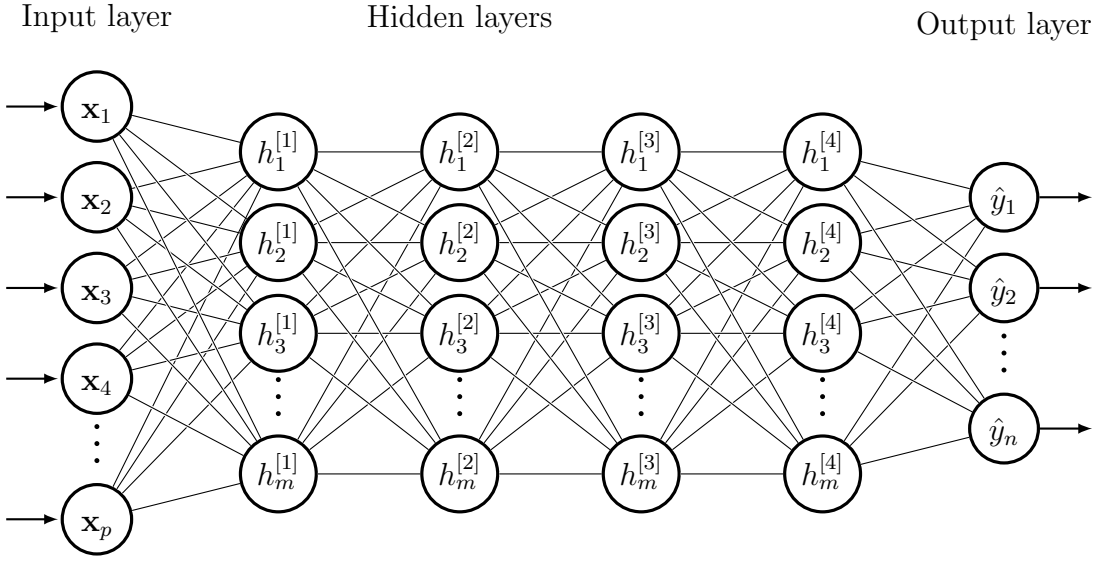


Figure 4: Artificial Neural Network Architecture

Note that the ANN architecture is composed of three layers: input layer, hidden layers and an output layer (predictions). ANN learn from data stemming from the feature vectors $\mathbf{x}_i$ in the input layer, process such information in each neuron located in the i^{th} hidden layer, and finally procure predictions $\hat{\mathbf{y}}$ in the output layer.

Neurons transmit information by forwarding data from the input layer to the output layer (*feed forward*). Then, $J(\mathbf{w}, \boldsymbol{\lambda})$ is computed and optimized by propagating it to the earlier layers (*back propagation*). This process is repeated until convergence is attained by either not reducing prediction error anymore or defining a maximum number of iterations.

Typically, *batches* (a fraction of the training data) are fed to the ANN for some cycles called *epochs* (number of times training data has been fully updated).

Unlike Decision Trees and Random Forests, ANN need to find minima of $J(\mathbf{w}, \boldsymbol{\lambda})$ using Gradient Descent algorithm. For the sake of avoiding vanishing gradients problem, data is standardized using the mean and standard deviation of the training set. Additionally, neurons are randomly initialized following He (2015) and a nonsaturating link function or activation function is applied to each hidden layer's ouput. Exponential Linear Unit (ELU) has been chosen as the preferred activation function as it forces negative outputs to lie close to zero, while not altering positive ones. This overcomes issues with Rectified Linear Unit function (ReLU), which is not differentiable for negative values. The intuition behind ELU is that it makes backpropagation easier and faster by having an average output closer to 0, which alleviates vanishing gradients problem (Clevert et al., 2015).

The optimization algorithm used to train the ANN was Nesterov-accelerated Adaptive Moment Estimation (Nadam), which includes an adaptive learning rate to classical Adam algorithm called Root Mean Squared Propagation (RMSProp) which increases performance and convergence speed. Additionally, Elastic Net Regularization was applied to penalize overconfident predictions. Algorithm 8 in the appendix provides further technical details on the training process of the ANN.

The architectural complexity of ANNs makes them ideal to solve intricate predictive problems. However, when trained in medium-sized (less than 100,000 observations) tabular datasets, Grinsztajn et al. (2022) showed that tree-based models like Random Forests

and XGBoost tend to outperform ANNs. Even when ANNs procure more accurate predictions, training takes longer and the number of parameters is astronomical compared to the aforementioned algorithms. Indeed, the resulting ANN is composed of 1,591,532 parameters, a total overkill considering it does not even beat Bayesian RF.

Drawbacks like training time and algorithm complexity might deter AI practitioners from adapting ANNs to Bayesian Statistics, since the latter is associated with demanding compute resources. Nevertheless, the possibility of quantifying uncertainty, better predictive performance and above all, Variational Inference (VI), have made it possible to create Bayesian Neural Networks (BNNs).

A BNN is a stochastic ANN trained using Bayesian Inference. BNNs can be built following a similar logic to the Frequentist one by defining prior distributions on the weights $p(\mathbf{w})$, updating them according to evidence $p(\mathbf{y}|\mathbf{w}, \mathbf{X})$ so as to quantify uncertainty around predictions $p(\hat{\mathbf{y}}|\mathbf{X}, \mathbf{w})$ (posterior predictive distribution). In VI, a variational distribution parametrized by learnable weights $q_\phi(H)$ is approximated to the true posterior distribution $p(H|\mathbf{y}, \mathbf{X})$ with a modified Stochastic Gradient Descent algorithm (Blei et al., 2016).

As BNNs introduce penalization with prior distributions, only two hidden layers were needed with no Elastic Net Regularization.

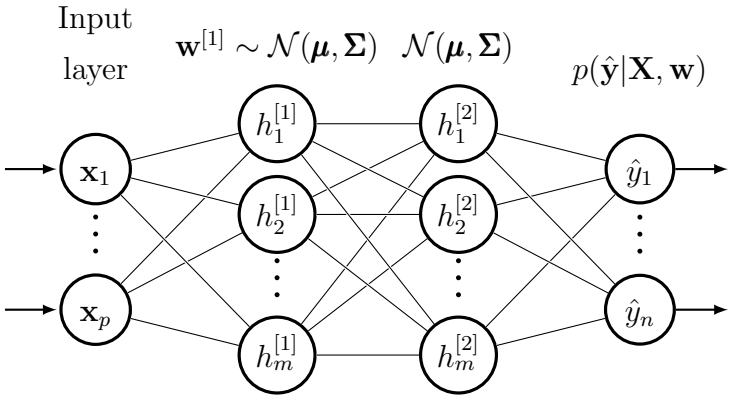


Figure 5: Bayesian Neural Network Architecture

The Frequentist ANN performed worse than the BNN on test data (0.4305 vs 0.4024 RMSE). As of now, the preferred model is the BNN in terms of predictive accuracy.

6.5 XGBoost

Similarly to Random Forest, XGBoost (eXtreme Gradient Boosting) combines weak algorithms into a stronger learner. It achieves such results by sequentially training Decision Trees, each improving its predecessor. Concretely, the i^{th} predictor is fit into the residual errors made by the previous one, and $\hat{\mathbf{y}}$ is the joint result of every $\hat{y}_i$ made by weak learners. Algorithm 7 in the appendix summarizes how XGBoost works.

Although Gradient Boosting Machine (GBM) algorithms also follow this logic, Chen and Guestrin (2016) enhanced GBM by increasing its scalability and regularization capacity. In fact, XGBoost can be trained on GPUs rather than CPUs, considerably reducing training time. Additionally, unlike GBMs, XGBoost natively introduces L1, L2 and Elastic Net regularization, leading to better generalization. XGBoost is one of the most powerful ML algorithms, and a proof of this claim is that 17 out of 29 winning solutions in Kaggle competitions used it for training (Harasymiv, 2015). Not coincidentally, it arises as the champion model of this project as its RMSE on test data is the lowest: 0.3682.

6.6 Opening the black box: XAI methods

Although procuring accurate predictions already helps economic agents maximize their utility, informed decision-making requires understanding model outputs. A first step into interpreting the champion model is identifying the most important features, namely, those variables that contribute the most to final predictions. There are two ways in which importance was defined in this thesis: by 'gain' (average RMSE reduction across all splits the feature was used) and 'weight' (number of times a feature was used to split the data across trees). Figure 13 in the appendix provides a graphical representation of the most relevant variables according to XGBoost. Intuitively, the number of bathrooms, bedrooms, beds, the geographical location, neighborhood and room type turned out to be essential.

Despite the significance of detecting said features, further analysis is needed to gain deeper insights. Thus, XAI methods were employed. The first metric to be expounded was developed by Lundberg and Lee (2017), who proposed to use SHapley Additive exPlanations (SHAP) values to interpret model outputs. SHAP values are a combination

of Statistics and Game Theory. In terms of the latter, the cooperative game of interest is the outcome of the model, while players are represented by the power set of features (different coalitions of players). The goal is to estimate the marginal contribution of each feature to $\hat{\mathbf{y}}$, accounting for both, overall effect on predictions and individual impact on each Airbnb listing.

For every coalition in the power set a predictive model has to be built and SHAP values are then calculated, keeping hyperparameters and data constant across each training iteration. The overall marginal contribution of the j^{th} feature to $\hat{\mathbf{y}}$ is then estimated by considering the weighted average of $\hat{y}_i$ differences between interconnected coalitions where the j^{th} feature is present (see Figure 14 in the appendix). Mathematically:

$$\phi_f(i) = \sum_{S \subseteq F \setminus \{f\}} \frac{|S|!\,(|F| - |S| - 1)!}{F!} \left[g(S \cup \{f\}) - g(S) \right], \tag{5}$$

where f is the j^{th} feature, i is the i^{th} observation of the dataset, $\phi_f(i)$ is the SHAP value associated with f for i, S is the subset of F where j is not present, F represents the set with all features and $g()$ is the output of a predictive model (in this case XGBoost).

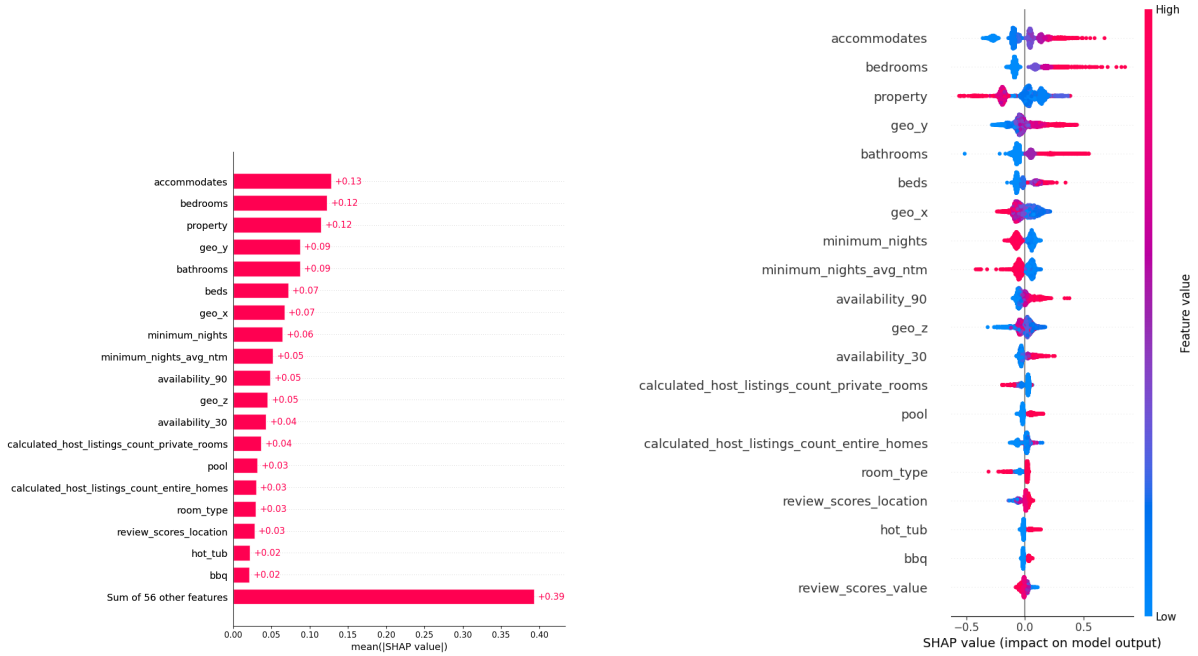


Figure 6: Global Interpretability with SHAP

Figure 6 provides an illustration of SHAP values at the global level. Since the marginal effect of features is not specifically evaluated at each observation, the graphs above are similar to Figure 13 in the sense that they represent feature importance. In general, the most relevant features based on the mean SHAP value coincide with those identified by XGBoost. Nevertheless, additional information is available, as the overall effect of each feature on $\hat{\mathbf{y}}$ is displayed. For instance, additional number of beds and bathrooms are associated with a positive impact on the predicted price. Airbnb houses closer to the beach (`geo_x` and `geo_y`) are also positively related with price. Lastly, it seems that requiring minimum nights tends to decrease price, which intuitively can be interpreted as risk-averse hosts assuring longer stays at the expense of a lower fee.

Due to differences in each Airbnb listing, features may not affect prices uniformly across observations. SHAP values can also be used to study local level effects, that is, the impact of features on the predicted price for each Airbnb house. Economic agents are thus given the opportunity to specifically target a concrete asset and understand spread in prices, which is of great use when comparing similar houses and looking for sources of competitive advantage.

As an example, two apartments are analyzed in Figure 7: the first one falls below median price and the second one exceeds it. Lacking a TV, receiving a somewhat mediocre location score and having only one bedroom leads to a dominant negative effect on price for the first instance. On the other hand, high availability in a 30 and 90-day period, two bedrooms, three beds and a maximum capacity of six people is associated with a more expensive rental.

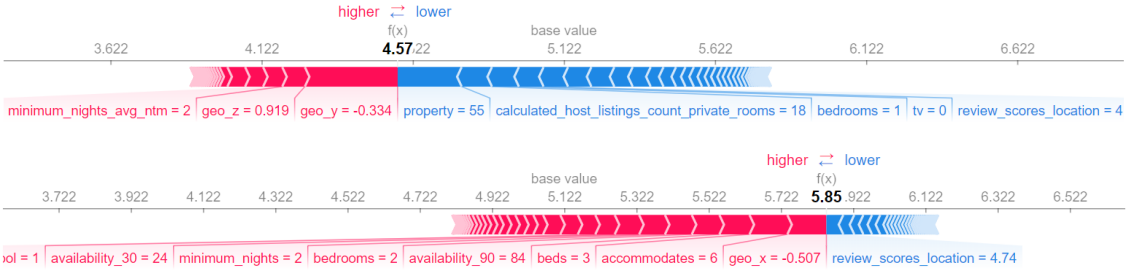


Figure 7: Local force plots with SHAP

Leveraging that SHAP values share the same units of measurement as the target variable, contributions to final predictions can be easily interpreted (not in causal terms, only associatively). In Figure 8 below it can be appreciated how for the left hand side Airbnb rental, the maximum capacity limit is associated with a 0.1 increase effect in the predicted price (ceteris paribus). Likewise, geographical locations and amenities such as swimming pool, hot tub and barbeque are also linked with rises in price. Specifically, negative score ratings seem to drag price down by 0.03 and a low number of reviews decrease it by 0.02. With respect to the second listing, being a shared room type with only one bedroom with no amenities but a hot tub resiñts in a low-price prediction. Concretely, the property type negatively affects the transformed price by 0.39 units, and limiting it to be a short-term stay further decreases price by 0.05 and 0.04 (90 and 30-day period, respectively).

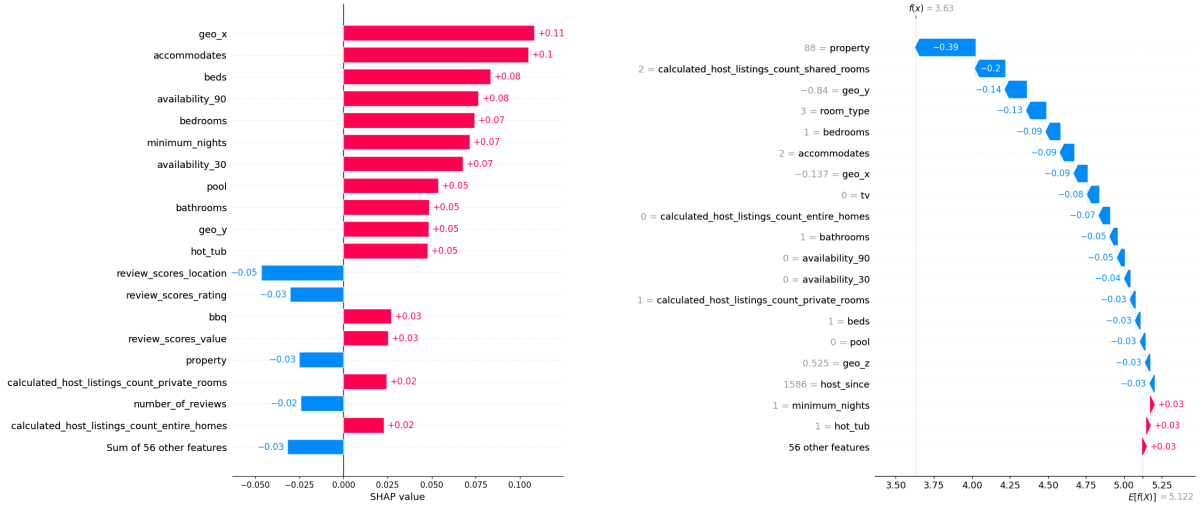


Figure 8: Local Interpretability with SHAP

Another XAI method, Partial Dependence Plots (PDP) developed by Friedman (2001), also allows to estimate marginal effects of features on predictions. PDPs show the relation between $\mathbf{y}$ and a set of inputs of interest $\mathbf{x}_S$, isolating the latter from the rest of features $\mathbf{x}_C$ (complement features). Note that these plots are meant to provide a general idea of the relationship between $\mathbf{x}_S$, $\mathbf{y}$ and should be interpreted with caution since for discrete data they might display implausible continuous values. Conveniently, economic agents can

quickly check whether such relationship is linear, monotonic or more complex with these graphs. Mathematically, the Partial Dependence of a Machine Learning model f at point $\mathbf{x}_S$ is expressed as:

$$\psi(\mathbf{x}_S) = \int f(\mathbf{x}_S, \mathbf{x}_C) p(\mathbf{x}_C) \, d\,\mathbf{x}_C \tag{6}$$

It can be seen that PDPs marginalize over $\mathbf{x}_C$ to estimate the impact of $\mathbf{x}_S$ on the target variable. Let $\mathbf{x}_S = \{\texttt{host\_response\_time}, \texttt{host\_is\_superhost}\}$, where the first variable represents the time it takes for a host to answer guests' messages and the second one identifies experienced Airbnb hosts.

The LHS graps below represent PDPs for such choice of $\mathbf{x}_S$. Note how the expected value of price falls as hosts take longer to answer messages. In addition, renting with superhosts is associated with higher prices. The joint effect of both features is shown in the third graph: superhosts that quickly respond to guests' demands charge higher prices. Intuitively, the most experienced Airbnb hosts charge higher prices for a faster service. Note that these should be

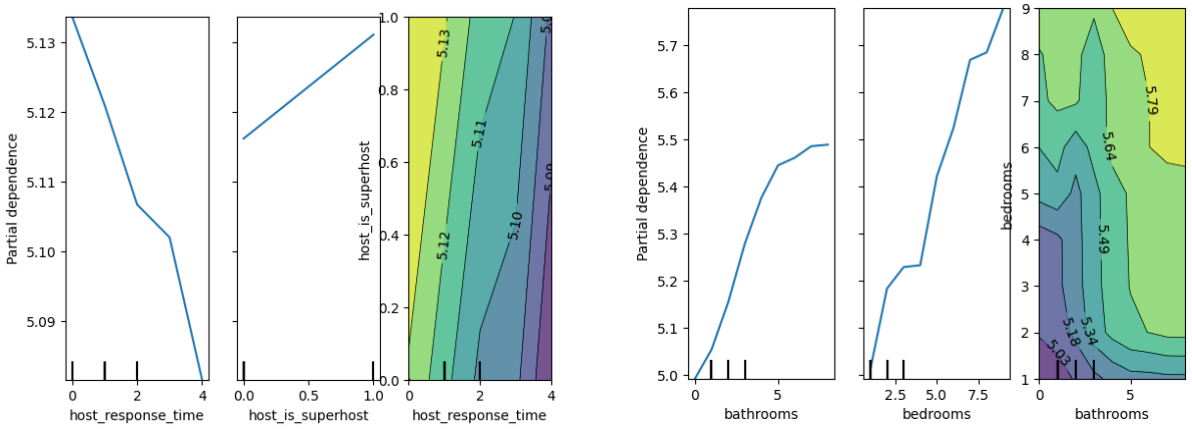


Figure 9: Global Interpretability with PDPs

In the RHS graphs, $\mathbf{x}_S = \{\texttt{bathrooms}, \texttt{bedrooms}\}$, and as expected, the number of beedrooms and bathrooms is positively related with price. What is interesting is that for `bathrooms` there seems to be some 'diminishing returns'. Nonlinearities like this cannot be so easily identified with linear regression models.

Just like SHAP values, it is possible to estimate local level effects with PDPs. In this case, they are called ICE plots (Individual Conditional Expectation plots) and were developed by Goldstein et al. (2014). Similarly to PDPs, ICE plots show the dependence of $\mathbf{y}$ with respect to $\mathbf{x}_S$, but also display several lines corresponding to individual observations. Mathematically:

$$\Psi(\mathbf{x}_S) = \int f(\mathbf{x}_S, \mathbf{x}_C^{(i)}) p(\mathbf{x}_C)\ d\,\mathbf{x}_C \tag{7}$$

ICE are defined as a single $f(\mathbf{x}_S, \mathbf{x}_C^{(i)})$ evaluated at $\mathbf{x}_S$. The aforementioned lines representing Airbnb listings can be appreciated along with the average marginal effect in Figure 10. In the LHS, graphs for an already explained choice of $\mathbf{x}_S$ is presented. With ICE plots, it is possible to identify which specific Airbnb listing is more sensitive to changes in the number of bathrooms and bedrooms. In fact, extreme individual nonlinearities in the case of `bathrooms` can be identified at the very top and bottom of the partial dependence axis.

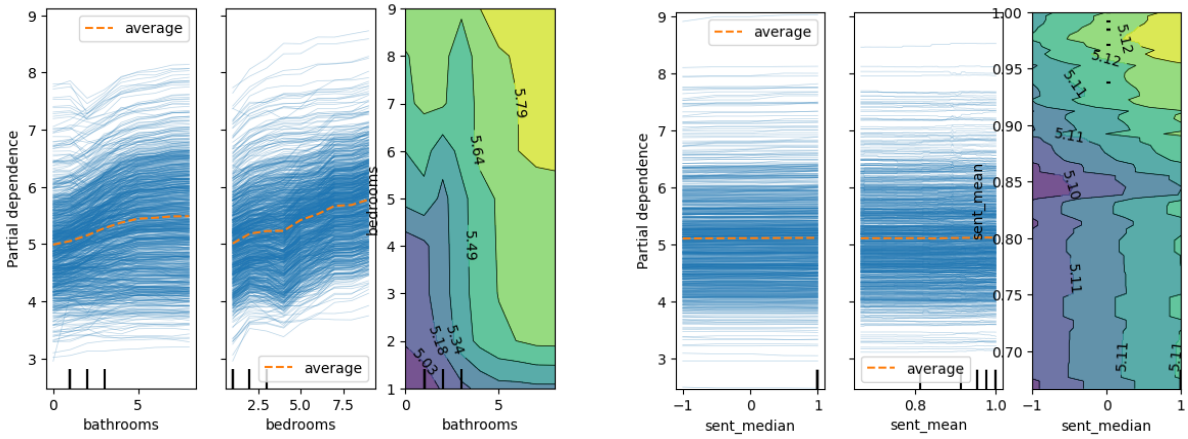


Figure 10: Local Interpretability with ICE plots

For $\mathbf{x}_S = \{$`sent_median`, `sent_mean`$\}$, where 'sent' stands for the estimated users' sentiment expressed in the reviews dataset, it is concluded that there is no noticeable overall impact of reviews' score on the predicted price. There is a weakly positive relation between average and median sentiment and prices at the global level. Nevertheless, thanks to ICE plots it can be clearly seen how various listings are more affected by sentiment.

7 Limitations & Conclusions

7.1 Limitations

Despite procuring accurate predictions based on RMSE on test data, note that these are only valid for June 2022. If the champion model was to be deployed to production, it should accommodate time-awareness by introducing past periods and possibly some lagged features. Moreover, it might be wise to consider continuously updating Airbnb data due to recent error correction or web scrapping enhancements. Data augmentation by resorting to alternative sources might also increase model performance (like private data from AirDNA). Additionally, rental rules and regulations should be taken into account, as certain shocks might be explained by modifications in tenant-landlord laws.

Economic agents should either provide further information about relevant subjective factors that could affect final predictions or properly adapt the champion model to incorporate such beliefs, since current predictions rely mainly on data.

Although XAI methods have been applied, only associative relations can be derived, as causal interpretation requires stricter assumptions on the joint distribution of $\{\mathbf{x}_i, y_i\}$ and the application of Causal Machine Learning techniques.

7.2 Conclusion

This thesis has shown the potential of using AI techniques to predict Airbnb prices. A scalable and accurate statistical toolkit has been developed to provide real estate users with a means of better understanding and predicting the prices of Airbnb listings. By applying Bayesian Inference for hypothesis testing and using XGBoost and XAI methods, predictive performance was improved over traditional econometric techniques and insights into the decision-making processes were gained. XAI methods were specifically used to understand the contributing factors to model's predictions, providing a way to overcome the AI black box problem. Overall, the results show that AI techniques can be powerful tools for predicting Airbnb prices, and it is hoped that this work will be useful for real estate users interested in making informed decisions in the Big Data era.

References

Angrist, J. (2020). Rajk College for Advanced Studies Interview. Retrieved 13/11/202.

Angrist, J. and Frandsen B. (2019). "Machine Labor". *National Bureau of Economic Research*, 40, 97-140.

Bergstra, J., Bardenet, R., Bengio, Y., Kégl, B. (2011). "Algorithms for Hyper-Parameter Optimization". *Advances in Neural Information Processing Systems*, 24, 2546-2554.

Bishop, C. (1995). *"Neural Networks for Pattern Recognition"*. Oxford University Press.

Bishop, C. (2006). *"Pattern Recognition and Machine Learning"*. Springer.

Blei, D., Kucukelbir, A., McAuliffe, J. (2020). "Variational Inference: A Review for Statisticians". *Journal of the American Statistical Association*, 112, 859-877.

Brownlee, J. (2020). "Difference Between Algorithm and Model in Machine Learning". Retrieved from *Machine Learning Machinery* (04/11/2022).

Chai, T. (2014). "Root mean square error (RMSE) or mean absolute error (MAE)? Arguments against avoiding RMSE in the literature". *Geoscientific Model Development*, 7, 1247–1250.

Chen, T. and Guestrin, C. (2016). "XGBoost: A Scalable Tree Boosting System". *KDD '16: Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 785–794.

Clevert, D., Unterthiner, T., Hochreiter, S. (2015). "Fast and Accurate Deep Networks Learning by Exponential Linear Units". *International Conference on Learning Representations*.

Davidson, A. and Gleim, M. (2020). "Female Airbnb hosts earn thousands less per year than male hosts ". *The Conversation*. Retrieved 10/10/2022.

Dube, A. Cengiz, D., Lindner, A., Zentler-Munro, D. (2019). "Seeing Beyond the Trees: Using Machine Learning to estimate the impact of Minimum Wages on Labor Market Outcomes". *National Bureau of Economic Research*, 40, S203-S247.

Embaye, W., Zereyesus, Y., Chen, B. (2021). "Predicting the rental value of houses in household surveys in Tanzania, Uganda and Malawi: Evaluations of hedonic pricing

and machine learning approaches". *Public Library of Science*.

Friedman, J. (2001). Greedy Function Approximation: A Gradient Boosting Machine. *The Annals of Statistics*, 29, 1189-1232.

Gelman, A., Carlin, J., Stern, H., Dunson, D., Vehtari, A., Rubin, D. (1995). *"Bayesian Data Analysis"*. Chapman and Hall/CRC.

Géron, A. (2019). *"Hands-On Machine Learning with Scikit-Learn, Keras & TensorFlow"*. O'Reilly.

Geurts, P., Ernst, D., Wehenkel, L. (2006). "Extremely Randomized Trees". *Machine Learning*, Springer Verlag, 36, 3-42.

Goldstein, A., Kapelner, A., Bleich, J. Pitkin, E. (2014). "Peeking Inside the Black Box: Visualizing Statistical Learning with Plots of Individual Conditional Expectation". *Journal of Computational and Graphical Statistics*, 44-65.

Grewal, P. (2014). "A Critical Conceptual Analysis of Definitions of Artificial Intelligence as Applicable to Computer Engineering". *Journal of Computer Engineering*, 16, 9-13.

Grinsztajn, L., Oyallon, E., Varoquaux, G. (2020). "Why do tree-based models still outperform deep learning on tabular data?". *NeurIPS 2022 Track Datasets and Benchmarks Program Chairs*.

Hansen, S., McMahon, M., Prat, A. (2019). "Transparency and Deliberation Within the FOMC: A Computational Linguistics Approach". *The Quarterly Journal of Economics*, 133, 801-870.

Harasymiv, V. (2015). "Lessons from 2 Million Machine Learning Models on Kaggle". Retrieved from KDNuggets

He, K., Zhang, X., Ren, S., Sun, J. (2015). "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification". *Microsoft Research*, 11026-1034.

Ho, T. (1998). "The Random Subspace Method for Constructing Decision Forests". *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 20, 832-844.

Hutto, C. and Gilbert E. (2014). "VADER: A Parsimonious Rule-Based Model for Sentiment Analysis of Social Media Text". *Eighth International AAAI Conference on*

Weblogs and Social Media, 8, 216-225.

Inside Airbnb. Data retrieved from Inside Airbnb website.

Kayode, E. and Tang, B. (2021). "The Role of Econometrics Data Analysis Method in the Social Sciences (Education) Research". *Journal of Education and Practice*, 4, 159-167.

Koehrsen, W. (2018). A Conceptual Explanation of Bayesian Hyperparameter Optimization for Machine Learning. Retrieved from Towards Data Science

Limsombunchai, V., McMahon, M., Prat, A. (2019). "House Price Prediction: Hedonic Price Model vs. Artificial Neural Network". *American Journal of Applied Sciences*, 1, 193-201.

Lundberg, S. and Lee, S. (2017). "A Unified Approach to Interpreting Model Predictions". *Conference on Neural Information Processing*, 30.

Michael, J. (2006). "Gender-verification by forename". *Authokey forum.*

Mitchell, T. (1997). "*Machine Learning*". McGraw Hill.

Ng, A. (2019). "Machine Learning and AI via Brain Simulations". *Stanford University.*

Pandey, P. (2020). "*A Guide to Handling Missing values in Python*". Kaggle Notebook.

Pernagallo, G. and Torrisi, B. (2019). "A theory of information overload applied to perfectly efficient financial markets". *Review of Behavioral Finance.*

Rudin, C. (2019). "Stop Explaining Black Box Machine Learning Models for High Stakes Decisions and Use Interpretable Models Instead". *Nature Machine Intelligence*, 1, 206-215.

Serengil, S. (2020). "LightFace: A Hybrid Deep Face Recognition Framework". *2020 Innovations in Intelligent Systems and Applications Conference (ASYU)*, 1-5.

Taddy, M., Chen, C., Yu, J., Wyle, M. (2015). "Bayesian and Empirical Bayesian Forests". *International Conference on Machine Learning*, 37, 967-976.

van Buuren, S. and Oudshoorn, K.G (2011). "mice: Multivariate Imputation by Chained Equations in R". *Journal of Statistical Software*, 45, 1-67..

Zou, H. and Hastie, T. (2005). "Regularization and Variable Selection via the Elastic Net". *Journal of the Royal Statistical Society*, 67, 301-320.

Appendix

1. Technical details

The entire project has been carried out using the Python programming language. Code is publicly available in a **GitHub repository**. Any questions regarding methodological or algorithmic aspects can be made to the author via josejaendelgado@gmail.com.

All Machine Learning and Deep Learning algorithms have been trained on a computer with six CPU cores (Intel i7-10750H) and a GPU (NVIDIA GeForce RTX 2060). The argument `n_jobs` in the models built with `scikit-learn` allows to specify the number of cores used for training. To speed learning time, it was set to -1 (using all available CPU cores). When possible, algorithms were trained on GPU as it is significantly faster. Concretely, XGBoost and all Deep Learning models were trained on it. XGBoost natively permits GPU usage by setting the `tree_method` argument to `gpu_hist`. For Deep Learning, `TensorFlow` directly detected and used GPU with NVIDIA's `CUDA` toolkit, while in the `Pytorch` environment it was necessary to specifically select `CUDA` as the working device. `TensorFlow` was used to build the Frequentist ANN and `Pytorch` was preferred for BNN as it has a simple and extensible Variational Inference library called `BLiTZ`.

The preferred data imputation algorithm was chosen by comparing performance of different imputers on synthetic data stemming from the test set. Firstly, every column were at least one missing value was present was discarded as to obtain all k-nearest neighbors. Then, some already known values of the test dataset were replaced by null values and imputed with the custom k-NN algorithm and MICE. The absolute value of the difference between real values and predictions was compared for each feature. Finally, it was found that k-NN predictions were closest to the real values.

For Bayesian Inference, the Gamma conjugate prior chosen for testing whether female hosts charged less than men was an uninformative prior ($\alpha = 0.01$, $\beta = 0.01$). With respect to Bayesian Ridge Regression however, optimal hyperprior and prior values were estimated from data. Unlike the frequentist ANN, there was no need to resort to TPE for tuning BNN as the prior distribution already regularized $\mathbf{w}$. Concretely, $p(\mathbf{w}) = \mathcal{N}(\mathbf{0}, \mathbf{I}_p)$.

Unfortunately, the Bayesian Ridge Regression class in `scikit-learn` lacks a method for calculating credible intervals. Thus, Figure III was produced by resorting to a Hamiltonian Monte Carlo (HMC) sampling algorithm called No U-Turn Sampler (NUTS) available through the probabilistic programming package `PyMC3`. In order to reduce computational complexity, features were standardized with the mean and standard deviation of $\mathcal{T}$.

Due to conflicting libraries and environment issues not all models have been trained in the same workspace. For instance, developers of `PyMC3` thoroughly recommend installing it with `conda` rather than `pip`, the general-purpose Python manager that was used with the rest of packages. Consequently, the Bayesian Linear Regression model was built in a separate virtual environment than the rest of ML models. Likewise, in order to train BNN on a GPU with `Pytorch`, it was necessary to set up `CUDA` in the aforementioned `conda` environment, as CPU was the only working device detected.

Implementing TPE with `TensorFlow` was far from being smooth. The `hyperas` package combines `keras` API with `hyperopt`, the library used for Bayesian Hyperparameter Optimization in the rest of ML models trained with `scikit-learn`. Since `hyperas` has not been recently updated, some `numpy` methods were deprecated. Thus, some updates had to be carried out in the source code to get it properly working.

All of the XAI methods have been applied to test data rather than training instances since the latter were already taken into account by selecting the champion model as the mapping function from which to calculate SHAP values, PDPs and ICE plots. Only some random observations were chosen to serve as examples of how imperative Responsible AI is, but as previously noted Python code is publicly available for those who want to interpret a higher number of predictions.

It has to be noted that Causal Inference was beyond the scope of this thesis. Instead, focus was shifted to solving an economic problem related with optimal decision-making. The solution to such problem was found by providing accurate predictions and trying to make sense of them with XAI.

It is believed that one of the main underlying goals of showing how econometricians can benefit from applying AI techniques has been attained.

2. Airbnb Data

Inside Airbnb collected information that was of little predictive use such as listings' ID, rentals' licenses, internet URLs to houses and hosts' pictures, etc. Nevertheless, some of these features were instrumental in creating new ones or merging the listings and reviews datasets (i.e, through a SQL left join on listings' ID). A list of the final predictive variables is presented below:

- `description`: Whether hosts provided a brief description of their listing or not.
- `host_since`: Number of days hosts offering their listing in Airbnb. Formula: `days(last_scraped)` - `days(host_since)`.
- `host_response_time`: Categorical value with five distinct ascending levels related to how much it took hosts to respond messages.
- `host_response_rate`: Percentage of questions answered by hosts.
- `host_acceptance_rate`: Percentage of times a host accepted guests.
- `host_is_superhost`: Binary variable discerning whether a host has been given the status of superhost or not.
- `host_listings_count`: Number of listings the host has.
- `host_has_profile_pic`: Boolean feature denoting if hosts posted a profile picture.
- `host_identity_verified`: Dummy variable identifying if the host is verified.
- `room_type`: Categorical value with four distinct values indicating if the listing is a private room, an entire apartment, shared room or hotel room.
- `accommodates`: Maximum capacity of the listing.
- `bathrooms`: Number of bathrooms.
- `bedrooms`: Total number of bedrooms.

- `beds`: Number of beds in the listing.
- `minimum_nights`: Minimum number of nights set by the host to rent the listing.
- `maximum_nights`: Maximum number of nights per stay.
- `minimum_nights_avg_ntm`: Average minimum number of nights in a one-year period.
- `maximum_nights_avg_ntm`: Average maximum number of nights in a one-year period.
- `has_availability`: Binary variable indicating whether the listing can be rented or not.
- `availability_30`: Availability of the listing in the next 30 days.
- `availability_90`: Availability of the listing in the next 90 days.
- `availability_365`: Availability of the listing for the next year.
- `number_of_reviews`: Total number of reviews received by the listing since it was posted.
- `number_of_reviews_ltm`: Number of reviews in the last month.
- `first_review`: Days since the listing was firstly reviewed.
- `last_review`: Number of days elapsed since the last review.
- `reviews_month`: Average number of reviews per month.
- `review_scores_rating`: Average rating of the listing (0 - 5 stars).
- `review_scores_accuracy`: Average rating related with accurate description of the listing.
- `review_scores_checkin`: Average rating with respect to the check-in process.

- `review_scores_communication`: Average rating related with how smooth the communication with the host was.
- `review_scores_location`: Location average rating.
- `review_scores_value`: Average rating about how fairly priced the listing was.
- `instant_bookable`: Binary variable indicating if the listing can be instantly bookable or not.
- `calculated_host_listings_count`: Total number of listings the host has in Airbnb.
- `calculated_entire`: Total number of entire apartment listings type the host has in Airbnb.
- `calculated_private`: Total number of private room listings type the host has in Airbnb.
- `calculated_shared`: Total number of shared room listings type the host has in Airbnb.
- `neighborhood`: Categorical variables with the encoded cluster of neighborhoods.
- `neighborhood_group`: Categorical feature with specific neghborhood names.
- `inactive`: Indicates if the listing has not been reviewed.
- `responds`: Binary variable detecting whether hosts answers messages or not.
- `geo_x`: $\cos(\texttt{latitude}) \times \cos(\texttt{longitude})$
- `geo_y`: $\cos(\texttt{latitude}) \times \sin(\texttt{longitude})$
- `geo_z`: $\sin(\texttt{latitude})$
- `property`: Encoded property type.
- `price`: $\ln(\texttt{price})$. Nonlinear transformation to increase predictive accuracy by decreasing skewness in the distribution of `price`.

- `sent_mean`: Mean estimated sentiment of reviews using NLP.
- `sent_median`: Median estimated sentiment of reviews using NLP.
- `sent_mode`: Mode estimated sentiment of reviews using NLP.
- `amenities`: Binary variables indicating whether the listing has certain items such as TV, Netflix, elevator, gym, oven, patio, bbq, etc (22 amenities in total).
- `nlp_gender`: Estimated gender using NLP.
- `cv_gender`: Estimated gender using CV (ultimately discarded as `nlp_gender` procured better results).

3. Sentiment Analysis

As mentioned in Section 4.3.2, sentiment was estimated using VADER. Such pretrained NLP model was fine-tuned by introducing additional linguistic rules and expressions. Final sentiment is the result of normalizing the sentiment score in the lexicon, after applying user-defined rules. Since there were three possible values for sentiment, namely, positive, neutral and negative, this was a multiclass classification task. Some examples showing the performance of the model are presented below:

> "Not my favorite place i've airbnb at. Maybe ok for a night or two. Long term maybe not"
>
> (Neutral review)

> "A good place to stay close to Marina del Ray, Venice Beach, Santa Monica. The house is poorly isolated so you need to heat it more than what should be necessary during the nights. Only mugs to do the second B in BnB - Breakfast. No bowls for cereal, no cutting board to cut your bread on, etc which was disappointing. Otherwise nice place. You need an ber account to travel to most things"
>
> (Neutral review)

"Very unreliable host. Wasn't helpful I found mold in the bathroom and the owner blamed it on me when it is lack of upkeep of his property very rude and lazy. I would not recommended staying here"

(Negative review)

"The host canceled my reservation 42 days before arrival. Bay stated that her vacation plans had changed and so she had to cancel my reservation. My boyfriend and I spent a lot of time at the Air BnB website trying to find the perfect place. Until Bay canceled we though we had found the perfect place. Now we have to repeat the whole process. Very disappointing"

(Negative review)

"i had a wonderful stay. Everything from start to the end was perfect. Will come back again"

(Positive review)

"Staying with Chas was an absolute pleasure. He was very accommodating and respectful of personal space. He is truly a nice person. He was very helpful and I am grateful that he opened up his home to me. I am glad to have met him"

(Positive review)

4. MLOps: AI Model Deployment

A price prediction web app was developed for the sake of simulating how a service based on this work could assist economic agents. `streamlit`, an open-source app framework for web development was the perfect tool for deployment the champion model to production. Firstly, it allows users to create web apps in pure Python, avoiding to resort to programming languages like JavaScript. Secondly, it provides free (but limited) compute resources to showcase the app. While not being the main point of the thesis, this app will be maintained and improved as long as it does not incur in any monetary cost.

An accessible link to the app can be found **here**.

5. Algorithms

Algorithm 2: Iterative Imputer Algorithm

Input: Incomplete dataset $\mathcal{D}$

Output: Imputed dataset $\mathcal{D}^*$

Create m copies of $\mathcal{D} = \{\mathcal{D}_1, \ldots, \mathcal{D}_m\}$

Iteratively sample $p(\mathbf{y}_i|\mathbf{y}_{-i}, \boldsymbol{\theta}_i)$ from each $\mathcal{D}_j$

Draw $(\boldsymbol{\theta}^{(t)}, \mathbf{y}^{(t)})$ with Gibbs sampler:

for $t = 1, \ldots, n$ **do**

$$
\begin{aligned}
\boldsymbol{\theta}_1^{(t)} &\sim p(\boldsymbol{\theta}_1|\mathbf{y}_1^{\text{obs}}, \mathbf{y}_2^{(t-1)}, \ldots, \mathbf{y}_p^{(t-1)}) \\
\mathbf{y}_1^{(t)} &\sim p(\mathbf{y}_1|\mathbf{y}_1^{\text{obs}}, \mathbf{y}_2^{(t-1)}, \ldots, \mathbf{y}_p^{(t-1)}, \boldsymbol{\theta}_1^{(t)}) \\
&\vdots \\
\boldsymbol{\theta}_p^{(t)} &\sim p(\boldsymbol{\theta}_p|\mathbf{y}_p^{\text{obs}}, \mathbf{y}_1^{(t)}, \ldots, \mathbf{y}_{p-1}^{(t)}) \\
\mathbf{y}_p^{(t)} &\sim p(\mathbf{y}_p|\mathbf{y}_p^{\text{obs}}, \mathbf{y}_1^{(t)}, \ldots, \mathbf{y}_p^{(t)}, \boldsymbol{\theta}_p^{(t)})
\end{aligned}
$$

Estimate $Q^{(1)}, \ldots, Q^{(m)}$ and pool results: $\bar{Q} = \text{pool}(\hat{Q}^{(1)}, \ldots, \hat{Q}^{(m)})$

Algorithm 3: Random Forest Algorithm

Input: Training dataset $\mathcal{F}$

Output: Predictions $\hat{\mathbf{y}}$

for $i = 1, \ldots, k$ **do**

`/* Randomly partition data into` $\mathcal{D}^{(1)}, \ldots, \mathcal{D}^{(k)}$ `*/`

$$\mathcal{D}^{(i)} \sim \frac{\Gamma\left(\sum_{i=1}^{n} x_i + 1\right)}{\prod_{i=1}^{n} \Gamma(x_i + 1)} \prod_{i=1}^{n} \theta_i^{x_i}$$

Select (j, t_j) in each $\mathcal{D}^{(i)}$ and obtain final estimate $\hat{\mathbf{y}}$:

for $i = 1, \ldots, k$ **do**

$$j^{(i)} \sim \mathcal{U}[1, p]$$

$$j^{(i)}, t_j^{(i)} \leftarrow \arg \min_{j^{(i)}, t_j^{(i)}} \frac{n_l}{n} \sum_{l} \left(\mathbf{y}^{(i)} - \hat{\mathbf{y}}_l^{(i)}\right)^2 + \frac{n_r}{n} \sum_{r} \left(\mathbf{y}^{(i)} - \hat{\mathbf{y}}_r^{(i)}\right)^2$$

$$\hat{\mathbf{y}} = \frac{1}{n} \sum_{i} \hat{\mathbf{y}}^{(i)}$$

Note: Subscripts l, r denote the left-hand and right-hand split, respectively.

Algorithm 4: Bayesian Random Forest Algorithm

Input: Training dataset $\mathcal{F}$

Output: Predictions $\hat{\mathbf{y}}$

for $i = 1, \ldots, k$ **do**

`/* Randomly partition data into` $\mathcal{D}^{(1)}, \ldots, \mathcal{D}^{(k)}$ `*/`

$$\mathcal{D}^{(i)} \sim \frac{\Gamma\left(\sum_{i=1}^{n} x_i + \alpha_i\right)}{\prod_{i=1}^{n} \Gamma(x_i + \alpha_i)} \prod_{i=1}^{n} \theta_i^{x_i + \alpha_i - 1}$$

Select (j, t_j) in each $\mathcal{D}^{(i)}$ and obtain final estimate $\hat{\mathbf{y}}$:

for $i = 1, \ldots, k$ **do**

$$j^{(i)} \sim \mathcal{U}[1, p]$$

$$j^{(i)}, t_j^{(i)} \leftarrow \arg \min_{j^{(i)}, t_j^{(i)}} \frac{n_l}{n} \sum_{l} \left(\mathbf{y}^{(i)} - \hat{\mathbf{y}}_l^{(i)}\right)^2 + \frac{n_r}{n} \sum_{r} \left(\mathbf{y}^{(i)} - \hat{\mathbf{y}}_r^{(i)}\right)^2$$

$$\hat{\mathbf{y}} = \frac{1}{n} \sum_{i} \hat{\mathbf{y}}^{(i)}$$

Algorithm 5: Extremely Randomized Forest Algorithm

Input: Training dataset $\mathcal{F}$

Output: Predictions $\hat{\mathbf{y}}$

for $i = 1, \ldots, k$ **do**

`/* Randomly partition data into` $\mathcal{D}^{(1)}, \ldots, \mathcal{D}^{(k)}$ `*/`

$$\mathcal{D}^{(i)} \sim \frac{\Gamma\left(\sum_{i=1}^{n} x_i + 1\right)}{\prod_{i=1}^{n} \Gamma(x_i + 1)} \prod_{i=1}^{n} \theta_i^{x_i}$$

Select (j, t_j) in each $\mathcal{D}^{(i)}$ and obtain final estimate $\hat{\mathbf{y}}$:

for $i = 1, \ldots, k$ **do**

$$j^{(i)} \sim \mathcal{U}[1, p], \quad t_j \sim \mathcal{U}[\min\{\mathbf{X}_j\}, \max\{\mathbf{X}_j\}]$$

$$j^{(i)} \leftarrow \arg \min_{j^{(i)}} \frac{n_l}{n} \sum_{l} \left(\mathbf{y}^{(i)} - \hat{\mathbf{y}}_l^{(i)}\right)^2 + \frac{n_r}{n} \sum_{r} \left(\mathbf{y}^{(i)} - \hat{\mathbf{y}}_r^{(i)}\right)^2$$

$$\hat{\mathbf{y}} = \frac{1}{n} \sum_{i} \hat{\mathbf{y}}^{(i)}$$

Algorithm 6: Extremely Randomized Bayesian Forest Algorithm

Input: Training dataset $\mathcal{F}$

Output: Predictions $\hat{\mathbf{y}}$

for $i = 1, \ldots, k$ **do**

`/* Randomly partition data into` $\mathcal{D}^{(1)}, \ldots, \mathcal{D}^{(k)}$ `*/`

$$\mathcal{D}^{(i)} \sim \frac{\Gamma\left(\sum_{i=1}^{n} x_i + \alpha_i\right)}{\prod_{i=1}^{n} \Gamma(x_i + \alpha_i)} \prod_{i=1}^{n} \theta_i^{x_i + \alpha_i - 1}$$

Select (j, t_j) in each $\mathcal{D}^{(i)}$ and obtain final estimate $\hat{\mathbf{y}}$:

for $i = 1, \ldots, k$ **do**

$$j^{(i)} \sim \mathcal{U}[1, p], \quad t_j \sim \mathcal{U}[\min\{\mathbf{X}_j\}, \max\{\mathbf{X}_j\}]$$

$$j^{(i)} \leftarrow \arg\min_{j^{(i)}} \frac{n_l}{n} \sum_{l} \left(\mathbf{y}^{(i)} - \hat{\mathbf{y}}_l^{(i)}\right)^2 + \frac{n_r}{n} \sum_{r} \left(\mathbf{y}^{(i)} - \hat{\mathbf{y}}_r^{(i)}\right)^2$$

$$\hat{\mathbf{y}} = \frac{1}{n} \sum_{i} \hat{\mathbf{y}}^{(i)}$$

Algorithm 7: XGBoost Algorithm

Input: Training dataset $\mathcal{F}$, Regularization function $\Omega(f(\mathbf{x})_i)$

Output: Predictions $\hat{\mathbf{y}}$

\# Initialize

$f_0(\mathbf{x}_i) \leftarrow \arg\min_{f(\mathbf{x}_i)} \sum_{i=1}^{n} l(y_i, f(\mathbf{x}_i))$

for $t = 1, \ldots, T$ **do**

`/* Find optimal split */`

$$\hat{y}_i^{(t+1)} \leftarrow \arg\min_{f(\mathbf{x}_i)} \sum_{i=1}^{n} l\left(y_i, \hat{y}_i^{(t-1)} + f(\mathbf{x}_i)^{(t)}\right) + \Omega(f_t)$$

Algorithm 8: Artificial Neural Network Algorithm

Input: Training dataset $\mathcal{F}$, number of layers L, batches M, epochs T)

Output: Predictions $\hat{\mathbf{y}}$

Set hyperparameters $[\alpha,\ \beta_1,\ \beta_2,\ \varepsilon]' = [0.002,\ 0.9,\ 0.999,\ 10^{-7}]'$

Initialize $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \frac{2}{n_l})$

for $t = 1, \ldots, T$ **do**

Iterate over T epochs

for $i = 1, \ldots, m$ **do**

Iterate over m batches

Forward pass

$$\hat{\mathbf{y}}^{(t,i)} \leftarrow \begin{cases} \mathbf{X}^{(i)}\mathbf{w}_l^{(t)} & \text{if } \mathbf{X}^{(i)}\mathbf{w}_l^{(t)} \geq 0 \\ \alpha\left(\exp\{\mathbf{X}^{(i)}\mathbf{w}_l^{(t)}\} - 1\right) & \text{if } \mathbf{X}^{(i)}\mathbf{w}_l^{(t)} < 0 \end{cases}$$

$$J(\mathbf{w}_l^{(t)}, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) \leftarrow \tfrac{1}{n_l}\left(\mathbf{y}^i - \hat{\mathbf{y}}^{(t,i)}\right)'\left(\mathbf{y}^i - \hat{\mathbf{y}}^{(t,i)}\right) + \boldsymbol{\lambda}_1\|\mathbf{w}_l^{(t)}\|_1 + \boldsymbol{\lambda}_2\|\mathbf{w}_l^{(t)}\|_2^2$$

Backward pass

$$\nabla\mathbf{w}_l^{(t)} \leftarrow \frac{\partial J(\mathbf{w}_l^{(t)}, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)}{\partial \mathbf{w}_l^{(t)}}$$

$$\hat{m}^{(i)} \leftarrow \frac{1}{1-\tilde{\beta}_1}\left(\beta_1 m_{-1}^{(i)} + (1-\beta_1)\nabla\mathbf{w}_l^{(t)}\right)$$

$$\hat{v}^{(i)} \leftarrow \frac{1}{1-\tilde{\beta}_2}\left(\beta_2 v_{-1}^{(i)} + (1-\beta_2)\nabla{\mathbf{w}_l^{(t)}}^2\right)$$

$$\mathbf{w}_l^{(t)} \leftarrow \mathbf{w}_l^{(t)} - \alpha\frac{1}{\sqrt{\hat{v}^{(i)}} + \varepsilon}\left(\beta_1\hat{m}^{(i)} + \frac{1-\beta_1}{1-\tilde{\beta}_1}\nabla\mathbf{w}_l^{(t)}\right)$$

Algorithm 9: Bayesian Neural Network Algorithm

Input: Training dataset $\mathcal{F}$

Output: Predictions $\hat{\mathbf{y}}$

1 Define prior distribution: $p(\mathbf{w}) \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_p)$

2 Calculate posterior distribution and get predictions:

$$p(\mathbf{w}|\mathcal{T}) = \frac{p(\mathbf{y}|\mathbf{x}, \mathbf{w})p(\mathbf{w})}{\int_{\mathbf{w}} p(\mathbf{y}|\mathbf{x}, \mathbf{w}')p(\mathbf{w})'\,\partial\mathbf{w}'}$$

for $i = 1, \ldots, p$ **do**

$\mathbf{w}^{(i)} \sim p(\mathbf{w}|\mathcal{T})$

$\hat{\mathbf{y}}^{(i)} = \boldsymbol{\Phi}_{\mathbf{w}^{(i)}}(\mathbf{x})$

6. Models Summary

Table 2: Algorithms Performance Summary

Algorithm	RMSE on test data
Bayesian Ridge Regression	0.5104
Elastic Net	0.5232
Frequentist Random Forest	0.4551
Bayesian Random Forest	0.4205
Frequentist Extremely Randomized Forest	0.4507
Bayesian Extremely Randomized Forest	0.4572
Artificial Neural Network	0.4305
Bayesian Neural Network	0.4024
XGBoost	0.3682

7. Bayesian Inference

Marginal probability distribution of prices $f(y)$

$$f(y) = \int_0^{+\infty} f(y|\theta) f(\theta)\ \partial\theta$$

$$= \int_0^{+\infty} \frac{\omega\theta y^{-(\omega+1)}}{(1+y^{-\omega})^{\theta+1}} \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\theta\beta}\ \partial\theta$$

$$= \frac{\omega y^{-(\omega+1)} \beta^\alpha}{\Gamma(\alpha)} \int_0^{+\infty} \theta^\alpha e^{-\theta\beta} \frac{1}{(1+y^{-\omega})^{\theta+1}}\ \partial\theta$$

$$= \frac{\omega y^{-(\omega+1)} \beta^\alpha}{\Gamma(\alpha)} \int_0^{+\infty} \theta^\alpha e^{-\theta\beta} \exp\left\{\ln\left[\frac{1}{(1+y^{-\omega})^{\theta+1}}\right]\right\}\ \partial\theta$$

$$= \frac{\omega y^{-(\omega+1)} \beta^\alpha}{\Gamma(\alpha)} \int_0^{+\infty} \theta^\alpha e^{-\theta\beta} \exp\left\{-(\theta+1)\ln(1+y^{-\omega})\right\}\ \partial\theta$$

$$= \frac{\omega y^{-(\omega+1)}}{1+y^{-\omega}} \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^{+\infty} \theta^\alpha \exp\left\{-\theta[\beta + \ln(1+y^{-\omega})]\right\}\ \partial\theta$$

$$= \frac{\omega y^{-(\omega+1)}}{1+y^{-\omega}} \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha^*)}{\beta^{*\alpha^*}} \int_0^{+\infty} \frac{\beta^{*\alpha^*}}{\Gamma(\alpha^*)} \theta^{\alpha^*-1} e^{-\theta\beta^*}\ \partial\theta$$

$$= \frac{\omega y^{-(\omega+1)}}{1+y^{-\omega}} \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\alpha\Gamma(\alpha)}{[\beta + \ln(1+y^{\omega})]^{\alpha+1}}$$

$$= \frac{\alpha\beta^\alpha}{[\beta + \ln(1+y^{-\omega})]^{\alpha+1}} \left(-\frac{\omega y^{-\omega+1}}{1+y^{-\omega}}\right) \qquad \text{Let } z \coloneqq \ln(1+y^{-\omega})$$

$$= \frac{\alpha\beta^\alpha}{(\beta+z)^{\alpha+1}} \left(-\frac{\partial z(y)}{\partial y}\right) \implies y \sim \text{Pareto}(\alpha, \beta)$$

Posterior probability distribution $f(\theta|\{y_i\}_{i=1}^n)$

$$f(\theta|\{y_i\}_{i=1}^n) = \frac{l(y|\theta)f(\theta)}{\int_0^{+\infty} f(y|\theta)f(\theta)\ \partial\theta}$$

$$\propto \prod_{i=1}^n \frac{\omega\theta y_i^{-(\omega+1)}}{(1+y_i^{-c})^{\theta+1}} \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\theta\beta}$$

$$\propto \theta^{\alpha+n-1}e^{-\theta\beta}\prod_{i=1}^n \exp\left\{\ln\left[\frac{1}{(1+y_i^{-\omega})^{\theta+1}}\right]\right\}$$

$$\propto \theta^{\alpha+n-1}e^{-\theta\beta}\prod_{i=1}^n \exp\left\{-(\theta+1)\ln(1+y_i^{-\omega})\right\}$$

$$\propto \theta^{\alpha+n-1}e^{-\theta\beta}\exp\left\{-\sum_{i=1}^n(\theta+1)\ln(1+y_i^{-\omega})\right\}$$

$$\propto \theta^{\alpha+n-1}e^{-\theta\beta}\exp\left\{-\sum_{i=1}^n\theta\ln(1+y_i^{-\omega})\right\}$$

$$\propto \theta^{\alpha+n-1}\exp\left\{-\theta\left(\beta+\sum_{i=1}^n\ln(1+y_i^{-\omega})\right)\right\}$$

$$= \frac{\left(\beta+\sum_{i=1}^n\ln(1+y_i^{-\omega})\right)^{\alpha+n}}{\Gamma(\alpha+n)}\theta^{\alpha+n-1}\exp\left\{-\theta\left(\beta+\sum_{i=1}^n\ln(1+y_i^{-\omega})\right)\right\}$$

$$= \frac{\beta^{*\alpha^*}}{\Gamma(\alpha^*)}\theta^{\alpha^*-1}e^{-\theta\beta^*} \quad \Longrightarrow\ f(\theta|\{y_i\}_{i=1}^n) = \text{Gamma}\left(\alpha+n, \beta+\sum_{i=1}^n\ln(1+y_i^{-\omega})\right)$$

Bayesian Random Forest: Posterior probability distribution $f(\theta|\{x_i\}_{i=1}^p)$

Let $x|\theta \sim \text{Multi}(\theta)$ and $\theta \sim \text{Dir}(\alpha)$, then

$$f(\theta|x_i\}_{i=1}^p) = \frac{l(x|\theta)f(\theta)}{\int_0^{+\infty} f(x|\theta)f(\theta)\ \partial\theta}$$

$$\propto \prod_{i=1}^p \theta^{x_i} \frac{\Gamma\left(\sum_{i=1}^p \alpha_i\right)}{\prod_{i=1}^p \Gamma(\alpha_i)} \prod_{i=1}^p \theta_i^{\alpha_i - 1}$$

$$\propto \prod_{i=1}^p \theta_i^{\alpha_i + x_i - 1} \frac{\Gamma\left(\sum_{i=1}^p \alpha_i\right)}{\prod_{i=1}^p \Gamma(\alpha_i)}$$

$$= \frac{\Gamma\left(\sum_{i=1}^p \alpha_i + x_i\right)}{\prod_{i=1}^p \Gamma(\alpha_i + x_i)} \prod_{i=1}^p \theta_i^{\alpha_i + x_i - 1}$$

Consequently, $f(\theta|\mathbf{x}) = \text{Dir}(\alpha + \mathbf{x})$

8. Additional Figures

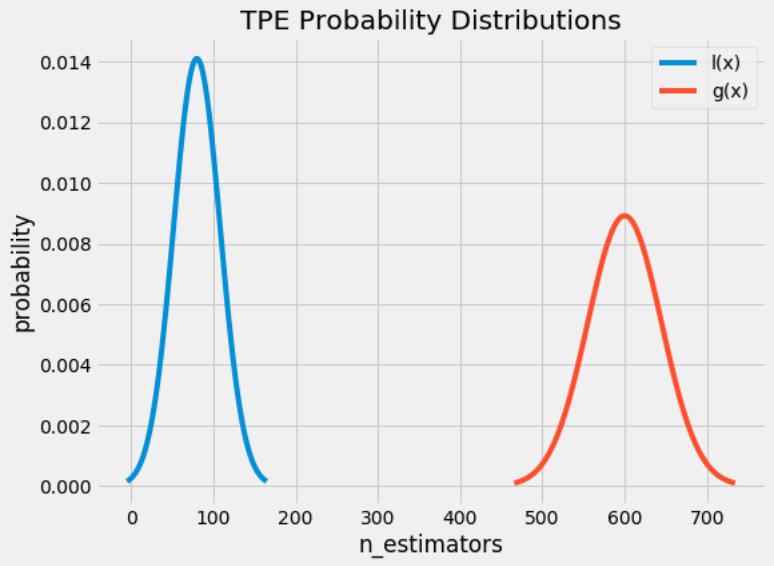


Figure 11: From Koehrsen (2018)

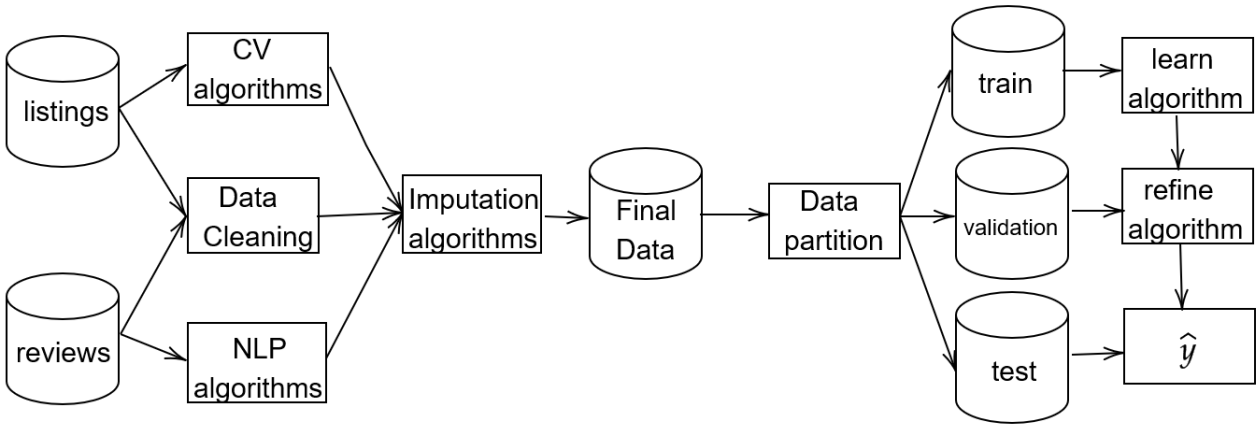
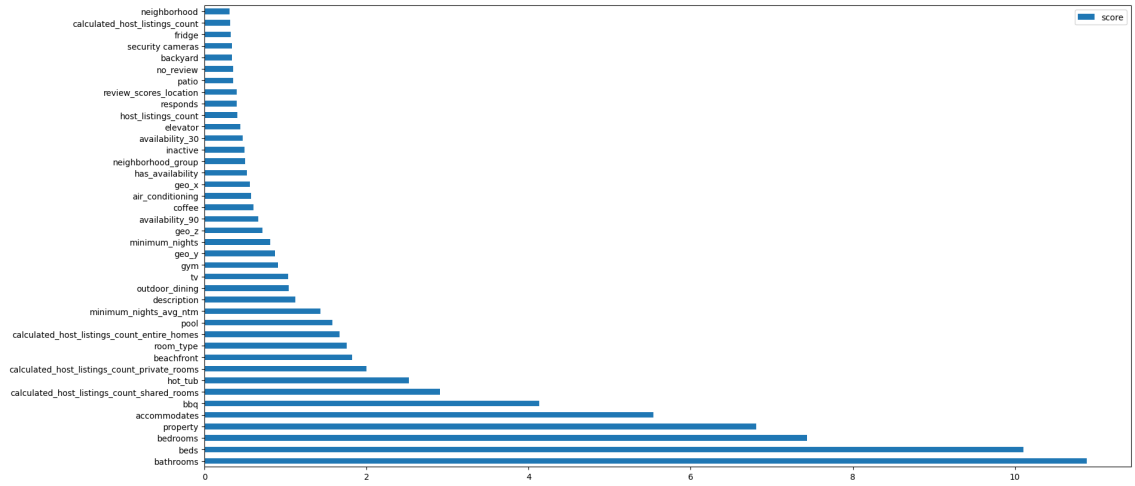
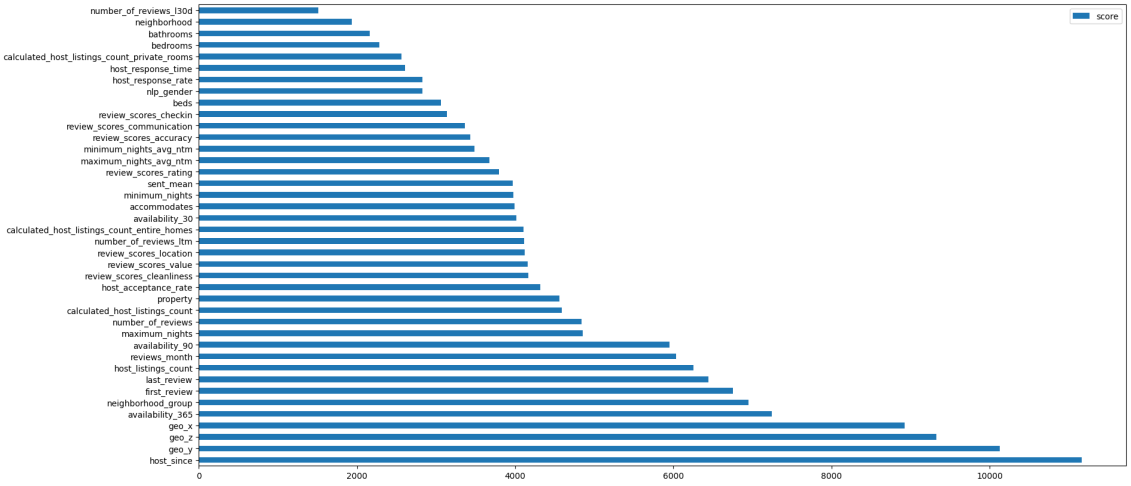


Figure 12: AI Framework



(a) Top 20 Features by Gain



(b) Top 20 Features by Split

Figure 13: XGBoost Feature Importance

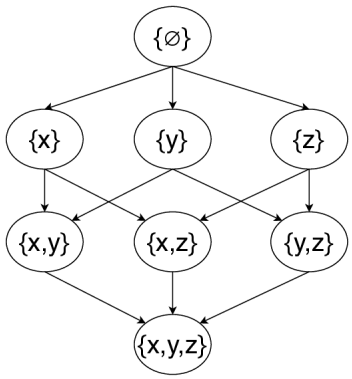


Figure 14: Power Set